

ON IMPROVING NORMAL BOUNDARY INTERSECTION METHOD FOR GENERATION OF PARETO FRONTIER†

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Final Abstract

The goal of multi-objective optimization is to find optimal solutions by simultaneously solving an optimization problem with multiple objective functions that are at least partly conflicting. These problems arise in many fields, including structural and multidisciplinary design optimization. For these problems, in contrast to single-objective optimization, there is typically a set of *Pareto* optimal solutions (or Pareto frontier). A solution point is Pareto optimal if there is no other feasible point that improves at least one objective function without detriment to one or more other objective functions.

In general, gradient-based methods for solving multi-objective optimization problems (e.g., weighted method, epsilon constraint method, Normal Boundary Intersection (NBI)) require solving at least one single-objective optimization problem for each Pareto optimal point, and thus solving many problems to find the Pareto frontier. These methods can be computationally expensive with an increase in the number of variables and/or constraints of the optimization problem. Moreover, they can lead to an unevenly clustered generation of solution points along the Pareto frontier. Among the gradient-based methods, the NBI method provides an even generation of Pareto optimal points, but is again computationally prohibitive. Reducing computational cost is essential for multi-objective optimization problems, as new applications require solving large problems while adding elements of uncertainty further complicate the formulations.

This presentation provides a modification to the original NBI algorithm so that a continuous Pareto frontier is obtained “in one go,” i.e., by solving only one single-optimization problem. Another advantage of this modified method over the original NBI method is that computational efficiency is preserved for nonlinear multi-objective optimization problems. The proposed method can solve problems with discontinuous Pareto frontiers using significantly fewer optimization problems than the original NBI and other popular gradient-based methods. In the proposed method, the multi-objective optimization problem is solved using a quasi-Newton method whose sequence of iterates is used to obtain points on the Pareto frontier. The proposed and the original NBI methods have been applied to a collection of 16 numerical test problems, including structural design and heat exchanger design problem.

Based on the test results, it is concluded that not only is the proposed modified NBI method computationally more efficient, but for some test problems also obtains better solutions as compared to the original NBI algorithm. In the proposed method, the number of function calls increases at a quadratic rate with an increase in number of variables and constraints for the numerical test problems considered. These problems are from diverse sources and of varying complexity, providing substantial evidence of the strength of the method. Future work includes extending this method to solve mixed-integer multi-objective optimization problems and multi-objective robust optimization problems.

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