

WHAT ARE THE CHARACTERISTICS OF THERMAL PUMPING?

It is well known that heat can be transferred by conduction, convection, and radiation. For most applications the transfer process involves either conduction or convection or a combination thereof. Pure conduction follows the familiar Fourier law -

$$\frac{dQ}{dt} = -kA \text{grad}(T)$$

, where dQ/dt is the heat flow rate in joules/sec, k the thermal conductivity in watts per meter-kelvin, A the cross-sectional area in square meters and $\text{grad}(T)$ the temperature gradient in degree kelvin per meter. Examples of conduction include heat flow from an electrically heated stove top to a cooking pot and the transfer of heat across an opaque wall whose opposite sides are maintained at different temperatures. Convection involves the actual movement of heated fluid elements through a conduit to a colder region such as water flowing from a hot water heater to the shower head. This form of heat transfer is given by-

$$\frac{dQ}{dt} = \rho c A U T$$

, where ρ is the fluid density in kilogram per meter cubed, c the specific heat in joules per kilogram degree kelvin, A the cross-sectional area in meters squared, U the average velocity in meter per second, and T the temperature in kelvin. For most cases convection is much more efficient in transferring heat than conduction so that the Nusselt number, which represents the ratio of convective to conduction heat transfer, will be high. There are exceptions to this fact when smaller dimensions are involved such as in nano-technology. Also some thirty years ago I discovered a new heat transfer method in which large conduction heat flow across very thin Stokes boundary layers in oscillatory flow combined with a phase shifted periodic axial velocity produces large axial convection heat flux without mass exchange between the bounding end reservoirs. Some references giving details of the process are the following-

***"Heat Transfer by High Frequency Oscillations; A New Hydrodynamic Technique for Achieving Large Effective Thermal Conductivities"*, (with L. Zhao), *Phys. Fluids* 27, 2624-2627, 1984.**

"Enhanced Heat Conduction in Fluids Subjected to Sinusoidal Oscillations," J. of Heat Transfer (ASME), 107, 459-462, 1985.

"Enhanced Heat Conduction in Oscillatory Viscous Flows within Parallel Plate Channels," J. Fluid Mech., 156, 291-300, 1985.

It has taken awhile for this new form of heat transfer to have become accepted and it is now familiar to most individuals working in the heat transfer area. Abstracts of these papers can be found on-line by going to Google Scholar and typing in U.H.Kurzweg.

The essence of thermal pumping involves the oscillation at angular frequency ω of fluid elements in a bundle of open ended capillary tubes (or flat plate channels) connected at their opposite ends to a T_H and a cold T_C fluid reservoir. The average axial temperature in the bundle has the linear variation $T=T_C+[(T_H-T_C)/L] x$ but there is also a periodic transverse temperature variation superimposed upon this axial value. The transverse temperature variation can produce large oscillatory conduction heat flows given approximately by-

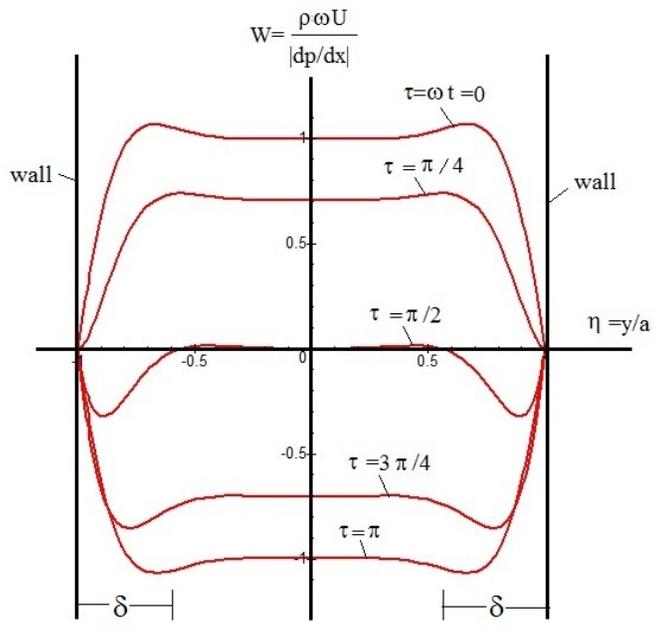
$$\frac{dQ}{dt} = -k(2\pi aL)N \frac{(T_H - T_C)}{\delta L} \Delta x \cos(\omega t)$$

Here 'a' is the radius of each one of the N capillaries, ω the angular oscillation frequency, Δx is the cross-stream averaged axial fluid displacement, and k the fluid thermal conductivity. The quantity $\delta = \sqrt{2\nu / \omega}$ represents the Stokes boundary layer whose thickness will be typically be less than the capillary radius. Here $\nu = \mu / \rho$ is the kinematic viscosity of the fluid expressed in meters squared per second. Also the axial displacement Δx must be kept at less than the bundle length L in order to prevent direct fluid exchange between the end reservoirs. Axial heat flows can reach very large values when the oscillation frequency increases and the axial displacement is large. For example , water oscillating at ten hertz has the Stokes boundary layer thickness of just –

$$\delta = \sqrt{2\nu / \omega} = \sqrt{\frac{2(9.8)10^{-7}}{20\pi}} = 1.8 \times 10^{-4} m = 0.18 mm$$

This is indeed quite thin and accounts for the large transverse temperature gradient produced near the capillary walls. The fact that the axial velocity field, produced by a piston in one of the end chambers and a flexible membrane in the other, varies in the transverse direction means that there will be a time-averaged coupling with the radial varying temperature to produce a net axial convective heat flux . This represents the essence of the thermal pumping process. Some idea of the transverse variation of the periodic axial fluid flow can be gotten by looking at the following graph-

PERIODIC VELOCITY DISTRIBUTION IN A
FLAT PLATE CHANNEL AT $\alpha=10$



In this simplified model we have replaced the flow in a capillary by laminar flow in a flat plate channel separated by distance $2a$. The graph shows the axial velocity over an oscillation half-cycle for the special case where the non-dimensional Womersley Number $\alpha = a\sqrt{\frac{\omega}{\nu}} = 10$. That is, where the Stokes layer has thickness $\delta = 0.44a$. The Stokes layers are clearly visible along both walls of the flat plate channel. Note that the core of the flow does not participate directly in any axial heat convection because its value averages out to zero when integrated over one cycle. The active axial convection lies in and around the Stokes layer. This fact suggests that one design the capillary (or flat channel) such that the frequency ω , the cross-stream distance 'a', and fluid kinematic viscosity ν have the combined property that $\omega a^2 / \nu$ be of order one. For a micro-channel bundle of stacked insulating plates with $a=10^{-4}$ m using air for which $\nu=1.5 \times 10^{-5} \text{m}^2/\text{s}$ requires an oscillation frequency of approximately $f = \omega/2\pi = 300$ hertz. For water in the same channel configuration the frequency can be less by an order of magnitude since the kinematic viscosity for water is just $\nu=0.98 \times 10^{-6} \text{m}^2/\text{s}$.

What is clear from the above discussion is that thermal pumping will work for any condition where the Stokes boundary layer thickness has a value comparable to the capillary radius or channel half-width. The main improvement will be found if the walls become thermal conductors so that heat can also be stored there and if the phase difference between fluid axial velocity $U(x,y,t)$ and transverse heat variation $T(x,y,t)$ is maximized. This will produce the desired large time-averaged product $U T$. The mathematics giving the actual axial convection induced in a thermal pump is rather complex when thermal storage within the capillary or flat plate channel walls are taken into account. The results (after some lengthy calculations) are that an optimum axial heat transfer occurs near $\alpha^2 \text{Pr} = \pi$, where α is the Womersley Number and Pr

$Pr = \frac{v}{\kappa}$ the fluid Prandtl number, with κ the fluid thermal diffusivity. Under such optimum conditions the axial heat transfer is found to have the rather large value of-

$$\frac{dQ}{dt} = \text{const.} \cdot \rho c \omega \Delta x^2 N A \frac{(T_H - T_C)}{L}$$

, where c is the fluid specific heat in joules/(kg-K) . The constant has a value depending on channel/ wall width ratio and on the thermal properties of both the fluid and wall. Typically it has a value around 3×10^{-2} when $N A = 1 \text{ m}^2$. This value can be thought of as a measure of the effectiveness with which the transverse heat conduction is being coupled with the axial convection. If this value could be increased the thermal pumping process would become even more effective than it already is.

Several variations on the thermal pumping process been explored by others since our 1985 studies. In the future I anticipate the thermal pumping process will find several additional new applications in the area of micro-fluid mechanics. The use of Thermal Pumping in heat removal form heat generating microchips has already received quite a bit of attention in the last decade. Here is a schematic of such a device we worked on in 2002-2003-

