High reliability estimation using $CVaR^+$

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Value at Risk (VaR) is the name used for inverse measure in finance engineering. Value at risk is essentially quantiles. Researchers have used VaR extensively in reliability estimation. However, VaR becomes volatile in the tail regions. This volatility leads to error in predictions for unknown reliability levels. Some works have reported using the concept of Conditional - VaR (CVaR) which has many advantages over VaR, for conservative reliability estimation. CVaR is a weighted sum of $CVaR^+$ and VaR. $CVaR^+$ is the mean responses strictly exceeding VaR and is sometimes referred to as upper CVaR. $CVaR^+$ is also referred to as buffer probability in structural reliability theory. It is observed that $CVaR^+$ is less volatile in tail portion and is smoother than the VaR and CVaR though conservative from both, in a failure prediction perspective. This work utilizes the fact that $CVaR^+$ is well behaved and smooth, to fit the relationship with its Cumulative Density Function (CDF).

Current status:

Using random scarce samples, VaR and $CVaR^+$ are computed. One can fit a distribution to a well behaved sample like $CVaR^+$ with more confidence than to VaR. The tail part of relationship between $CVaR^+$ and the empirical CDF is approximated using a Generalized Pareto Distribution (GPD). Once the parameters of the GPD are known, estimating the $CVaR^+$ at a lower failure probability is straight forward. The failure probability axis is converted to a reliability index axis and its relationship with the $CVaR^+$ is approximated using a family of polynomials. Once the coefficients of the polynomials are known, computing the $CVaR^+$ for a lower failure probability is straight forward. The extrapolated results for a set of true distributions like the normal, lognormal, exponential and Rayleigh are better compared to predicting VaR using the same procedures. The different distributions cover all the possible types of tails. Prediction at higher reliability indices are compared using box plots of the estimations. The variation of $CVaR^+$ is much lesser than that of VaR. However, the $CVaR^+$ is a conservative estimate compared to VaR. Here, we model the relationship between the VaR and $CVaR^+$ for the native scarce samples and try to extrapolate their relationship as well. In the true distributions tested, the difference is linear allowing a simple addition to the predicted $CVaR^+$ to obtain the VaR. The VaR computed by this approach is stable and closer to the actual VaR compared to the VaR predicted from approximating the relationship between VaR and its CDF.

Future Work:

The current work has developed methods of fitting $CVaR^+$ tails to predict high reliability better and it has been demonstrated on true distributions. The final paper will demonstrate the approach on practical engineering examples. The examples would include classical bench mark problems where traditional approaches will perform poorly. In addition to the reliability predictions, the paper will attempt to develop bounds around the predictions using bootstrap methods.

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