THE BINOMIAL EXPANSION AND THE PASCAL TRIANGLE

Consider the following expansions-

$$(a+b) = a+b$$
$$(a+b)^2 = a^2+2ab+b^2$$
$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

which can be generalized to the nth power to yield the well known **binomial expansion-**

$$(a+b)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} a^{n-k} b^k$$

first written down and used extensively by Newton. He applied this expansion also to non-integer n such as-

$$\sqrt{2} = \sqrt{(1+1)} = \sum_{k=0}^{\infty} \frac{\Gamma(3/2)}{k!\Gamma(3/2-k)} = 1+1/2-1/8+1/16- = 1.41421356..$$

and realized that a more rapid series convergence occurs when a>>b. Thus-

$$\sqrt{2} = \sqrt{(1.96+0.04)} = \sum_{k=0}^{\infty} \frac{\Gamma(3/2)}{k!\Gamma(3/2-k)}(1.96)^{1/2-k}(0.04)^k = 1.400000000+0.014285714-...$$

The binomial coefficient-

$$(\frac{n}{k}) = \frac{n!}{k!(n-k)!}$$

arising in the above expansion is also encountered in other areas such as in quantum statistics. It is known, for example, the number of different combinations of n different things taken k at a time is precisely the value of this coefficient. Thus if we have six apples and ask how many groupings of three each are possible, the answer would be 6!/(3!(3!)= 20. Also we can construct a triangle from the binomial coefficient starting with k=0 for the apex followed by k=1 for the next row, and so on. This leads to -

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

which is referred to as the **Pascal Triangle**. Note that each integer is constructed by adding the two numbers directly above it. Prior to Newton people used this triangle to construct expansions of the quantity (a+b)^n. We show you how this works for n=6 where the seventh row from the top reads [1, 6, 15, 20,15, 6, 1] and thus one has-

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

This way of obtaining a binomial expansion is seen to be quite rapid , once the Pascal triangle has been constructed. If one looks at the magnitude of the integers in the kth row of the Pascal triangle as k gets large one approaches the Gaussian distribution **[n!/((n/2)!)^2]* exp[-(2/n)*x^2].** You can see a support for this point by clicking on the title of the section containing this pdf file. It is probable that the Gaussian was discovered by such a comparison with the binomial coefficients.

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