

# **An Adaptive Surrogate-Assisted Strategy for Multi-Objective Optimization**

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Multi-Objective Optimization is typically conducted using a Direct Optimization approach such as the Non-Dominated Sorting Genetic Algorithm (NSGA-II). The NSGA-II algorithm developed by Deb can be used to obtain “exact” results but requires a very large number of simulations so can be expensive, especially in crashworthiness optimization where simulations involve highly detailed Finite Element models and costly nonlinear dynamic analysis.

MOO is also possible with surrogate-assisted methods. A naïve approach is to conduct a large number of simulations by using a Space Filling experimental design. This approach typically relies on uniform global sampling to construct the metamodel and can be faster than direct solution. The accuracy of the approach is severely affected by scaling as the number of simulations to obtain an accurate surrogate model in a large variable space can be very large. For single objective problems, this deficiency can be addressed by an iterative Domain Reduction (known as Sequential Response Surface Method = SRSM) approach in which the search domain size is gradually reduced for each iteration. With suitable heuristics this approach can provide a sufficiently accurate result, but so far has not been adapted to multi-objective problems in which multiple solutions are possible.

In this study a new adaptive domain reduction method known as Pareto Domain Reduction (PDR) is introduced for improving efficiency and accuracy. The method employs heuristics which are similar to those of SRSM but since multiple optimal solutions are possible, it uses the irregular subregion of the Pareto Optimal Frontier (POF) as a sampling domain. The size of this subregion is iteratively reduced in order to intensify the exploration in the neighborhood of the POF. The method is conservative in the sense that early sampling is global with a gradual convergence to the POF. Hence it can also be viewed as an adaptive sampling approach. The proposed method has the additional advantage that, if a multi-objective problem is posed so that it has only one optimal solution, it degenerates to SRSM which unifies and simplifies the methodology and user choice.

Since Radial Basis Function Networks are typically used as surrogates, a Space Filling approach is used to obtain a well spaced sampling in the reduced domain. This avoids point duplication and maximizes the accuracy in the POF neighborhood.

The algorithm is based on a simple concept consisting of the following major steps:

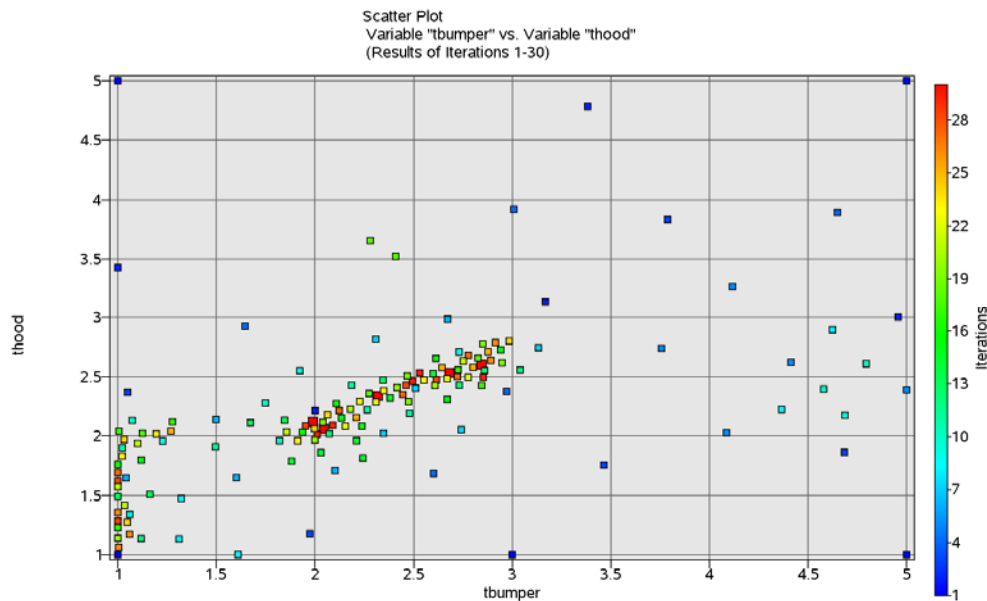
Initial conditions:

1.  $k := 1$ , choose  $m$  simulation designs in the full design space.
2. Conduct the simulations, build the surrogate model and construct an approximate POF.

For each iteration  $k$ :

1. Select  $m$  kernels from the POF.
2. Adapt the subregion size based on iteration ( $k-1$ )
3. Center a subregion on each kernel.
4. Populate each subdomain with a number of *diversity* design points.
5. Select  $m$  points for simulation from the diversity design points.
6.  $k := k + 1$
7. Conduct  $m$  simulations, build the surrogate model and construct an approximate POF using NSGA-II

Figure 1 shows a typical sampling adaptation to the Pareto Domain. In this case, 5 simulation points are added per iteration (which is the default for 2 variables). The red points represent the most recent simulations in the progression.



**Figure 1: Typical sampling pattern in the variable space for the Pareto Domain Reduction method (two variable problem).**

To validate the method, the paper will firstly present two standard benchmark examples with 30 variables each. Following this validation, a 7-variable Multi-objective MDO crashworthiness/ modal analysis example of a vehicle is presented. The FE model is not very large (about 30,000 elements), but sufficiently realistic and representative of a typical model used in industry.

The Direct GA requires 12000 simulations to converge while the PDR requires 490 (including 100 simulations of the POF). The results show (Figure 2) that the PDR method is highly accurate for this crashworthiness problem while being an order of magnitude cheaper than a direct approach. The new PDR method is also compared to the existing global sampling approach in which the design points for building the metamodel are globally uniformly distributed using a Space Filling sampling approach (490 simulations). The benefit of using the PDR method is clearly visible. Note that finer features such as discontinuities in the POF are not faithfully modeled by the uniform sampling approach, obviously because of a sparsity of simulation results near the optimum.

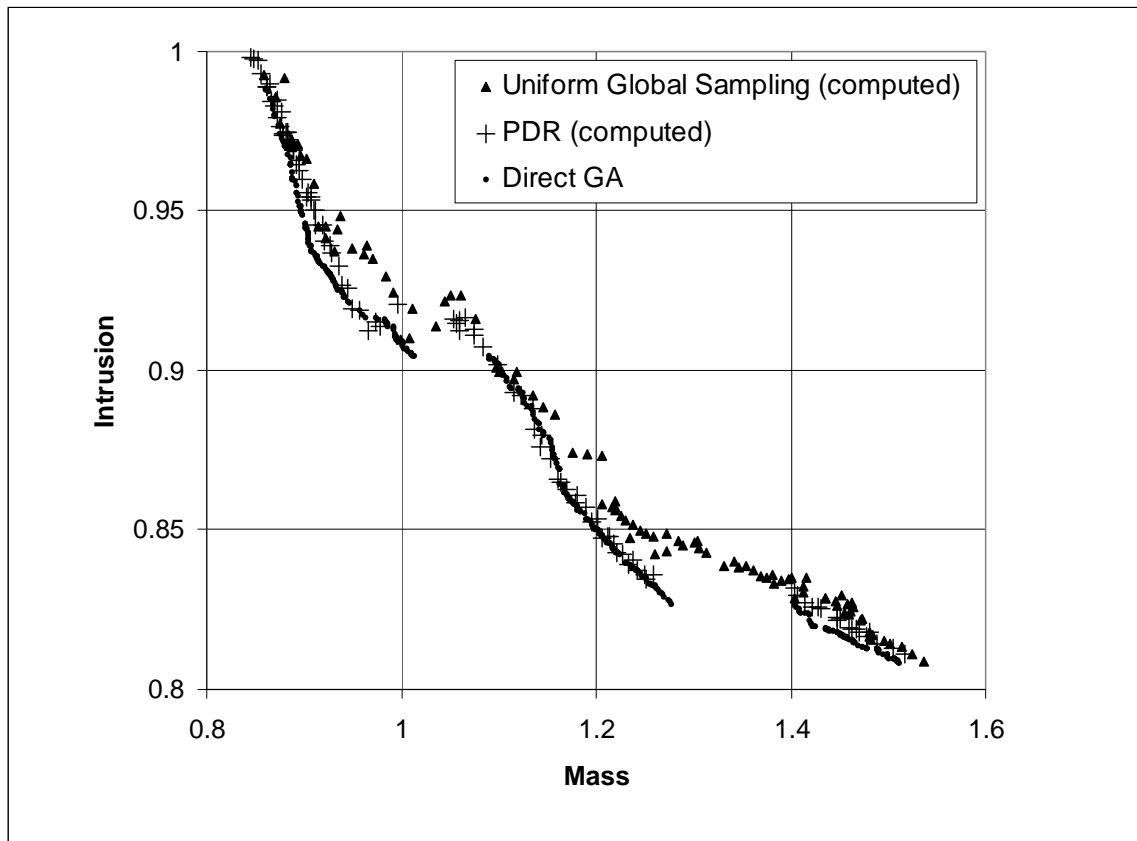


Figure 2: Pareto Optimal Front: Comparison of the FE simulation results for uniform global sampling (▲) and PDR sampling (+) with the reference solution obtained using the Direct NSGA-II algorithm (•).