Optimal Free-form Design of Shell Structure for Stress Minimization

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ABSTRACT

Shell structures with thinness and lightness are extensively used as the structural components in various industrial products, structures and constructions. In the design of shell structures, it is important to optimize their forms, or curvature distributions to achieve required mechanical performances such as stiffness, natural frequency, maximum stress and buckling load. In our previous works, we proposed a non-parametric free-form optimization method of shell structures, and applied it to stiffness design problems and vibration design problems. With the method, an optimum shell with smooth free-form surface can be obtained without any shape parameterization which is inevitable process in general parametric shape optimization methods.

In this paper, we newly present a free-form optimization method by applying this method to a maximum stress minimization problem as a strength design problem of shell structures. The issue of non-differentiability is inherent to this stress minmax problem because of the singularity of maximum stress, making it theoretically difficult to determine directly the sensitivity function of the local objective functional. This issue is avoided by transforming the local measure to an integral functional by using the Kreisselmeier-Steinhauser (KS) function to transform the local objective functional into the smooth differentiable integral functional. When the constant ρ is sufficiently large in the KS function, the maximum stress can be extracted.

This stress minmax problem is formulated in a function space. The maximum von Mises stress extracted by the KS function and a volume are defined as the objective and constraint functional, respectively. It is assumed that the shell is varied in the normal direction to the surface and the thickness is constant. The shape sensitivity function for this problem is theoretically derived using the material derivative method and the adjoint variable method. The shape sensitivity function derived is applied to the shell surface as the traction force to vary the shape under elastically supported condition, and then the displacements obtained as the optimum shape variation in this pseudo-elastic analysis are added to the original shape to update the shape. By repeating the sensitivity analysis and shape updating, the optimum shape is created while minimizing the objective functional, since this method is a gradient method in a Hilbert space. The advantages offered by this method are summarized as follows: (1) a smooth and natural surface can be obtained because the elastic tensor in the pseudo-elastic analysis serves as a mapping function and as a smoother for maintaining mesh smoothness, and its

positive definitiveness is the necessary condition for minimizing the objective functional. (2) An optimal free-form surface is created because all the nodes can be moved as the design variable. (3) Mesh smoothing is simultaneously implemented in the shape changing process. (4) It can be easily implemented in combination with a commercial FEM code.

Several design examples calculated by this method are demonstrated. The optimum shapes with beads and the iteration histories show the effectiveness of the proposed method as a solution to stress minmax problem.