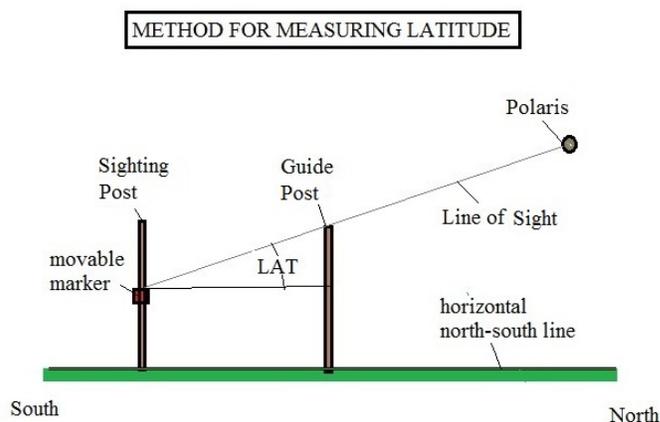


## FINDING LATITUDE AND DATE WITHOUT AID OF ELECTRONIC OR PRINT INFORMATION

In the days before GPS and other forms of electronic or print information, record keeping logs and calendars, people had to rely on astronomical readings to determine their latitude and the date throughout the year. The same would be true today if one were stranded on a desert island without means for any type of electronic reception or transmission. How would such an individual determine at what latitude he is located and what the approximate date is? We show you here how anyone with just an elementary knowledge of astronomy could answer these questions.

The simplest quantity to measure is the latitude (LAT). Assuming that the person is located somewhere in the northern hemisphere, his latitude can be determined by noting the angle the north star (Polaris) makes at night with respect to the local horizon. Polaris is located along a straight line extending from the front part of the Big Dipper ( Ursa Major ) at about five times the length of the distance between the two stars (Merak and Dubhe) making up the front of the dipper. Polaris is located at the North Celestial Pole along the earth's rotation axis and will remain fixed throughout the year. Someone at the north pole would see Polaris directly above while someone at the equator would locate Polaris along the local horizon. If someone is in the southern hemisphere one has no star sitting directly at the South Celestial Pole but one can still estimate the south pole location by making use of the intersection of the length of the Southern Cross with a line drawn perpendicular to two pointer stars in Centauries.

The simplest way to measure the local latitude (LAT) is to set up a primitive version of a stationary astrolabe. This can be accomplished by placing two vertical posts along a north-south line. The person would then move his eye up and down along the post further to the south until a line of sight is established between the eye, the top of the more northerly post and Polaris. A movable marker would be used to indicate the point at which this is true. A schematic of the measuring scheme follows-



With the posts a few feet apart quite accurate latitude measurements should be possible. If the accuracy is good to 1/10<sup>th</sup> degree then the error in position along a fixed longitude would be

about 11km( $\approx 10,000/900$ ). This latitude will remain constant throughout the year since the Polaris elevation does not change. There is no easy way for a person to establish his longitude without having an accurate chronometer. The reason zero degrees longitude goes through Greenwich, England is that the British were the worlds top maritime nation at the time when the first accurate chronometer construction became possible ( John Harrison, 1736). The top of Mt. Everest or the eastern tip of Brazil near Recife might today be better locations for a zero longitude great circle.

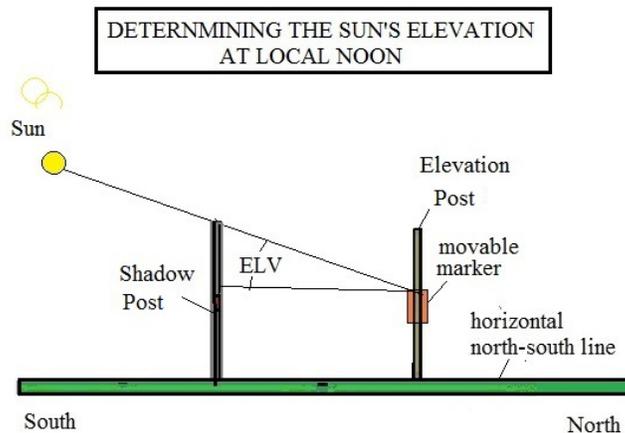
The approximate date can be established as follows. One knows that the sun at local noon lies directly above the earth's equator during the Vernal and Autumnal Equinoxes occurring March 21 and Sept 21, respectively. The sun's position at local noon on December 21 is directly above the Tropic Of Capricorn at  $LAT = -23.5\text{deg}$ . This date represents the Winter Solstice during which time the sun in Rio de Janeiro will lie at the local observer's zenith. It is also the time of year celebrated at Stonehenge in England by the ancient inhabitants as the time of yearly renewal. On June 21 the sun lies directly above the Tropic of Cancer at  $+23.5\text{ deg}$  above the equator. This is the Summer Solstice. Riyadh in Saudi Arabia lies approximately at this latitude. June 21 is also the time of mid-summer celebrations in Scandinavian countries.

Now the earth trajectory in its yearly journey about the sun follows a slightly elliptical path which can be well approximated as a circle. This means one can make the reasonable estimation that the suns declination relative to the earth's equator follows a sinusoidal time variation with a period of one year. One can approximate the declination (DEC) by the formula-

$$DEC = 23.5 \sin\left(\frac{2\pi}{365} t\right)$$

, where t represents the number of days since the Vernal Equinox on March 21. Nine months later ( $t = 3 \times 365/4$ ) during the Winter Solstice the sun will be located above the Tropic of Capricorn at  $DEC = -23.5\text{deg}$ . Using this declination formula will require some knowledge of  $\sin(x)$ . Most readers will recall that  $\sin(0) = 0$ ,  $\sin(\pi/6) = 0.5$ ,  $\sin(\pi/4) = 1/\sqrt{2}$ ,  $\sin(\pi/3) = \sqrt{3}/2$ , and  $\sin(\pi/2) = 1$ . The remaining angles can then be estimated by a curve passing through these points. More accurate values for DEC are found in almanacs, but we are assuming here that the stranded individual has no access to such a publication.

To determine the date our Robinson Crusoe can now obtain a good estimate for the number of days past March 31<sup>st</sup> by using his observation posts in reverse as shown-



The top of the shadow cast by the more southerly post will transit the elevation post at local noon. This transit path is noted by a movable marker. Should the sun at local noon be much higher than shown one can mark the shadow top on a north-south ground line between the two posts. Either way the altitude ALT at local noon is measurable. The observer also will recall from his knowledge of astronomy that at zero Hour Angle HA-

$$ALT = \frac{\pi}{2} - LAT + DEC$$

Solving for the date we get-

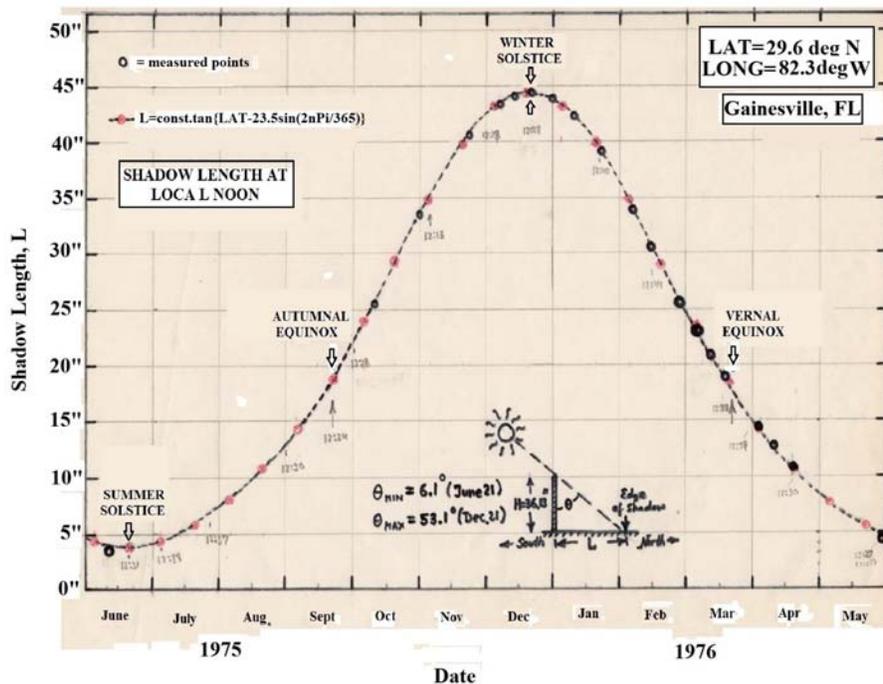
$$t = \frac{365}{2\pi} \arcsin \left\{ ALT + LAT - \frac{\pi}{2} \right\}$$

when all known angles ALT and LAT are expressed in radians. Now suppose the individual is sitting on some deserted island and he has measured the latitude as  $LAT=30\text{deg}=\pi/6$  rad and the sun's altitude as  $ALT=40\text{deg}=2\pi/9$  rad. Plugging into the formula he finds  $t=-20.71$ . So it must be some 21 days before the Vernal Equinox. That is, the date is February 28<sup>th</sup> assuming no leap year. Christmas is celebrated 4 days after the Winter Solstice and so t will be either -87.25 or +268.75 days from the Spring (Vernal) Equinox.

Should one want to measure the time before or after local noon one can mark on the ground the shadow line of the shadow post at hourly intervals from local noon provided one has a some means to measure time. Assuming our survivor has no wrist watch, he could establish the length of one hour by calibrating a rudimentary hour-glass to match the time the Big Dipper makes to rotate  $1/24^{\text{th}}$  of a full circle about Polaris. Note that local noon differs throughout the year because of the earth's slight elliptical trajectory about the sun. This difference between local noon and official noon will vary throughout the year. This follows from the fact that time-zones have discontinuous one hour jumps and the earth's path departs from a perfect circle. Local noon is very accurately determined by the present vertical post technique since the shadow transit time across a marked vertical line on the more

northerly post takes only a few seconds. The ancient Greeks used such vertical posts (Pelekinon) as sundials.

About three decades ago I set up a Pelekinon in my back yard to measure date and time . To determine the date I recorded the length of the top of the shadow on the ground being cast by a vertical 36" post and noting the point where the shadow crossed a horizontal north-south line on different days. Here are the results of one year's observations compared with the sinusoidal variation predicted by a modified form of the above arcsine formula at local noon-



From the resultant curve we see that shadow length at the equinoxes is  $L=19''$ . Since the Pelekinon height was  $H=36''$  we get the local latitude estimate to be  $LAT = \arctan(19/36) = 27.8$  deg. The actual latitude of my home is  $LAT=29.6$ deg, so the rather crude measurement, obtained without benefit of a watch or considering atmospheric effects, is off by just 1.8 deg. Also if one were living back in 2000BC at the time of Stonehenge's Beaker People, a graph of the above type would be an excellent indicator of when its time to plant, time to harvest, and for predicting the occurrence of the longest and shortest days of the year.