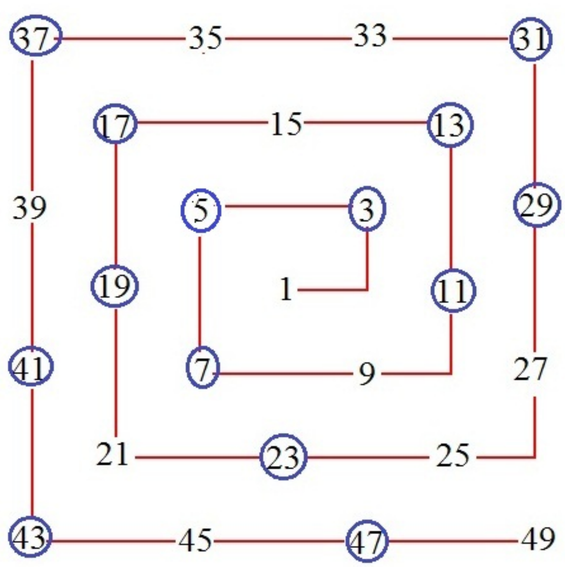
MORPHING OF THE ULAM SPIRAL

In 1963 Stanislav Ulam came up with a way to graph prime numbers which seemed to indicate some recognizable structure in a graph of the primes. People have since that time made elaborate graphs showing these patterns believing them to be of significance. **It has apparently not been recognized that all the patterns show is the obvious that all prime numbers above n=2 are odd numbers.** We demonstrate this fact here by morphing the Ulam spiral into a form where all odd numbers, both prime and composite, lie along a single diagonal. The result is somewhat reminiscent of our own work on an Integer Spiral defined by the intersections of the Achimedes spiral $z=n*\exp(i\ n\ \pi/4)$ and the lines $y=x$ and $y=-x$.

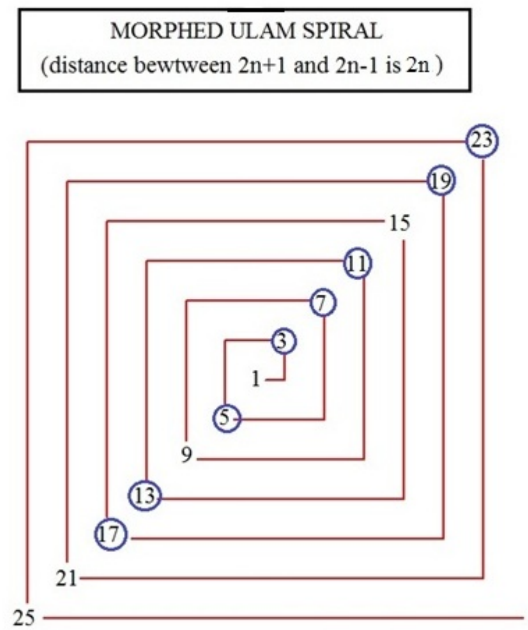
Let us start with the standard Ulam spiral constructed by writing down all odd numbers in sequential order forming the following spiral pattern -

ULAM SPIRAL SHOWING THE FIRST FEW ODD INTEGERS
(primes are circled in blue)



Stepping procedure is one unit per integer around a counter-clockwise spiral

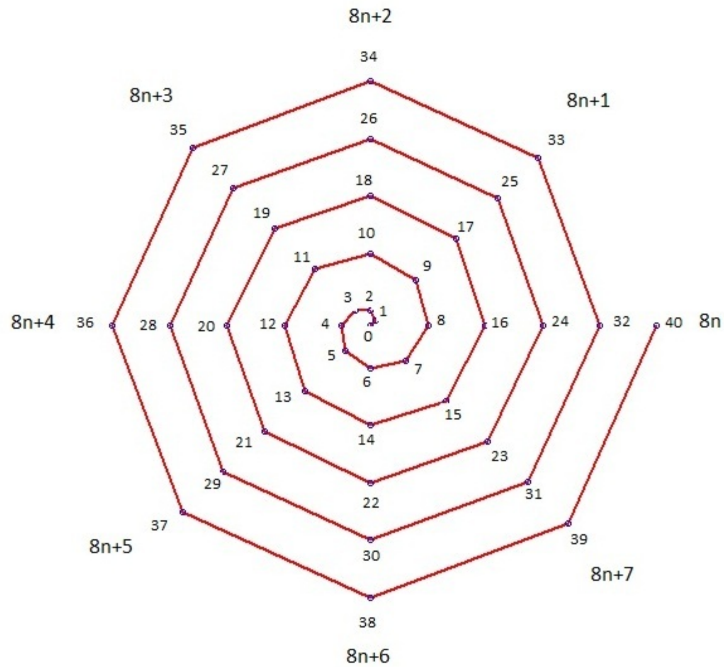
The prime numbers are circled in blue with the stepping about the counter-clockwise spiral taken as one unit between integer n+1 and integer n. The resultant pattern of primes indicates some diagonal structures and has led many to think that this spiral contains hidden information concerning prime numbers. **We show you here that this is not so**. For if we follow the same procedure but instead of leaving one unit between neighboring integers one increases the spacing between two neighboring odd numbers 2n-1 and 2n+1 by 2n units we get a completely different picture. The resultant morphing of Ulam's Spiral takes on the following much simplified form-



This figure clearly shows that any structure shown in the original Ulam Spiral is just a manifestation the fact that primes (with the exception of 2) are odd numbers and will fall along a single diagonal line, The odd numbers on this diagonal differ by a multiple of 4 from each other**. Note that a position along this diagonal says nothing about whether or not the number is prime.** I remember talking with Ulam years ago on combinatorics while he was a visiting professor here at the University of Florida, but unfortunately I never got to ask him about his views on his prime spiral.

Additional variations on the Ulam pattern can be gotten by using other spacings between neighbors when moving around the spiral. We can , for instance, get the odd integers to fall along four separate diagonals in the four quadrants of the x-y plane .
A related spiral which we found recently is the Integer Spiral -

INTEGER SPIRAL



It allows one to plot all positive integers in a very convenient manner by having the even integers lie along the x and y axes while all the odd integers , including the primes, lie along the diagonal lines y=±x. We discovered this spiral through some work we did in connection with the complex variable function $f(n)=(1+i)^n$ in our undergraduate analysis class. The Mersenne Primes $M(n)=2^{2n+1}-1$ lie along a diagonal line in the fourth quadrant and are of the form 8n+7.

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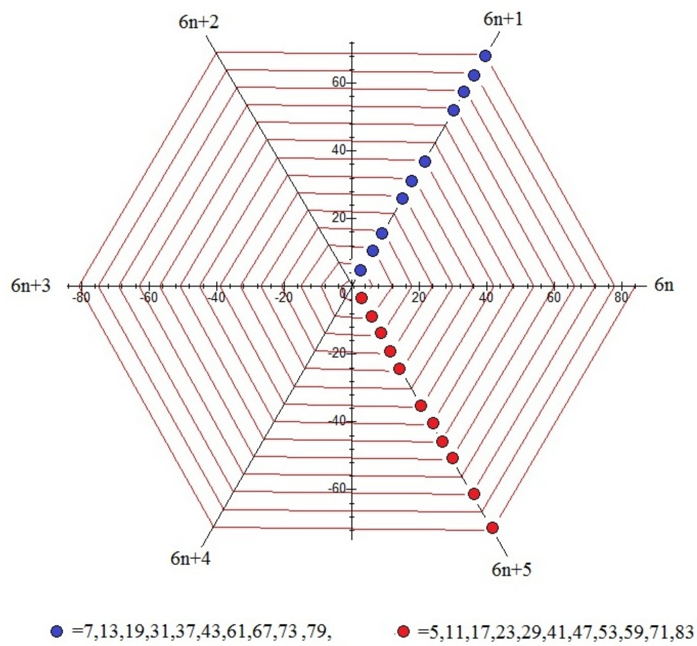
August 2008

Note added July 10, 2013- An even better way to project primes on an integral spiral is to define a hexagonal spiral with corners at r=N and θ=nπ/3. This way one gets the following graph-

INTEGER SPIRAL SHOWING THE FIRST FEW Q PRIMES



Here all primes above p=3 lie along the diagonals 6n+1 or 6n+5. We call these the Q Primes. Note that all odd numbers of the form 6n+3 must always be composite numbers. The spacing's between primes along a given diagonal are observed to be multiples of six.

The observed gaps between the blue primes occur when $N=p_n(p_n+6m)$ and the gaps between the red primes occur for $N=p_n(p_{n+1}+6m)$. Here p_n=5,7,11,13,17,.. sqrt(N) and m=0,1,2,3,.. .