Validation of models with correlated multiple responses

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Validation of models with correlated multivariate outputs involves the comparison of multiple quantities. Both univariate and marginal comparisons are incapable to incorporate the correlation information among multiple responses [1]. Besides, consistent conclusion is difficult to arrive by conducting univariate validation separately for each response. Therefore, an integrated validation metric is needed to characterize the overall mismatch between the multiple quantities of predictions and observations. As uncertainty inevitably exists in both the model predictions and experimental observations, the validation metric should also have the capability to capture the stochastic behavior of both the computational models and experimental data. Existing validation techniques for models with multiple responses either lack the capability to provide quantitative assessment of model accuracy or provide intuitive measures of model discrepancy. In this paper, an integrated multivariate area metric (IMAM) is proposed by extending the idea behind the single response area metric [2] to quantitatively assess the agreement between the multivariate uncertain quantities from both simulations and experiments. Using the multivariate CDF of the model responses and the multivariate empirical cumulative distribution (ECDF) of the observation data respectively, the proposed IMAM provides the measure of difference (distance) between the two distributions in the entire prediction space. By using methods of multivariate numerical integration, the overall agreement between the multiple correlated quantities of predictions and experimental data as well as the global predictive capability of the model over any specified domain of interest can be assessed. The proposed IMAM is easy to compute as it does not need transformation for non-normal data neither complex multivariate statistical analysis required by existing methods for multivariate model validations [1, 3]. The method inherits many good features of the single response area metric and can be used when predictions or experimental observations are sparse. Following the desired properties of validation metrics recommended by Oberkampf [4] and other authors [5, 6], the proposed metric is tested using illustrative examples. For comparison, two existing types of validation techniques for models with multiple responses, namely, the multivariate classical hypothesis testing and the multivariate Bayesian metric, are analyzed together with the IMAM to show their relative merits and disadvantages.

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