Introduction to Evolutionary Structural Optimization (ESO)

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Introduction

• The Evolutionary Structural Optimization (ESO) method is based on the concept of gradually removing unnecessary or inefficient material from a structure to achieve an optimal design.

• The ESO method is simple to understand and easy to implement and link with existing finite element analysis software packages (e.g. Nastran, Abaqus, Strand6/7, Ansys).
An example
(An apple hanging on a tree?)

Gravity

An object hanging in the air under gravity loading
The finite element mesh
Stress distribution of a “square apple”
Evolution of the object
Comparison of stress distributions

movie
General applicability of the method

The ESO method has been successfully applied to a wide range of engineering optimization problems, including

- minimizing stress, displacement;
- maximizing stiffness;
- maximizing frequency, buckling load;
- minimizing temperature in heat transfer;
- minimizing contact stresses, etc.
The scope of this presentation

• ESO for tension-only or compression-only structures

• Bi-directional ESO

• Examples of possible applications
Stresses at a point

\[ \sigma_x, \tau_{xy}, \tau_{xz}, \tau_{yx}, \tau_{yz}, \tau_{zx}, \sigma_y, \sigma_z \]
## Design Criteria

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Element stress $\sigma^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>von Mises stress</td>
<td>$\sigma_{VM}$</td>
</tr>
<tr>
<td>Tension 1</td>
<td>$\sigma_{11}$</td>
</tr>
<tr>
<td>Tension 2</td>
<td>$\sigma_{11} + \sigma_{22} + \sigma_{33}$</td>
</tr>
<tr>
<td>Compression 1</td>
<td>$-\sigma_{33}$</td>
</tr>
<tr>
<td>Compression 2</td>
<td>$-\sigma_{11} - \sigma_{22} - \sigma_{33}$</td>
</tr>
</tbody>
</table>

ESO will remove elements with the lowest $\sigma^e$
Design for tension-only structure

- Elements with the highest compressive stresses will be deleted from the structure first.

- Then the less tensile elements will be deleted as well.
Design for compression-only structure

- Elements with the highest tensile stresses will be deleted from the structure first.

- Then the less compressive elements will be deleted.
An example of tension-only structure: Catenary

Movie of the evolution history of the above structure using Tension Criterion 2
A 3D tensile example

Movie of evolution history of the above structure using Tension Criterion 2
Example of compression-only structure (using Compression Criterion 1)

Sagrada Familia Church Façade
Barcelona, Spain

Movie
3D print-out of the optimized structure

Wax prototype of the Passion Façade colonnade
ESO for structures with nonlinear materials

(a) Elastic-perfectly-plastic material model
(b) Bilinear material model
(c) Power-law material model
Example: A three-dimensional block

Power-law material model $\sigma = \varepsilon^{0.2}$
Strain energy criterion
Optimal Topologies ($V_f=21.4\%$)

Linear design  

Nonlinear design
Force vs. deformation relationships of different designs
Bi-directional ESO (BESO)

A 3D beam with a solid deck at the top

Initial design for the above structure

Movie
A stool?
A bridge-type structure

A 3D beam with a solid deck at the top and a gap in the middle

Initial design for the above structure
An optimal bridge-type structure

Movie
Space underground

$p = 100\text{N/m}^2$
Initial design
Final design
Load transfer structure for multi-storey atrium
Cellular matrix roof
Case 1: flat, four bottom sides supported

movie
Cellular matrix roof

Case 2: curved, four corners supported