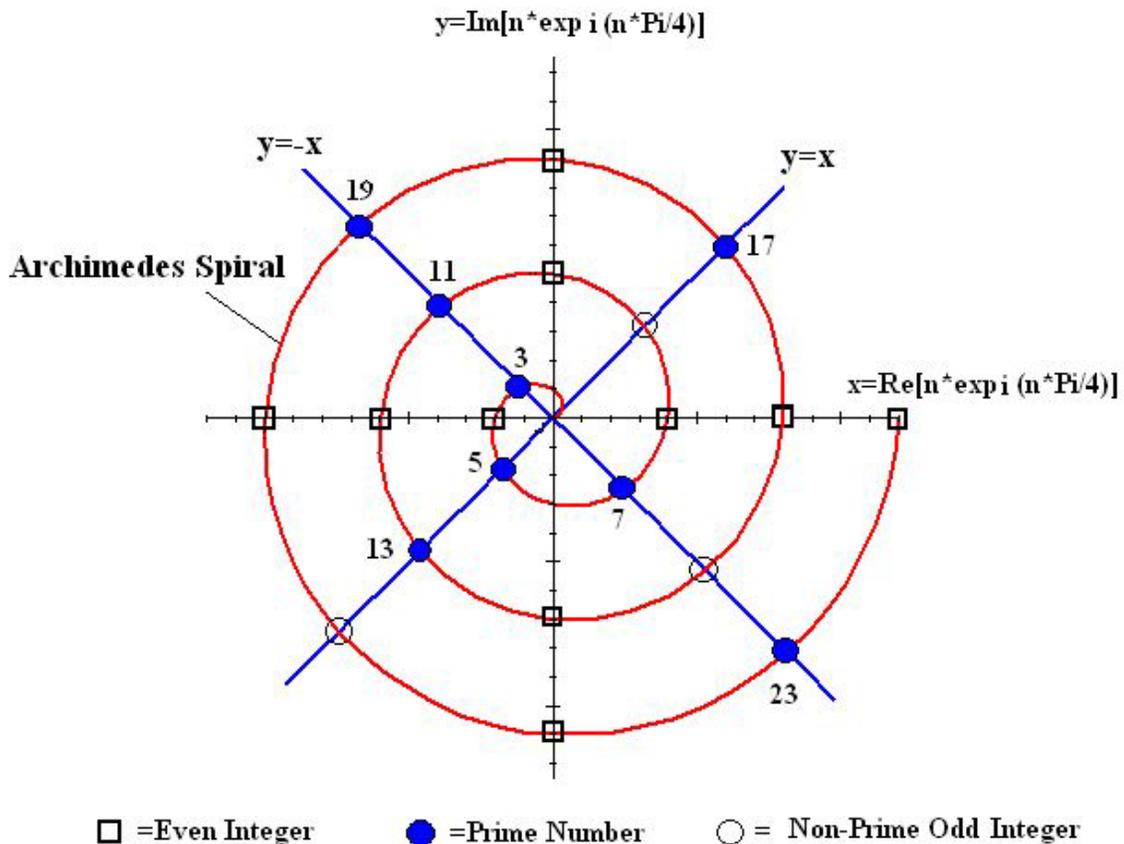


## EVEN, ODD, AND PRIME INTEGERS

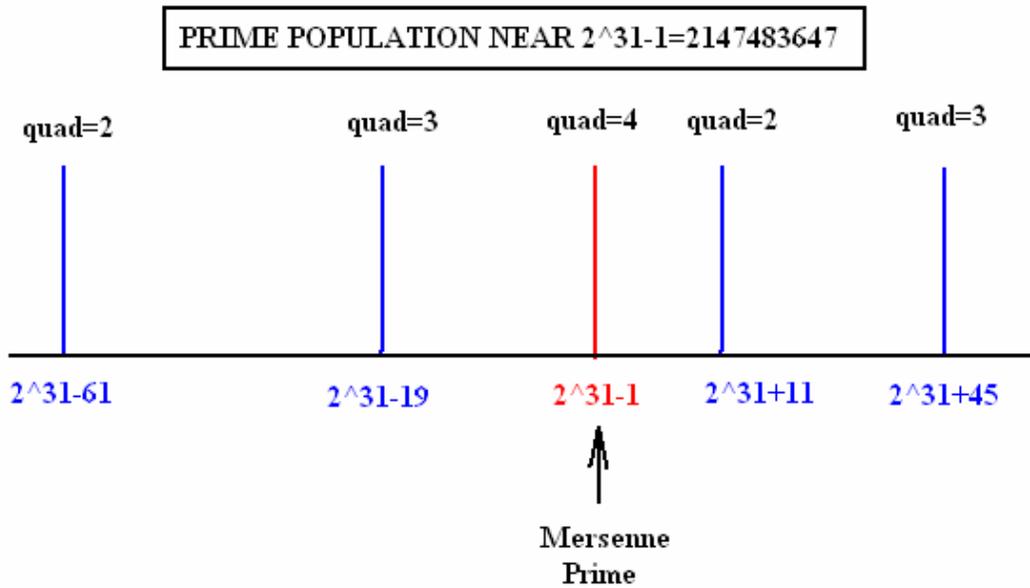
We have recently found a convenient way to represent all integers geometrically. The procedure, arising from an earlier observation (see <http://www.mae.ufl.edu/~uhk/ANALYSIS.html>) which showed that the locus of  $(1+i)^n$  in the complex plane has the form of a Bernoulli spiral, is to plot the Archimedes spiral  $z = x+iy = n \exp i (n \pi/4)$  and superimpose the diagonal lines  $y=x$  and  $y=-x$  on it. The resultant intersections locate all odd integers and the x and y axis intersections will represent all even integers. Prime numbers will be those odd numbers which do not factor. We summarize these observations with the following graph-



Note that the distance  $|z|$  from the origin to a given intersection is just the value of the integer  $n$  in question and that the distance between neighboring turns of the spiral along any diagonal line through the origin equals eight units. Thus, all odd integers lying along  $y=-x$  in the 4<sup>th</sup> quadrant are defined as  $N_4=8n+7$  with  $n=0, 1, 2, \dots$ . The odd integers along the line  $y=x$  in the 1<sup>st</sup> quadrant are defined as  $N_1=8n+1$ . Any integer power (greater 2) of 2 will be found along the positive x axis.

The well known Mersenne Primes have the form  $M[m]=2^m-1$  for  $m=3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, \dots$ . The first 44 Mersenne primes are presently known

and it seems there are an infinite number of these, although they are considerably more sparse than other primes of the form  $8n+7$ . We note that the Mersenne Primes all lie along the blue diagonal line in the 4<sup>th</sup> quadrant. Therefore  $8n+7=2^m-1$ , from which it follows that  $(M[m]-7)/8=2^{(m-3)}-1$  will always be an odd integer. The Fermat primes, defined as  $F[m]=2^{2^m}+1$  for  $m=1, 2, 3, 4$  lie along the line  $y=x$  in the first quadrant. Euler, and subsequent mathematicians, have shown that no Fermat primes exist for  $m=5$  and higher. One recognizes from the above geometry that the odd numbers  $8n+1, 8n+3, 8n+5$  and  $8n+7$  along the diagonals in the first, second, third, and fourth quadrant, respectively, form a much larger group than the possible primes among this group. In turn all the primes form a much larger group than the Mersenne and Fermat Primes. No one has been able to show whether the number of Mersenne Primes is finite or infinite. What is known from the prime number theorem is that prime numbers become quite sparse for large  $n$ . Here is a view of the prime number landscape between  $n=2^{31}+45$  and  $2^{31}-61$  which supports this point-



I have indicated in which of the four quadrants the five PC determined primes are located. Note that their number is quite consistent with what is given by the prime number theorem. This theorem predicts that the number of primes to be found in this range is-

$$N = \left[ \frac{(2^{31} + 45)}{\ln(2^{31} + 45)} \right] - \left[ \frac{(2^{31} - 61)}{\ln(2^{31} - 61)} \right] = 4.7$$

Finally, we look at the even number  $n=34,567 \times 98,764=3,413,975,188$ . Where is it located in the spiral graph given above? Clearly it will be along the x or y axis.

**As to which one, one observes that only  $(n-4)/8$  yields a whole integer, namely, 426746898. This implies that  $n$  occurs at the 426746898<sup>th</sup> turn of the Archimedes spiral at a distance  $n$  out along the negative  $x$  axis.**

**July 21, 2008**