A GRAPHICAL EXTENTION OF THE ERATOSTHENES SIEVE METHOD FOR FINDING PRIMES

One of the earliest , simplest and most effective ways to find smaller primes is the sieve method introduced by Eratosthenes (284-192BC) of the Greek school in Alexandria, Egypt. It starts essentially with the sequence of odd integers written down in ascending order-

3- 5-7-9-11-12-13-15-16-17-18-19-21-23-25-27-29-31-33-35-37-39-41-43-45-47-49-

and next strikes out all those numbers which are multiples of any of the numbers to the left of of a given odd number. This produces the sequence-

3-5-7-11-13-17-19-23-29-31-37-41-47-

which are the first few primes (number 2 excepted). What is clear from this procedure is that 3 takes out the odd numbers 9, 15, 21, 27, 33, 39, 45, etc and 5 takes out the odd numbers 15, 25, 35 45, etc. That is, any of the lower primes on the left of the prime sequence takes out an infinite number of odd integers to the right in a periodic manner. Thus the zeros of the sine function-

$$y[n,x]=\sin\left(\frac{\pi x}{ithprime(n)}\right)$$

gives the values of x which are integer multiples of the nth prime. From the standard designation we have-

ithprime(1)=2, ithprime(2)=3, ithprime(3)=5, ithprime(4)=7, etc

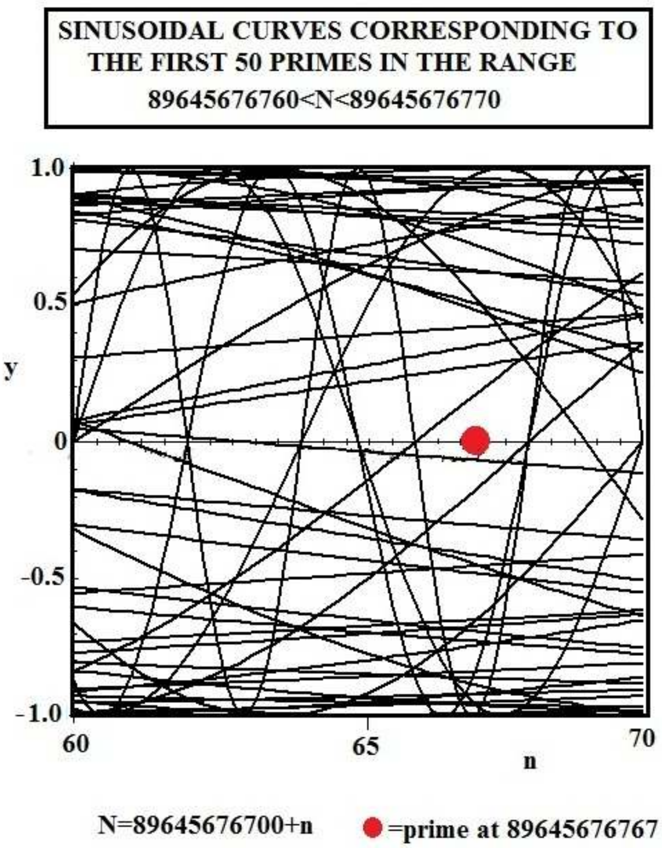
The values of the ithprime(n) up to several thousand are available in most computer math programs such as MAPLE. We have, for example, that ithprime(50)=229 and ithprime(1000)=7919. If an odd number is composite it is likely to more often than not to have its lowest prime number multiplier be a relatively low number rather than one near the square root of the number. For example taking the random odd numbers-

ifactor(1234567)=127 x9721 and ifactor(37993371)=3 x 13 x 974189

This fact suggests that if one superimposes the sine plots of a group consisting of the first n primes and looks at their collective curves in a narrow range about some larger odd number, a non-zero result very likely indicates the number is a prime. Let us demonstrate these thoughts by looking at the superimposed sine plots of the first 50 primes out in the range-

89646767760<x<89646767770

The resultant graph looks like this-



Note that x=N=89645676767 is the only integer not a zero by any of the fifty sine curves and hence is likely to be prime. A simple computer test confirms that this number marked in red is indeed a prime number.

Of course a non-zero for a given odd number need not always indicate a prime. This will happen when one of the lower prime factor of a number is greater than the largest ithprime(n) used in the sine curve generation. Look at the range 346890<x<346900. Here the graph shows two odd integers where no sine curve crosses zero. The first 346891 is indeed prime while the second is composite 346897=263 x 1319. The reason for the appearance of a composite number is the fact that 263>ithprime(50)=229. This graphical interpretation of the Eratosthenes sieve thus can be thought of as a number separator which eliminates all those odd numbers with factors below ithprime(n) used in our sine curve superposition from being a prime number. This will represent the majority of an infinite set. The procedure can also be used to quickly determine the ranges which contain no prime numbers and to help in locating double primes (ie-those at

2n+1 and 2n+3) . Some of the infinite number of larger primes one can find using the sine graph approach using up to the ithprime(50) curve are-

71423746921 19645679279 257546963 and 23746951

This type of graphical search method becomes impractical when the lowest prime factor in a composite number becomes very large such as in public keys where the lowest prime factor may be several hundred digits long. In that case one must abandon the Eratosthenes sieve method and rather generate a large composite number near the desired lowest prime number factor and then adjust a constant in the expression until a prime number is found. The form of this generalized prime number is-

$$p = a + \sum_{n=1}^{N} c_n \ b_n^{x_n}$$

See our earlier note "Finding-Large-Primes" of July 2010 concerning this type of representation. It can represent any prime number and contains the Fermat and Mersenne Primes as special cases. Just to demonstrate the utility of such an expansion consider the following 601 digit prime number we generated in a matter of five minutes by starting with the random choice p{a,[213,415],[227,167],[181,269]}=$a + 213 \cdot 227^{181} + 415 \ \cdot 167^{269}$ and then varying 'a' until the number becomes prime at a=-37. Here is the prime-

p= 3 37892 72042 10489 21226 28083 48728 44082 62513 02481 03854 08007 10649 39389 69400 46198 03304 75887 46650 21801 17745 00764 74641 35370 17894 84982 01249 88541 53028 77381 72240 87727 90499 64120 06916 72857 86409 26583 61074 26413 34795 26759 68949 78338 11765 80664 86873 56632 55480 41559 00433 68808 36297 51594 73370 54376 81824 17376 16050 13426 38150 83973 68730 08054 90092 28265 27529 96144 86870 12046 94535 90788 14298 13837 78467 63509 31733 43356 28086 88745 37473 53044 13343 36026 25315 51237 93533 65152 59871 36907 50626 16161 29806 15100 24420 35353 95522 64801 47669 13840 72549 73047 18891 82883 92376 44475 91585 33483 51333 58857 50357 03756 74273 57015 63408 56499 09271 70203 02029 32012 15619

The evaluation was accomplished by use of the big number calculator found at-

http://www.alpertron.com.ar/BIGCALC.HTM

June 2012