On the optimal topologies considering uncertain load positions

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The more than century old topology optimization has a relatively young new research direction, namely considering probabilistic data in the topology design. Uncertainty is typically limited to the loading, although recent works have considered extensions to support conditions, and material properties, etc.

In this paper the loading positions are taken as stochastic variables and all the other data are deterministic. The linearly elastic discretized structure is modelled mechanically by plane stress disk elements. To make the optimization method robust an equivalent deterministic problem is derived [1,2,3,5]. The elaborated technique can be described as follows: it is assumed that the load positions are given by either their distribution function, mean value and covariance matrix or by the simple values of the probability of the occurrence of a force at a certain location. The first case, where the statistical information (distribution functions, mean values and variations) are given, is always finished by a simple calculation which results in the probability values of the occurrence of a force at a certain location, that is practically in the second case. Hence each load is considered in an extended loading domain. Since the loading positions are not known precisely, an equivalent loading system should also be created around the expected location of each force to perform a "simulation". According to the original distribution assumption, mean value and variation of the point applications, an extended force system is set up for each possible loading domain with the original magnitude of the force and given (or calculated) probability values. (For the sake of simplicity here seven points as "base" points are used with adjustment to the original distribution.) Each load is independent and acts as an independent load case in the original loading domain. Applying these forces at these "base" points as loads the stochastic design problem becomes a deterministic one. By the use of the elements of this force system one by one, the displacement vectors can be calculated from the usual linear equilibrium equations with several load cases. Since the material is linearly elastic, isotropic and homogenious, the additive properties of the displacements and the *reciprocity theorem* can be applied. Using these vectors and the assigned probability values the expected displacement and its variation can be calculated. By the use of these data the original compliance value, which is probabilistic due to the position uncertainties, can be substituted with a deterministic one applying the Kataoka theorem [4]. Using this compliance formulation a min-max objective function is formed, which is composed by the expected compliance and a certain type of variance of the compliance. In the case of Gaussian distribution of the displacement field this objective function is simplified to a function minimization also due to this theorem [4]. The constraints are the volume limitation and the side values of the design variables. An extended SIMP type algorithm is elaborated for the solution method. To validate the model a deterministic minimum compliance truss design is performed analytically and numerically.

Several numerical examples are presented.

References

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