GENERATING LARGE PRIME AND LARGE NON-PRIME NUMBERS

Although most computational efforts in the literature involving prime numbers has been devoted to finding ever larger Mersenne prime numbers, comparatively little attention has been given to finding larger non-Mersenne prime numbers and also non-prime composite numbers. According to the prime number theorem there are approximately N/ln(N) primes present in the set of the first N integers of which the Mersenne primes constitute only a very small portion. In addition one is left with the additional fraction [1-N/ln(N)] of non-prime(composite) numbers. To generate the latter, one starts with a random collection of k even and odd integer n_1, n_2, n_3,…n_k and converts them to all odd integers via the operation $m_1=2n_1+1$, $m_2=2n_2+1$, etc. Next taking the product of the ms one has the finite product-

$$C=\prod_{i=1}^{k}(2n_i+1)$$

This number is seen to always be an odd composite number. As an example, consider the two even numbers $n_1=6{,}857{,}620$ and $n_2=2{,}954{,}236$ with k=2 . This produces the 14 digit composite number-

$$C=20{,}259{,}037{,}690{,}177$$

In searching the neighborhood of this number C with our MAPLE program, we find the closest prime to be P=20,259,037,690,187 which means that C-P=10 or 2.5 turns earlier when looking at things in terms of our previous discussion involving the Archimedes spiral and 45 deg diagonal lines. Indeed, we have that-

$$C=8x2{,}532{,}379{,}711{,}272+1$$

so that this number is located at the intersection of the 2,532,379,711,272 turn of the Archimedes spiral and the 45 deg diagonal in the first quadrant of the complex plane. The neighboring prime P is found along the diagonal in the third quadrant.

To be a prime number P, it is necessary that the above product formula have no possible solution involving all whole integer values of the n_is. As a simple example of this , consider the case of $13=(n_1+1)(n_2+1)$. This yields

$(n_1,n_2)=\{(1,11/2),(2,10/3),(3,9/4),(4,8/5),(5,7/6),(6,6/7)\}$.

which clearly shows that no whole integer solutions are possible and hence, P=13 is a prime number. Geometrically one has for k=2 that no points on the hyperbola $(2n_1+1)(2n_2+1)$ can fall on any of the integer coordinate points (n_1,n_2). A number will be composite if at least one of the coordinate points (n1,n2,n3….,nk) falls on the

k space hyperbolic surface. See if you can find the six whole integer n_is which show that-

$$111{,}222{,}333{,}444{,}555{,}666{,}777$$

is a composite number.

As another example of a non-prime number take n_1=232, n_2=1102, and n_3=2088. Here the product formula evaluates to-

$$C = 536{,}870{,}911 = 2^{29} - 1$$

which is one of the non-prime Mersenne numbers. It is interesting to note that 2^{29}-3 and 2^{29}+11 are prime numbers, suggesting that perhaps some extended Mersenne number of the form-

$$M = 2^{p_1} \pm p_2$$

,with p_1 and p_2 being prime numbers, might be worth investigating for its primeness or non-primeness. For, example-

$$2^{137} - 13 \quad , \quad 2^{191} + 5 \quad , \quad 2^{233} - 3$$

are prime numbers.