Filter in topology optimization based on linear diffusion equation

Qinghai Zhao[1] **Xiaokai Chen**[1*] **Zheng-Dong Ma**[2] **Yi Lin**[1]

([1] *School of Mechanical Engineering, Beijing Institute of Technology, Beijing,* 100081*, China*)

([2] *Department of Mechanical Engineering, University of Michigan, MI,* 48105*, USA*)

**Corresponding author: E-mail: chenxiaokai@263.net*

Since the epoch-making work of Bendsøe and Kikuchi, topology optimization of continuum structures has undergone considerable developments in the past two decades, which has been widely utilized in many industrial designs. Although topology optimization is becoming a matured field, the topological solutions are prone to result in checkerboard pattern and mesh dependency designs if no restriction scheme is applied. This phenomenon is caused by errors in the finite element formulation. In the past, various regularization methods have been proposed, such as, by perimeter control, and gradient constraint control, smooth phase-field approaches, a hierarchical neighborhood search method, regularized density control, and a family of image morphology-based black and white filters. However, each scheme has its advantages and disadvantages. Among these, the most common one is mesh-independent filtering techniques, which impose filtered variables obtained by averaging original values over neighbor domains on topology optimization process. Though the approach is heuristic, it leads to practical, useful topology designs, a well-behaved optimization problem and relatively quick computation. However, one drawback is clearly that there are large arrays of neighborhood information that need stored. Recently, a Helmholtz-type partial differential equation has been successfully applied as sensitivity filtering and density filtering, where the filtered variables implicitly deal with as a solution of the equation with homogeneous Neumann boundary conditions. The PDE filter is advantageous because it can utilize the existing computational framework of FEM and inherits parallel properties.

In this paper, a linear diffusion equation-based filtering techniques are introduced as alternative to standard density and sensitivity filtering, where the filtered variables explicitly/implicitly deal with as a solution of the equation with homogeneous Neumann boundary conditions. As a length parameter, time is introduced at the filtering operation to avoid the numerical instability phenomenon sufficiently. Here, under the design domain is discretized by finite elements, three numerical methods, including Finite Difference Method (FDM), Finite Element Method (FEM) and Finite Volume Method (FVM) are adopted to solve the equation respectively. The FDM is relative easy to implement and compute efficiently, but requires neighbor element information and need special treatment of mesh boundaries. The FEM could obtain global optimal solutions but with slow convergence speed. However, the FEM solver for the state problem can be re-utilized for solving the diffusion equation. The FVM has the advantage between the above two methods. The focus of this paper is to discuss the pros and cons of filtering based on these three methods for a detail description in simple 2D minimum-compliance optimization problems.

Though filtering plays a useful role in regularizing the basic problem into a well-behaved setting, the technique has the drawback of a smoothing effect around the boundary of the material domain. For this reason, one possible extensions is filtering designed based on anisotropic diffusion, Through the proper parameters implementation, the filtering could not only leads to a suitable problem regularization but also bring the desirable effects of boundary preservation, which will be preliminary discussed in this paper.