Approximations in structural optimization

- Local algebraic approximations
- Global and midrange algebraic approximations
- Fast reanalysis techniques
- Sequential approximate optimization
Local algebraic approximations

• Linear Taylor series
  \[ g_L(x) = g(x_0) + \sum_{i=1}^{n} (x_i - x_{mi}) \left( \frac{\partial g}{\partial x_i} \right)_{x_0} \]

• Intervening variables
  \[ y_i = y_i(x) \quad i = 1, \ldots, m \]

• Transformed approximation
  \[ g_I(y) = g(y_0) + \sum_{i=1}^{m} (y_i - y_{0i}) \left( \frac{\partial g}{\partial y_i} \right)_{y_0} \]

• Most common: \( y_i = 1/x_i \)
Beam example

Example 6.1.1

- Displacement constraint

\[ g = w_{\text{all}} - \left( \frac{23}{6} \right) \frac{pl^3}{EI_1} - \left( \frac{5}{6} \right) \frac{pl^3}{EI_2} . \]

- Intervening variables \( y_i = 1/I_i \)

\[ g = w_{\text{all}} - \left( \frac{23}{6} \right) \frac{pl^3}{E} y_1 - \left( \frac{5}{6} \right) \frac{pl^3}{E} y_2 . \]
Reciprocal approximation

- Exact for displacement and stresses in statically determinate trusses
- Exact for scaling of all cross-sectional areas of trusses under mechanical loading

\[ y_i = 1 / x_i \]

\[ g_R(x) = g(x_0) + \sum_{i=1}^{n} (x_i - x_{mi}) \left( \frac{x_{0i}}{x_i} \left( \frac{\partial g}{\partial x_i} \right) \right)_{x_0} \]
Conservative-concave approximation

- At times we benefit from conservative approximations

\[ g_L - g_R = \sum_{i=1}^{n} \frac{(x_i - x_{m_i})}{x_i} \left( \frac{\partial g}{\partial x_i} \right)_{x_0} \]

\[ g_C(x) = g(x_0) + \sum_{i=1}^{n} G_i (x_i - x_{0i}) \left( \frac{\partial g}{\partial x_i} \right)_{x_0} \]

\[ G_i = \begin{cases} 
1 & \text{if } x_{0i} \frac{\partial g}{\partial x_i} \leq 0 \\
\frac{x_{0i}}{x_i} & \text{otherwise} 
\end{cases} \]

- All second derivatives of \( g_C \) are negative
- Convex linearization obtained by applying reverse to objective function
Other local approximations

- Quadratic approximations
- Approximations in $x^p$
- Method of moving asymptotes

\[
\bar{f}_i^{(k)}(x) = \sum_{e=1}^{k} \left( \frac{P_{ie}}{U_e - x^e} + \frac{q_{ie}}{x^e - L_e} \right) + r_i
\]

(4.17)

If \( \frac{\partial \bar{f}_i}{\partial x^e} > 0 \) at \( x^{(k)} \) then: \( p_{ie} = (U_e - x^{e(k)})^2 \frac{\partial \bar{f}_i}{\partial x^e} \) \& \( q_{ie} = 0 \)

If \( \frac{\partial \bar{f}_i}{\partial x^e} < 0 \) at \( x^{(k)} \) then: \( q_{ie} = -(x^{e(k)} - L_e)^2 \frac{\partial \bar{f}_i}{\partial x^e} \) \& \( p_{ie} = 0 \)
Schematic representation
Three-bar truss example

Figure 6.1.2 Three bar truss.
Stress constraint on member C

- Stress in terms of areas
  \[ \sigma_C = p \left( -\frac{\sqrt{3}}{3A_A} + \frac{2}{A_B + 0.25A_A} \right) \]

- Stress constraint
  \[ g = 1 - \frac{\sigma_C}{\sigma_0} = 1 - \frac{p}{\sigma_0} \left( -\frac{\sqrt{3}}{3A_A} + \frac{2}{A_B + 0.25A_A} \right) \]

- Using non-dimensional variables
  \[ x_1 = A_A \sigma_0 / p, \quad x_2 = A_B \sigma_0 / p, \]
  \[ g = 1 + \frac{\sqrt{3}}{3x_1} - \frac{2}{x_2 + 0.25x_1} \]
Results around (1,1)

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<th>$g$</th>
<th>$g_L$</th>
<th>$g_R$</th>
<th>$g_C$</th>
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Questions to think about

• What are intervening variables? There are also cases when we use “intervening function” in order to improve the accuracy of a Taylor series approximation. Can you give an example?

• What is conservative about the conservative approximation? Why is that a plus? Why is it useful that it is concave?
Reading assignment

• Section 6.3: Sequential linear programming, including examples!