

## Optimization of nonlinear structural vibrations using the incremental harmonic balance method

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### Abstract

We present an accurate and efficient method for optimizing steady-state nonlinear vibrations of mechanical structures subjected to time-periodic loads. The combination of the finite element method (FEM) and the incremental harmonic balance method (IHB) is used to compute the steady-state structural response including the effect of essential geometrical nonlinearities [1-2]. The finite element model facilitates a shape/topology design parametrization which we use in conjunction with a gradient-based topology optimization method [3].

We have currently studied the applicability and the efficiency of the proposed method by considering shape optimization of simple beam structures with the essential nonlinearity arising from midplane stretching. In our studies we consider a doubly clamped beam structure which is commonly encountered in microelectromechanical systems (MEMS) structures, e.g. for energy harvesting applications [4]. We use the beam width as design variable which is straightforward in the implementation of the optimization procedure and can easily be reproduced in a manufacturing process. We consider a number of design problems related to the nonlinear vibrational response: We optimize the width distribution in order to minimize the primary response at the first natural frequency and compare the designs obtained with linear and nonlinear models in order to stress the importance of including nonlinearities in the optimization procedure. Furthermore, we maximize super-harmonic resonances and demonstrate that these higher-order resonances can be amplified significantly by using an optimized beam width distribution. Finally, we show how it is possible to tune the softening/hardening nonlinearity by optimizing the backbone of the primary resonance, also known as a frequency-amplitude representation of nonlinear normal mode.

We are currently extending the application areas to deal with shape optimization of complex beam frame structures by implementing a geometrically exact beam theory and we further plan to extend the work to topology optimization of 2D continuum structures with a nonlinear Green-Lagrange strain measure in the near future.

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