

## Boundary effects in phase-field based topology optimization

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### 1. Abstract

It is well known that the SIMP approach is lacking a length scale which results in a mesh dependent design when solved using the finite element method. A remedy to the problem has been to add constraints on e.g. the perimeter or to include a filter such that the density is averaged over a neighborhood. An alternative approach to regularize the problem is offered by the phase-field approach where the gradient of the density field is penalized in the objective functional. In the phase-field approach gray designs are explicitly penalized in the objective functional. The phase-field approach restricts the total amount of interfaces via a volume integral which implies that interfaces at the boundary of the design domain is left un-penalized. As a result material along the boundaries is preferred. Moreover, the phase-field based topology optimization approach will require the design boundary to be perpendicular to the design domain. This restriction on the optimal design stems for the fact that homogeneous Neumann conditions is employed for the density field. In the present contribution, a penalty for interfaces along design boundaries is introduced. The penalization enters the objective functional via a surface integral along the design domain. Local stationarity of the objective functional reveals that a mixed, Robin type, boundary condition results in penalization of interfaces along the design domain. The standard phase-field approach is recovered by letting the surface 'energy' vanish. The problem formulation consists of two coupled PDE's that are formulated in weak format and discretized using the finite element method. The two field equations are solved using a staggered approach, i.e. the mechanical balance is solved for fixed densities and the densities are thereafter updated for fixed displacements. The diffusion type solution procedure for the density problem is based on a steepest descent algorithm. If a smooth penalization for gray densities is used, the density can take values outside the  $[0, 1]$  range and to avoid this effect we make use of an obstacle formulation such that the  $[0, 1]$  limits are strictly enforced. Since obstacles are used to strictly avoid non-admissible densities, the optimization problem is non-smooth. The problem is formulated as a max-min problem and solved by using the Howard policy iteration scheme which frequently is used in financial modeling. The algorithm is implemented in a finite element code and the talk is closed by numerical examples. To resolve the interfaces between void and full material properly an adaptive finite element procedure is used. The numerical simulations clearly illustrates the effect of the regularization, i.e. the length of the perimeter of the structure can be controlled. Furthermore, the influence of the penalization of interfaces along the design domain is investigated and it is demonstrated that the amount of material along the boundary of the design domain can be controlled by increasing the effect of the new penalization.