STRUCTURAL OPTIMIZATION USING SHAPE-SENSITIVITY ANALYSIS AND A LEVEL-SET BASED MESH EVOLUTION METHOD

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We propose an original framework for structural optimization in which all the considered shapes are exactly meshed at each stage of the iterative process. The problem of mesh deformation is addressed by combining the versatility of the level set method in the context of geometric evolution problems with meshing tools specialized in dealing with implicitly defined geometries.

The main difference between the presented approach and the original idea of G. Allaire, F. Jouve and A.-M. Toader is that the shape is exactly meshed at each iteration of the process ; starting from an initial domain Ω^0 with the aim to minimize a function of the domain $J(\Omega)$, all the admissible shapes Ω^n are enclosed in a larger computational domain D. Each step $n = 1, 2, ...$ of the sequence starts with a simplicial mesh $\mathcal{T}^n$ of D in which Ω^n is explicitly discretized - i.e. a mesh of Ω^n appears as a submesh of $\mathcal{T}^n$ - then rolls out as follows :

(1) a level set function ϕ^n associated to Ω^n is generated at the vertices of $\mathcal{T}^n$, as an approximation of the signed distance function to Ω^n.

(2) Mechanical computations are held on the mesh of Ω^n, relying on the finite element method, and a descent direction θ^n for the objective function $J(\Omega)$ is inferred.

(3) ϕ^n is advected along the velocity field θ^n on the mesh $\mathcal{T}^n$ of D, for a time step τ^n, which gives rise to a level set function ϕ^{n+1} associated to the new shape Ω^{n+1}, still defined on $\mathcal{T}^n$.

(4) A mesh $\mathcal{T}^{n+1}$ of D, which contains Ω^{n+1} as a submesh is eventually obtained from the datum of $\mathcal{T}^n$ and ϕ^{n+1} . This involves two successive operations :

   (i) the 0 level set of ϕ^{n+1} is explicitly discretized into $\mathcal{T}^n$, resorting to patterns. Unfortunately, doing so produces a very ill-shaped mesh $\widetilde{\mathcal{T}^{n+1}}$ of D, which is inappropriate for finite element computations.

   (ii) Local mesh modifications (e.g. edge splits, edge collapses,...) are performed to transform $\widetilde{\mathcal{T}^{n+1}}$ into a well-shaped mesh $\mathcal{T}^{n+1}$ of D in which Ω^{n+1} is still explicitly discretized.

The key point is that, at no stage a new mesh is generated out of nothing (which is costly, and sometimes impossible, notably in three space dimensions) : we always start with an existing, possibly ill-shaped mesh, which is then improved as best possible owing to local mesh modification techniques.

We have hitherto developed a two and three-dimensional layout software to validate this approach on the model test-case of linear elastic structures. In particular, this led us to devise numerical schemes for generating the signed distance function to a discrete contour, and for solving the level set advection equation on an *unstructured* computational mesh (which forbids the use of finite difference schemes), as well as a meshing algorithm of the negative subdomain of a level set function.