

Topology Optimization with Vanishing Constraints

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1. Abstract

Recent years have seen some progress in the treatment of so-called Mathematical Problems with Vanishing Constraints (MPVC). These problems form a class of nonlinear optimization problems with constraints which must be neglected (i.e., vanish) at certain points of the feasible domain. Standard algorithms of Nonlinear Programming are hardly applicable and usually fail because certain regularity conditions are not satisfied.

We consider different problem formulations from topology optimization formulated as MPVC. We focus, e.g., on local stress constraints and/or slenderness constraints in classical 2D topology optimization and in truss topology optimization. Our intention is the numerical calculation of feasible structures with a certificate, i.e., structures satisfying mathematical necessary conditions of local optimality. In view of this, basic mathematical properties, like continuity and differentiability, must be guaranteed for all functions in the problem formulation at all points of the feasible domain, i.e., also at structures where vanishing constraints are active. As a consequence, our problem formulations are stated in both, design as well as state variables being related through equilibrium constraints.

Recently, some numerical perturbation methods for general MPVC formulations have been proposed possessing useful mathematical convergence properties. More precisely, under standard assumptions convergence can be proved to points satisfying first order necessary optimality conditions when the perturbation is driven to zero. We apply these methods to the above mentioned problem formulations of topology optimization. It turns out that these techniques work well for problems of medium size, at least. In contrast to known perturbation methods working in the design variables only, our approach in design and state variables allows the application of the mathematical convergence theory mentioned before. The price for the use of these novel methodologies, however, is the treatment of highly non-convex problem formulations.

The results in this talk are based on joint work with T. Hoheisel, Ch. Kanzow, and Ch. Schürhoff.

2. Keywords: Topology optimization, vanishing constraints, stress constraints, optimality conditions, continuation methods