

## Ensemble of Unified Reliability Formulations (EURF)

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### 1. Abstract

Various Reliability-Based Design Optimization (RBDO) methods have been developed and widely used to solve design optimization problems with the existence of design uncertainties. The general problem formulation states that the objective function is minimized while the failure probabilities of the performance constraints are subjected to the allowable probability levels. RBDO algorithms derive and formulate various approximate probabilistic constraints with respect to the means of the randomly distributed design variables in replace of the evaluations of failure probabilities using the integral of joint probability density functions. However, there is a huge diversity of approximate probabilistic formulations from various RBDO algorithms. The goodness of fit of each approximate model is problem dependent but highly affects the accuracy and efficiency of the optimization process. In this paper, a Unified Reliability Formulation (URF) is derived from the fundamental aspect of the linear expansion with allowable reliability level to provide a general category of first-order RBDO methods. The URF is determined by the linear expansion at an Allowable Reliability Point (ARP) with the sensitivity analysis associated with a Gradient-based Transformation Point (GTP). The reliability of the ARP is exactly equal to the allowable probability. The GTP is the chosen expansion point of the approximate probabilistic constraint in each RBDO algorithm. The derived URF not only provides a comprehensive understanding of approximate probabilistic constraint but also an insightful acknowledgment of how various RBDO algorithms can be unified into one general equation. For instance, the various formats of the URFs for the existing RBDO algorithms are demonstrated. The accuracy of each URF depends on the evaluations at the GTPs. Therefore, an Ensemble of Unified Reliability Formulations (EURF) is formed to group together the approximate probabilistic formulations from various RBDO algorithms. The intersection of the URFs from each RBDO algorithm is considered when the limit state is a convex function; on the other hand, the union of the URFs is considered for the concave limit state function. EURF covers a wider range of reliability analyses than any individual method. The benchmark examples show that the EURF requires fewer iteration to finds the optimal solutions than either RIA or PMA when dealing with highly nonlinear constraints.

**2. Keywords:** ensemble of probabilistic constraints; Unified Reliability Formulation (URF); reliability analysis; Allowable Reliability Point (ARP); Gradient-based Transformation Point (GTP); Chance Constrained Programming (CCP); Reliability Index Approach (RIA); Performance Measure Approach (PMA).