A Numerical Form Finding Method for Minimal Surface of Membrane Structure

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ABSTRACT

Membrane structures are extensively used for artifacts, in particularly for the roof of the structures which creates a big space such as stadium, mall and station. Membrane structures contribute to economy, safety, and aesthetic sense. Moreover it's becoming more practical structures due to the development of the advanced materials. A distinctive mechanical characteristic of membrane structures is that they keep their shapes only with in-plane tension stress. This makes themselves difficult to maintain their shape as required. Therefore the form-finding is highly important in the design process. For the membrane structure, equally tensioned surface is supposed to be ideal one. It's well known that it conforms to a minimal surface under a geometric boundary condition which has zero mean curvature throughout surface. The minimal surface can be found through the physical experiment using soap-film or through numerical methods using computer, which has become the mainstream. For instance, Bletzinger et al. proposed a form-finding method with mesh regularization for isotropic and anisotropic prestressed membranes.

In this paper, we propose a new and convenient numerical form-finding method based on the variational method for the minimal surface or the equally tensioned surface of the membrane structure with a specified arbitrary boundary. Area minimization problems are formulated in a function space. An internal volume or a perimeter is added as a constraint condition for pneumatic or suspension membrane structures, respectively. It is assumed that the membrane is varied in the normal and/or the tangential direction to the surface, and the thickness is constant during the deformation. The shape sensitivity function for this problem is theoretically derived using the material derivative method. The shape sensitivity function derived is applied to the pseudo-elastic shell surface as the traction force to vary the shape. This analysis is conducted under the elastically supported condition in order to restrain the rigid motion and stabilize the convergence. The displacements obtained as the optimum shape variation in this pseudo-elastic analysis are added to the original shape to update the shape. By repeatedly updating the reference shape, it converges to the optimal one while minimizing the objective function. Advantages of the method are as follows: (1) A free-form

surface can be obtained without shape parameterization which is inevitable process in general parametric shape optimization because all nodes can be moved as the design valuable. (2) A smooth and natural surface can be obtained because the elastic tensor in the pseudo-elastic analysis serves as a mapping function and as a smoother for maintaining mesh regularization smoothness, and its positive definitiveness is the necessary condition for minimizing the objective functional. (3) Regardless of the initial shape, it robustly converges to a final one. (4) Mesh smoothing is simultaneously implemented in the shape changing process. (5) It can be easily combining with a standard FEM code.

This method was applied to several examples involving suspension membrane structure, pneumatic membrane structure, and frame membrane structure. The results will show the effectiveness and practical utility of the proposed method for the form-finding of membrane structures.