

A gradient-based multiobjective optimization technique using an adaptive weighting method

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1. Abstract

A process of compromise that addresses conflicting objective functions such as performance and cost is often involved in real-world engineering design activities. If such conflicting relationships among objective functions exist in a multiobjective design optimization problem, no single solution can simultaneously minimize all objective functions, and the solutions of the optimization problem are obtained as a set of design alternatives called a Pareto optimal solution set. Various multiobjective optimization methods based on meta-heuristic techniques have been used to obtain Pareto optimal solution sets in many applications, but doing so requires a cumbersome number of evaluations of objective functions and constraints, since design sensitivities are not utilized in such methods. Here, to solve this difficulty, we propose a multiobjective optimization method using a mathematical programming technique based on the multiple point technique and the weighting coefficient method, where weighting coefficients are adaptively assigned for each searching point considering its position in the objective function space. The weighting coefficients are determined based on the Data Envelopment Analysis technique (DEA), a tool used in economics to evaluate the relative performance of decision-making units in multi-input and multi-output environments. In DEA, the efficiency of decision-making units is calculated by solving a linear programming (LP) problem that evaluates relative performance. When a unit is located at the Pareto frontier, the efficiency value is one and the value decreases with increasing distance from the Pareto frontier. Since the variables of this LP problem are weighting coefficients, optimal weighting coefficients are obtained for each unit by solving this LP problem, and the obtained weighting coefficients are used in our multiobjective search. Sequential Linear Programming (SLP) is used to update design variables, since it is stable when dealing with optimization problems that include many design variables. In this process, design sensitivities for all points are calculated and a linearly approximated optimization problem for each point is formulated. The approximated LP problem is then solved using an LP solver. The procedure for the proposed gradient-based multiobjective optimization technique is the followings. 1. Multiple starting points are given, and the objective and constraint functions are evaluated. 2. Weighting coefficients are calculated using DEA. 3. Design sensitivities are calculated. 4. The approximated LP problem is solved to update design variables. 5. Return to Step 2. This procedure implies that two LP problems for each point must be solved during each iteration. This method can utilize the sensitivities of the objective functions and constraints to update the position of searching points so that non-dominated solutions can be effectively obtained even in large scale problems that have many objective functions and many design variables. We apply the proposed method to several numerical examples, including structural optimization problems, to illustrate its effectiveness and usefulness.

2. Keywords: Design optimization, Multiobjective optimization, Gradient-based optimization, Adaptive weighting coefficient