Question-1

(Problem 2.3 of Arora’s Introduction to Optimum Design): Design a beer mug, shown in fig, to hold as much beer as possible. The height and radius of the mug should be not more than 20 cm. The mug must be at least 5cm in radius. The surface area of the sides must not be greater than 900cm² (ignore the area of the bottom of the mug and ignore the mug handle—see fig). Formulate the optimal design problem.

Solution:

Design variables: \( X = (R, H) \)

Objective function: maximize volume, \( \pi R^2 H \) OR
Minimize, \( f(X) = - \pi R^2 H \)

Subjected to:
1) \( H \leq 20 \) OR \( g_1(X) = H - 20 \leq 0 \)
2) \( 5 \leq R \leq 20 \) OR \( g_2(X) = R - 20 \leq 0 \)
\( g_3(X) = 5 - R \leq 0 \)
3) \( 2\pi RH \leq 900 \) OR \( g_4(X) = 2\pi RH - 900 \leq 0 \)
4) \( H \geq 0 \)
**Question-2**

(Problem 2.16 of Arora’s Introduction to Optimum Design): Design of a two-bar truss. Design a symmetric two bar truss (both members have the same cross section) shown in figure to support a load W. The truss consists of two steel tubes pinned together at one end and supported on the ground at the other. The span of the truss is fixed at s. Formulate the minimum mass truss design problem using height and the cross-sectional dimension as design variable. The design should satisfy the following constraint.

1. Because of the space limitation, the height of the truss must not exceed $b_1$, and must not be less than $b_2$.
2. The ratio of the mean diameter to thickness of the tube must not exceed $b_3$.
3. The compressive stress in the tubes must not exceed the allowable stress $\sigma_a$ for steel.
4. The height, diameter, and thickness must be chosen to safeguard against member buckling.

Use the following data: $W=10\text{kN}$; span $s=2\text{m}$; $b_1 = 5\text{m}$; $b_2 = 2\text{m}$; $b_3 = 90$; allowable stress, $\sigma_a = 250 \text{ MPa}$; modulus of elasticity, $E = 210 \text{ GPa}$; mass density, $\rho = 7850 \text{ kg/m}^3$; factor of safety against buckling, $FS = 2$; $0.1 \leq D \leq 2 \text{ (m)}$; and $0.01 \leq t \leq 0.1 \text{ (m)}$.

**Solution:**

F₁ and F₂ are the forces in the two truss bars.
Let L be the length of each bar and s be the distance between two end points of bars (as shown in fig).
Let t be the thickness of cross section.
Due to the symmetry, $F_1 = F_2 = F$
Also, $\sum F_y = 0 \rightarrow -2Fcos(\alpha) + W = 0$
$\Rightarrow F = W/2cos(\alpha)$

Assuming $t \ll D$, cross-section area of truss bar, $A = \pi Dt$
Length of bar $L = \sqrt{H^2 + (.5s)^2}$
Stress in truss member, $\sigma_c = F/A = W/[2cos(\alpha) \pi Dt]$
For pinned-pinned column, critical buckling load, $P_{cr} = \pi^2 E I / L^2$
Where, moment of inertia, $I = \pi D^3t/8$ and $E$ is Young's modulus

**Design variables:** $X = (D,t)$

**Objective function:** Minimize the mass of truss

Minimize, $f(X) = 2\rho AL$

**Subjected to:**

1) $b_2 \leq H \leq b_1$ OR $g_4(X) = H - b_1 \leq 0$
   OR $g_5(X) = b_2 - H \leq 0$

2) $D/t \leq b_3$ OR $g_3(X) = D - b_3t \leq 0$

3) $\sigma_c \leq \sigma_a$ OR $g_4(X) = \sigma_c - \sigma_a \leq 0$

4) $F \leq P_{cr}/2$ OR $g_5(X) = F - P_{cr}/2 \leq 0$

5) $0.1 \leq D \leq 2$

6) $0.01 \leq t \leq 0.1$
Question-3

(Problem 2.17 of Arora’s Introduction to Optimum Design): A beam of rectangular cross section as shown in figure is subjected to maximum bending moment of $M$ and maximum shearing of $V$. The allowable bending and shearing stresses are $\sigma_a$ and $\tau_a$ respectively. The bending stress in the beam is calculated as $\sigma = \frac{6M}{bd^2}$ and average shear stress in the beam is calculated as $\tau = \frac{3V}{2bd}$

Where $d$ is the depth and $b$ is the width of the beam. It is also desired that the depth of the beam shall not exceed twice its width. Formulate the design problem for minimum cross sectional area using the following data: $M = 140kN$, $V=24kN$ $\sigma_a = 165MPa$ $\tau_a = 50MPa$

Solution:

Bending stress, $\sigma = \frac{6M}{bd^2}$

Shear stress, $\tau = \frac{3V}{2bd}$

Design variables: $X = (d, b)$

Objective function: minimize the cross sectional area

Minimize, $f(X) = bd$

Subjected to:

1) $d \leq 2b$ OR $g_1(X) = d - 2b \leq 0$

2) $\sigma \leq \sigma_a$ OR $g_2(X) = \sigma - \sigma_a \leq 0$

3) $\tau \leq \tau_a$ OR $g_3(X) = \tau - \tau_a \leq 0$

4) $b, d \geq 0$
Question 4

(Problem 2.17 of Arora’s Introduction to Optimum Design): Design a hollow circular beam shown in figure for two conditions when \( P = 50 \text{KN} \), the axial stress \( \sigma \) should be less than \( \sigma_a \), and when \( P = 0 \), deflection \( \delta \) due to self weight should satisfy \( \delta \leq 0.001 \text{l} \). The limits for dimension are \( t = 0.10 \) to 1.0 cm, \( R = 2.0 \) to 20.0 cm. Formulate the minimum weight design problem and transcribe it into the standard form. Use the following the data: \( \delta = 5wl^4/384EI; \ w = \) self weight force/length \((N/m)\); \( \sigma_a = 250 \text{MPa} \); modulus of elasticity, \( E = 210 \text{GPa} \); mass density; \( \rho = 7800 \text{kg/m}^3 \); \( \sigma = P/A \); gradational constant, \( g = 9.9 \text{m/s}^2 \); moment of inertia, \( I = \pi R^4t \) \((m^4)\).

![Beam Diagram](image)

**Solution:**

Part 1: \( P = 50 \text{KN} \)

Part 2: \( P = 0 \)

- Beam cross section area, \( A = 2\pi Rt \)
- Axial stress, \( \sigma = P/A \)
- \( w = \) self weight force/length \( = \rho g*2\pi Rtl/l = 7800*9.9*2\pi Rt \)
- Moment of inertia \( I = \pi R^4t \)
- Displacement under self weight: \( \delta = 5wl^4/384EI \)

**Design variables:** \( X = (R, t) \)

**Objective function:** minimization of weight \( = \rho g*2\pi Rt \)

Minimize, \( f(X) = \)

**Subjected to:**

1. \( \sigma \leq \sigma_a \) OR \( g_1(X) = (\sigma/\sigma_a) - 1 \leq 0 \)
2. \( \delta - 0.001l \leq 0 \) OR \( g_2(X) = (\delta/0.001l) - 1 \leq 0 \)

**NOTE:** IT IS GOOD TO WORK WITH NON-DIMENSIONAL NORMALIZED CONSTRAINTS

3. \( 0.1 \leq t \leq 1 \)
4. \( 2 \leq R \leq 20 \)
**Question-5**

Consider a laminate made from graphite/epoxy with $E_1=128$ GPa, $E_2=13$ GPa, $G_{12}=6.4$ GPa, and $v_{12} = 0.3$. Formulate the design problem of obtaining an 8-ply symmetric balanced laminate with maximum $E_x$, such that $G_{xy}$ is at least 25 GPa, and $v_{xy}$ is not bigger than 1.

**Solution:**

$E_1 = 128$ GPa, $E_2 = 13$ GPa, $G_{12} = 6.4$ GPa and $v_{12} = 0.3$

$v_{21} = E_2 \cdot v_{12}/E_1 = 0.030$

Also,

$Q_{11} = E_1/(1 - v_{12} \cdot v_{21}) = 128\cdot10^9/(1-0.009) = 129.16\cdot10^9$

$Q_{22} = 13.11\cdot10^9$

$Q_{12} = 3.93\cdot10^9$

$Q_{66} = 6.4\cdot10^9$

Also we can find the material invariants by:

$U_1 = (3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})/8 = 57.53\cdot10^9$

$U_2 = (Q_{11} - Q_{22})/2 = 58.02\cdot10^9$

$U_3 = (Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})/8 = 13.6\cdot10^9$

$U_4 = (Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})/8 = 17.55\cdot10^9$

$U_5 = (Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})/8 = 20\cdot10^9$

Thickness of individual ply is assumed to be constant = t

For balanced laminate we will have the following stacking sequence, $(\theta_1/-\theta_1/\theta_2/-\theta_2)$.

$V_{1A} = 4t [\cos 2\theta_1 + \cos 2\theta_2 ]$

$V_{3A} = 4t [\cos 4\theta_1 + \cos 4\theta_2 ]$

And $V_{1A}/h = V_{1A}/8t = [\cos 2\theta_1 + \cos 2\theta_2 ]/2$

And $V_{3A}/h = V_{3A}/8t = [\cos 4\theta_1 + \cos 4\theta_2 ]/2$

Then

$A_{11}/h = [U_2 + U_2 \cdot V_{1A} + U_3 \cdot V_{3A}]$

$A_{22}/h = [U_1 - U_2 \cdot V_{1A} + U_3 \cdot V_{3A}]$

$A_{12}/h = [U_4 - U_2 \cdot V_{1A} + U_3 \cdot V_{3A}]$

$A_{66}/h = [U_2 - U_3 \cdot V_{3A}]$

Design variables: $X = (\theta_1, \theta_2)$

Objective function: Maximize $E_x = [(A_{11}/h) \cdot (A_{22}/h) - (A_{22}/h)^2]/(A_{22}/h)$  OR  Minimize, $f(x) = - [(A_{11}/h) \cdot (A_{22}/h) - (A_{22}/h)^2]/(A_{22}/h)$

Subjected to:

1) $G_{xy} = (A_{66}/h) \geq 25\cdot10^9$  OR  $g_1(X) = 1 - [(A_{66}/h)/25\cdot10^9] \leq 0$

2) $v_{xy} = A_{12}/A_{22} \leq 1$  OR  $g_2(X) = (A_{12}/A_{22}) -1 \leq 0$

3) $0 \leq \theta_1 \leq 90$

4) $0 \leq \theta_2 \leq 90$