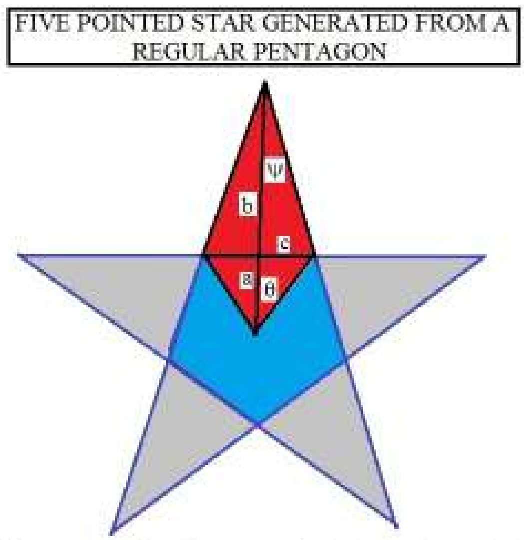
STAR CONSTRUCTION

If one takes a regular polygon of N sides and extends the sidelines until they intersect one obtains a star structure. I demonstrate this in the following figure using a pentagon as the basis-



red kite represents the basic construction element of any regular star

You will notice that this five pointed star , as all regular stars, is composed of N kite structures of height H=a+b and width W=2c. Two important half angles of the kite are θ and ψ as shown. The area of the red kite is c(b+a) so that the area of any regular N pointed star will be A=cN(b+a). Take now any regular N pointed star. One has at once from the geometry that-

$$\theta = \frac{\pi}{N}\,, \qquad \frac{b}{a} = \frac{\tan(\theta)}{\tan(\psi)} \quad and \quad c = a\tan(\theta)$$

These results hold for both even N and odd N, but there will be difference between them as far as the construction used below is concerned. We note that the simple line extention between neighboring sides of the base polygon will not yield a very interesting star structure when N is above about N=6. Therefore we choose rather to construct our N pointed stars from kite structures whose side extends in a straight line to kite whose tip lies nearest to the point lying directly across the star. This condition imposes a difference between even and odd pointed stars. For all even N stars we have that a=b so that θ=ψ while for odd N stars one finds that θ=2ψ. This difference will be made clearer by the examples to be given below.

Let us begin with **even pointed** regular stars having hexagon, octagon, or decagon central shapes. For these stars the basic kite element reduces to a standard rhombus with a height

H=a+b and width W=2a tan(π/N). To plot the N vertexes and N valleys of such a star we can make use of the list plot properties of certain canned mathematics programs such as MAPLE which we are using here. We can define these 2N points via the polar coordinates[r,θ] as -

$$r = f + (H - f)\cos(\frac{\pi n}{2})^2 \quad \text{and} \quad \theta = \frac{\pi n}{N} \quad \text{where f=} \frac{a}{\cos(\frac{\pi}{N})}$$

Connecting these points via a listplot operation produces the following even pointed stars corresponding to N=6, 8, and 10 -

The red and blue Rhomboids indicate the Basic Construction Elements

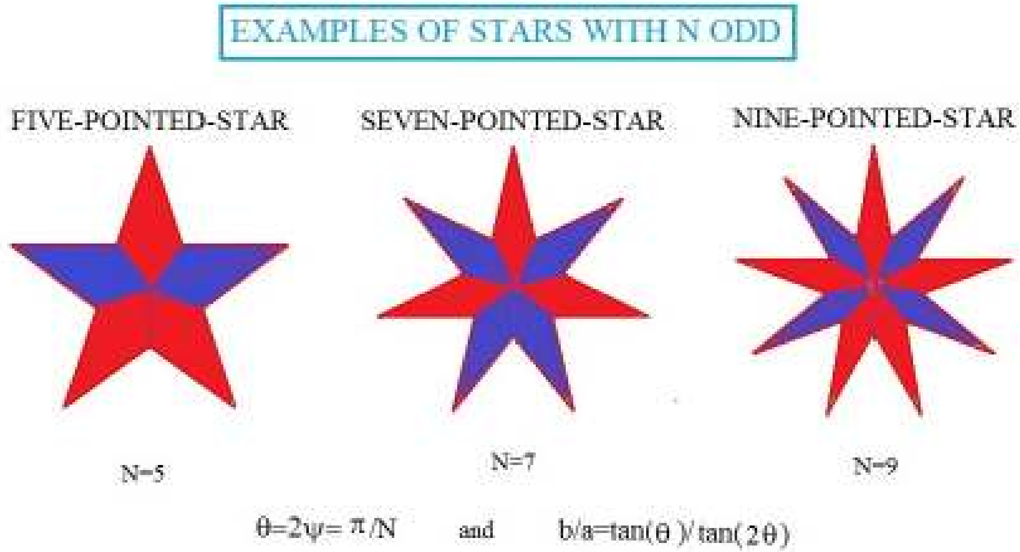
True to our basic definition the edge of the top rhomboid lies along the same straight line as the edge of the rhomboid lying next to the rhomboid directly across from the first rhomboid. The simple program used to generate these three examples reads-

```
listplot([seq([f+(H-f)*cos(Pi*n/2)^2,Pi/(2*N)+Pi*n/N],n=0..2*N)],coords=polar,color=red,

thickness=2,scaling=constrained,axes=none);
```

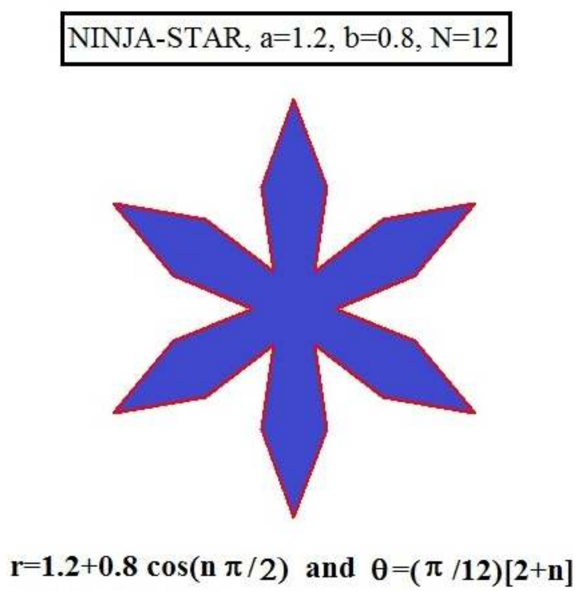
The color enhancement was achieved via Microsoft Paintbrush.

Looking next at stars with an **odd number of points** we find the following examples corresponding to N=5, 7 and 9-



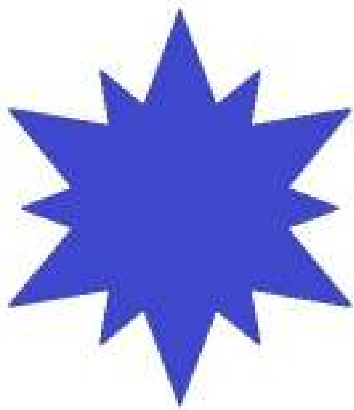
We notice this time the width of the star points are a little narrower than for the even N cases.

Nothing of course prevents one from looking at other types of stars using different values of a ,b, and c from those required earlier. For example, taking a=1.2, b=0.8, and N=12 we find the following-



This figure is very reminiscent of the Ninja Star used by Japanese Samurai as a concealed weapon. Note in this case we haver a radial postion $r=1.2+0.8\cos(n\pi/2)$ and an angle $\theta=(\pi/12)[2+n]$ with n=0.1.2,…,24. That is, the figure is composed of 24 straight lines and 24 corners. One can also construct stars where the star point length alternates in length. The simplest way to achieve this is to superimpose two solutions as shown here-

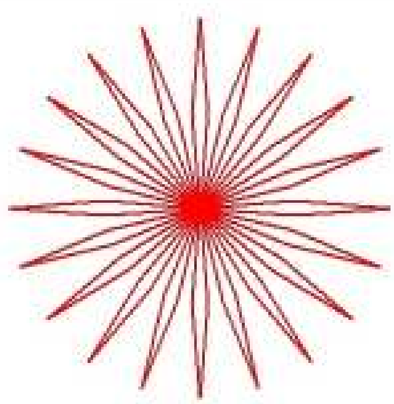
TWELVE POINTED IRREGULAR STAR



Produced by superimposing a regular six pointed star and a smaller six pointed regular star rotated by 30 degrees

Finally, see if you can figure out what values of a, b ,c, and N which I used to generate the following hyperstar pattern. It looks very much like a flower and has a large perimeter relative to its area-

TWENTY PETAL FLORAL PATTERN



Such a configuration would make an interesting compact air filter for a vacuum cleaner or home-air conditioner.

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