Convex Conic Optimization of Eigenvalue Bandgap Problems, with an Application to Bandgap Structures Design

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Abstract

Optimization involving eigenvalues bandgaps is an important class of problems that arise often in engineering design, but are often non-convex, as they require the maximization of the higher indexed eigenvalue and the minimization of the lower indexed eigenvalue. A novel algorithm [1, 2] has been previously developed to reformulate the original nonlinear, non-convex optimization problem to an iteration-specific fractional semidefinite program (SDP). This algorithm separates two consecutive eigenvalues – effectively, maximizing bandgap/bandwidth – by separating the gap between two orthogonal subspaces, which are comprised columnwise of "important" eigenvectors associated with the eigenvalues being bounded. By doing so, the computation of eigenvalue gradient can be efficiently approximated by the computation of gradient of affine matrices with respect to the decision variables. In this work, we propose an even more efficient algorithm based on fractional linear program (LP). The new formulation is obtained via approximation of the semidefinite cones by judiciously chosen linear bases, coupled with "delayed constraint generation". We apply the two convex conic optimization methods, namely, fractional SDP and fractional LP, to solve the bandgap optimization problems of both phononic crystals and photonic crystals. By comparing the two methods, we demonstrate the efficacy and efficiency of the LP-based algorithm in solving the category of eigenvalue bandgap optimization problems. We also suggest the applicability and necessity of the LP-based algorithm in our concurrent work on fabrication-adaptive optimization.

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