Design Optimization under Aleatory and Epistemic Uncertainty

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Abstract

Design optimization under aleatory uncertainty is a well-researched topic in the literature. However, when epistemic uncertainty is considered, the existing methods address only parts of the entire problem scope. This paper considers epistemic uncertainty due to both data sources and model errors. In uncertainty propagation analysis, model error is usually quantified at a particular value of inputs, and this process can be computationally expensive in itself. The design optimization setting however poses additional challenges, because each iterate of the optimization represents a different realization of input for uncertainty propagation, and the model errors are therefore potentially different at different regions of the design space.

This paper develops a design optimization methodology that includes three sources of uncertainty: physical variability (aleatory); data uncertainty (epistemic) due to sparse or imprecise data; and model uncertainty (epistemic) due to modeling errors/approximations. A Bayesian approach is used to combine multiple formats of information and to probabilistically quantify these uncertainties using non-parametric probability density functions (PDF).

The constraint and objective functions are assumed to be available only through computationally expensive simulation models. The computationally expensive physics model is replaced by a

Gaussian process (GP) surrogate model for the sake of computational efficiency. Two types of model errors are considered: model form error and numerical solution error, each of which is a function of the design variable that are changing at each iteration of the optimization. Model form error is quantified based on validation data. Two types of numerical solution error are quantified: (1) Discretization error, and (2) surrogate model error. Gaussian process surrogate models are also constructed for efficient quantification of discretization error using Richardson extrapolation. This treatment yields a probability distribution of the output that accounts for various sources of uncertainty. The use of a probabilistic approach to include both aleatory and epistemic uncertainties allows for their efficient integration into the optimization framework. The contributions of this paper can be summarized as follows:

1. Development of a probabilistic framework to include natural variability, data uncertainty and model uncertainty within the design optimization problem

2. Quantification of various types of model errors as functions of design variable values including model form error, discretization error, and surrogate model error

3. Uncertainty quantification in model output, through probability distributions, due to variability, data uncertainty, and model errors.

The proposed methods are illustrated using design optimization of a three-dimensional aircraft wing using fluid-structure interaction analysis.