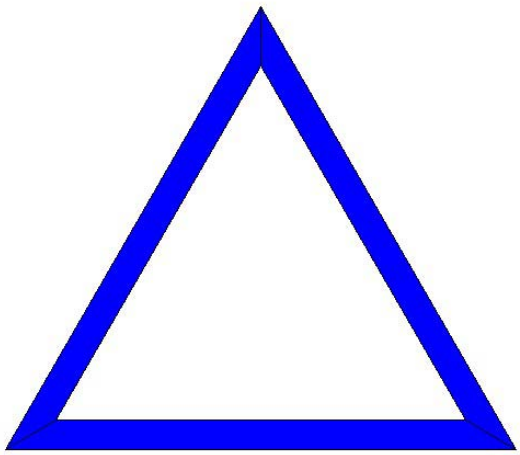
SYMMETRIC PATTERNS GENERATED FROM SUB-ELEMENTS

Many designs and historically significant symbols can be constructed by simple sub-elements rotated multiple times by angle increments $2\pi/m$ to produce m fold symmetric figures. Good examples of such patterns include the Maltese Cross, the Teutonic Cross, the Swastika, and the Star of David. We show here how such m-fold symmetric figures are generated. Let us begin with one of the simplest, namely that of a quadrangle sub-element defined by-

```
F:=polygonplot([[0,1] ,[sqrt(3)/2,-1/2],[0.8*sqrt(3)/2,-0.8/2],[0,0.8]],
    scaling=constrained, color=blue);
```

If we rotate this element by angle $2\pi/3$ and angle $4\pi/3$ about the origin at [0,0] we get the Greek capital letter delta as shown-

GREEK UPPER CASE DELTA



Example of THREE-FOLD Symmetry

The MAPLE command for achieving this is-

```
display(F, rotate(F,2*Pi/3), rotate(F,4*Pi/3));
```

Note the figure has a three-fold symmetry since the delta figure remains unchanged when looked at after multiple rotations of $2\pi/3$ radians. If we rotate the above element instead by multiples of just $\pi/3$ the six-fold symmetric Star of David will result.

Take next the sub-element defined by the corners-

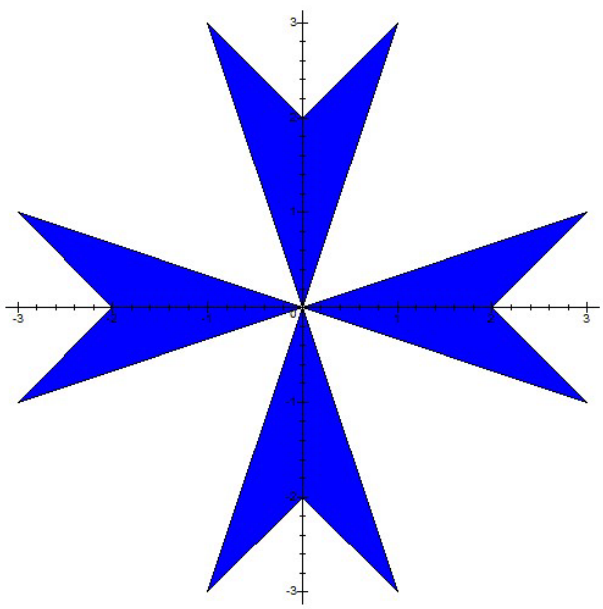
G:=polygonplot[[0,0],[3,1],[2,0],[3,-1]], scaling=constrained, color=blue);

which represents an oblique triangle and its reflection about the x-axis. If we rotate this element about the origin [0,0] at π/2 intervals, reflected by the command-

display(seq(rotate(G, nπ/2),n=0..3));

, we get the four-fold symmetric figure of a Maltese Cross as shown-

MALTESE CROSS CONSTRUCTED FROM POLYGONPLOT([[0,0],[3,1],[2,0],[3,-1]])



To construct a regular m sided polygon one starts with the sub-element triangle defined by-

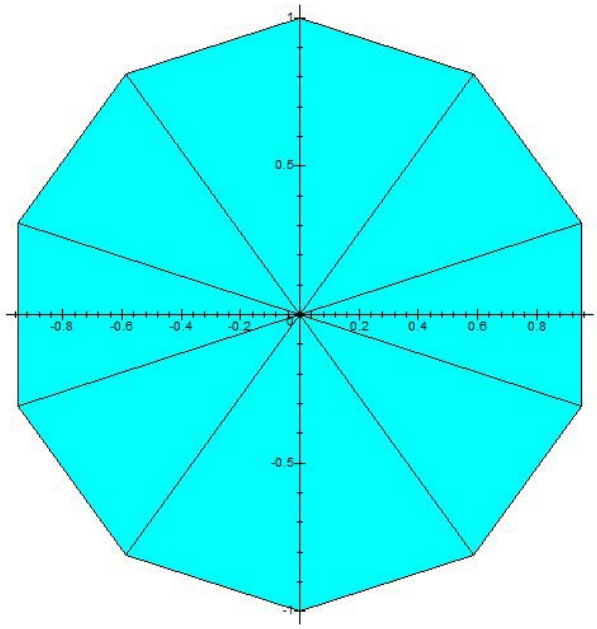
H:=polygonplot([[0,0],[cos(π/m,sin(π/m)],[cos(π/m),-sin(π/m)]],scaling=constrained, color=cyan);

and then replicates the triangle with m triangles produced by m rotations at 2π/m increments. This replication is produced by the command-

display(H, seq(rotate(H, 2*n*π/m),n=1..m-1));

To demonstrate, consider the decagon with m=10. It produces the figure-

TEN FOLD SYMMETRY OF A DECAGON



The vertex angle of the isosceles sub-triangles here is 2π/10 rad=36 deg, their two equal sides have length L=1 each and the base has width c=sqrt{2[1-cos(π/5)]}=0.6180339.. .

When dealing with sub-element areas with curved sides, the polygonplot command will no longer work and one must rather use descriptions involving the product of Heaviside Step Functions and a chosen non-linear functions of x. Take as an example a leaf shaped sub-element. In Maple language one defines the leaf borders as-

```
f:=0.5*x*(1-x)*(Heavisde(x-0)-Heaviside(x-1)): g:=-f:
```

To construct a eight-fold symmetric figure from this sub-element, we first plot the leaf as-

```
S:=plot({f, -f},x=-1..1, scaling=constrained, color-red):
```

and then type-

```
Display(S, seq(rotate(S,n*Pi/4), n=1..7));
```

to produce the flower pattern shown-

FLOWER PATTERN

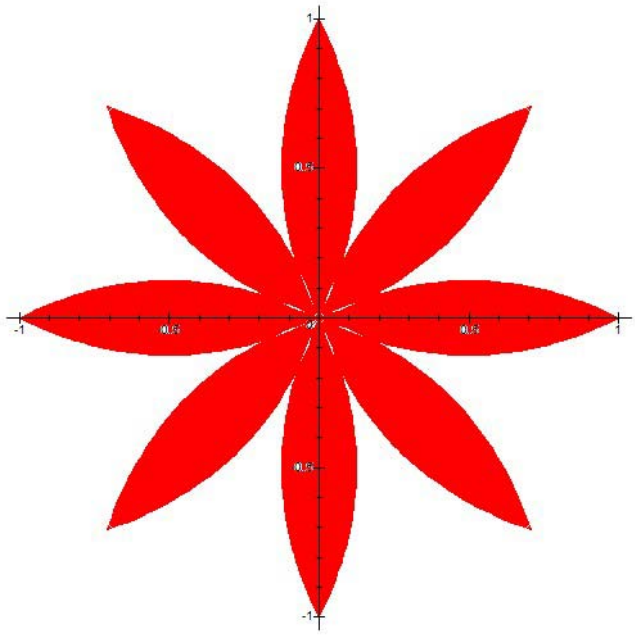
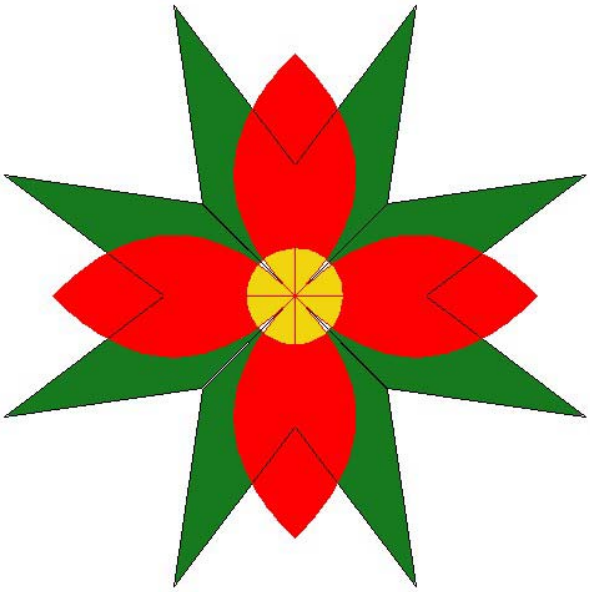


Figure has Eight-Fold Symmetry and Generated by a Leaf-Shaped Sub-Element

To fill in the leaf patterns with red we used the standard Microsoft Paint Program. One can readily modify this result to produce more complicated symmetric patterns. In keeping with the spirit of the Christmas season, here is a computer generated variation on the above pattern-



MERRY CHRISTMAS

December 2010