10th World Congress on Structural and Multidisciplinary Optimization
May 19 - 24, 2013, Orlando, Florida, USA

A new Second Order Reliability Method and its integration in Reliability Based Design Optimization

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1. Abstract

Research in the field of reliability based design is mainly focused on two sub-areas: the computation of the probability of failure and its integration in the reliability based design optimization (RBDO) loop. Typically, new methods for the computation of the probability of failure improve both accuracy and efficiency, and the methods in optimization investigate approximations that aim at decoupling or eliminating the double loop arising in probabilistic optimization. Two methods are presented in this work, representing a contribution to each sub-area.

In the first method, a new Second Order Reliability Method (SORM) is presented. As opposed to the most commonly used SORM methods, the presented approach is not limited to hyper-parabolic approximation of the performance function at the Most Probable Point (MPP). Instead, a full quadratic fit is used leading to a better approximation of the real performance function and therefore more accurate values of the probability of failure. Furthermore, the presented formulation is free of singularities and is generally more accurate than the existing methods, even for the hyper-parabolic case. Also, the method is more computationally efficient since it does not depend on the principal curvature at the MPP and therefore do not involve a transformation of the Hessian in a rotated space with a Gram-Schmidt orthogonalization, as does the most widely used SORM methods. The method is based on an asymptotic expansion of the sum of non-central chi-square variables taken from the literature, which applied to the current case yields an easy-to-use and a computationally efficient closed form expression for the probability of failure.

The second method focuses on the integration of the above described formulation in RBDO. One important feature of the presented expression for the probability of failure is that, for the special case of quadratic performance functions in normal variables, it does not involve locating the MPP. Therefore, if a quadratic surrogate model is valid in the whole region of interest, the use of the proposed expression reduces the RBDO problem to a single loop with efficiency comparable to a deterministic optimization. This is not the case for general non-quadratic problems in normal variables. Therefore, a sequential single loop method is presented. Quadratic surrogate models are fitted at the mean value point and probabilistic constraints are constructed using the proposed expression. The optimization problem is thereafter solved in a single loop, i.e. without any MPP search. In the next cycle, the surrogate model is updated around the found solution using a zoom-and-pan strategy and the RBDO problem is solved again. The process is repeated until convergence which is typically achieved after 3 updates of the surrogate model. Furthermore, the method can easily handle the case of varying variance of design variables using the same efficient formulation.