MULTIPLIER METHODS FOR OPTIMIZATION PROBLEMS WITH LIPSCHITZIAN DERIVATIVES

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ABSTRACT

In this study we obtain sharp local convergence results for the augmented Lagrangian method and for the linearly constrained Lagrangian method. Both of these two methods gave rise to successful software. The augmented Lagrangian method is used by LANCELOT [7], ALGENCAN [1], and PENNON [9] software packages; while the linearly constrained Lagrangian method is the base of MINOS [8] and filterSD [4]. Nevertheless, the local convergence theory of these methods has had several drawbacks that we remedy in the study.

Specifically, we extend the sharpest known theorems about local convergence of the two methods [2, 6] to the case of problems with Lipschitzian derivatives dropping the assumption of twice differentiability. This extension has been highly desirable since none of the methods in question involve second derivatives in their design. In addition, this extension makes it possible to apply the augmented Lagrangian method and the linearly constrained Lagrangian method to the lifted reformulations of MPCC [5], which find several applications in engineering including optimal design of elastoplastic structures [3].

Concerning the linearly constrained Lagrangian method, we also improve the result from [6] with respect to the rate of convergence, replacing the superlinear convergence rate estimate by the quadratic one.

In our analysis, the augmented Lagrangian method and the linearly constrained Lagrangian method are viewed as special cases of an abstract Newtonian iterative framework that we develop and study first. We believe that this framework may serve as a convenient tool for local convergence analysis of many other algorithms as well.

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