Efficient Optimization of Nonlinear Dynamic Systems Under Uncertainty Using Parallel Decomposition

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Optimization of dynamic systems has proven to be an effective method for improving design and operation in many industries. However, optimization of large-scale discretized systems can be challenging to perform efficiently, especially as we increase model rigor, system size and complexity, and the need to consider uncertainty in the design optimization. The increasing size of these optimization problems continues to outpace the capability of serial solution approaches to solve these problems on modern desktop computers, which drives the development of efficient parallel solution algorithms.

Large-scale optimization problems are almost always inherently structured, and we can exploit this structure in the development of efficient parallel algorithms. For example, optimal design of dynamic systems under uncertainty has induced structure due to both the discretization of the model, and the multi-scenario nature of the problem. Parallel solution of these types of problems can be arranged into two groups: problem level decomposition, which includes techniques like Bender's decomposition or Lagrangian Relaxation, and internal decomposition, which focuses on parallelizing the linear algebra of existing nonlinear programming algorithms. Our work focuses on the development of internal decomposition strategies built on nonlinear interior-point methods.

In this work, we focus on two levels of structure induced when considering optimal design of dynamic systems under uncertainty. First, we reformulate the dynamic optimization problem using the simultaneous discretization approach, discretizing all the variables and including the discretized model as constraints in a large-scale optimization problem. This dramatically increases the overall size of the problem, however, improved performance may be possible since the differential equation model is solved simultaneously with the optimization problem. The discretization induces the first level of structure in the problem. When considering uncertainty, it is common to formulate a multi-scenario optimization problem, seeking a single design that is feasible over all the scenarios while being optimal in some statistical measure over the scenarios. This multi-scenario formulation induces a second level of structure within the optimization problem.

Nonlinear interior-point methods provide a framework for efficient solution of these problems since the linear systems that are solved to find the step maintain a consistent structure from iteration to iteration. The structure of the KKT system can be exploited to allow for efficient solution in parallel. Here, a Schur-complement decomposition is used to exploit the structure of the KKT system, decouple individual blocks, and allow for efficient solution in parallel. In this presentation, we will discuss our parallel algorithm and show parallel timing results from several optimization problems. Additionally, we will discuss current research into the use of iterative linear solvers for solution of the Schur-complement system that avoid the expense of explicitly forming and solving the Schur-complement.