Density Filter for Control of Thickness-to-Length Change of Composite Structures

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ABSTRACT

To improve on the design and performance of composite structures we have formulated a composite structure optimization (CSO) method for fiber reinforced composite structures. The CSO method is a two phase optimization method that consists of two optimization problems that are performed in sequence. In the CSO method we are only considering unidirectional fiber reinforced composite materials with a fixed set of fiber orientations:

$$\Theta = \left\{0^\circ,\, +45^\circ,\, -45^\circ,\, 90^\circ\right\}. \tag{1}$$

In the first phase of the CSO method we are using a homogenized material optimization problem, which is based on a sizing optimization formulation, see [1]. For each finite element Ω_e we uniformly distribute the material properties associated with each fiber orientation $\theta \in \Theta$ across the thickness of the element, thereby achieving a homogenization of the material properties. The statement of the homogenized material optimization problem of the CSO method is given in (HMO), where the objective function is to minimize the compliance of the structure, $\mathbf{F}^T\mathbf{u}$. The design variable vector $\boldsymbol{\delta}$ contains the amounts of material associated with each fiber orientations $\theta \in \Theta$ for all elements Ω_e of the composite structure. Design constraints are the total volume of the composite structure V, and upper and lower bounds on the total amount of material for a given fiber orientation δ_e^θ, in an element Ω_e. We have implemented an optimality-criteria (OC) scheme, see [1], to solve the problem:

$$(HMO)\begin{cases} \min\limits_{\mathbf{u},\boldsymbol{\delta}} \mathbf{F}^T\mathbf{u} \\ s.t \quad \begin{cases} \mathbf{K}\left(\boldsymbol{\delta}\right)\mathbf{u} = \mathbf{F} \\ \sum\limits_{e=1}^{N_e}\sum\limits_{\theta} a_e\delta_e^\theta = V \\ \underline{\delta_e^\theta} \leq \delta_e^\theta \leq \overline{\delta_e^\theta} \end{cases} \end{cases}.$$

Here $\mathbf{K}\left(\boldsymbol{\delta}\right)$ is the stiffness matrix of the composite structure that depends on $\boldsymbol{\delta}$, $\mathbf{F}$ is the force vector, $\mathbf{u}$ the displacement vector and a_e is the area of a finite element Ω_e. (HMO) generates an optimal composite structure by means of finding the best distribution of material for a given fiber orientation $\theta \in \Theta$ across the entire design domain. To address a manufacturing constraint

that controls the thickness-to-length change of the composite structure, we have implemented a density filter in (HMO), see [2, 3]

$$\tilde{\delta_e^\theta} = \frac{\sum\limits_{i \in N_e} \omega\left(\mathbf{x}_i\right) \delta_i^\theta}{\sum\limits_{i \in N_e} \omega\left(\mathbf{x}_i\right)}, \tag{2}$$

where $\tilde{\delta_e^\theta}$ is the filtered version of the design variable δ_e^θ, N_e is the neighborhood of element Ω_e, δ_i^θ are the design variables belonging to N_e and $\omega\left(\mathbf{x}_i\right)$ is the weighting function given by a linearly decaying, cone-shaped function

$$\omega\left(\mathbf{x}_i\right) = R - \left\|\mathbf{x}_i - \mathbf{x}_e\right\|, \tag{3}$$

where R is the filter radius, $\mathbf{x}_i$ is the spatial (central) location of an element Ω_i belonging to the neighborhood N_e and $\mathbf{x}_e$ is the central location of element Ω_e . To investigate the influence of the density filter on the result of (HMO) we are going to perform simulations for different settings of the design constraints, mesh density and filter radius and by doing so, we find how these parameters affects the thickness-to-length change for the composite structure.

The second phase of the CSO method is a lay-up optimization problem based on a integer optimization formulation, see [4]. In the lay-up optimization problem we use as a starting point the design obtained from (HMO) to derive a manufacturable lay-up sequence of discrete composite plies for the entire composite structure.

References

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