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Optimization of Bolt Stress

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1. Abstract
The state of stress in bolts and nuts with ISO metric thread design is examined and optimized. The assumed failure mode is fatigue so the applied preload and the load amplitude together with the stress concentrations define the connection strength. Maximum stress in the bolt is found at, the fillet under the head, at the thread start or at the thread root. To minimize the stress concentration shape optimization is applied.
2. Keywords: Metric ISO thread, Design, Stress concentration, Optimization, Contact, FE

3. Introduction
Bolted connections play a key role in the function of many practical designs. Improvement in fatigue life of bolts can be achieved in three principally different ways

- Improving the joint stiffness factor by minimizing the bolt stiffness or/and maximizing the clamped material stiffness
- Improving the load distribution along the thread, by design changes made to the nut
- Minimizing the stress concentration factor in the bolt by applying shape optimization to the bolt design

The first bullet is a matter of practical design and selecting appropriate bolt nominal diameter relative to the thickness of the material clamped between the bolt head and the nut. The second bullet deals with the practical problem that for a traditional thread design the load is not evenly distributed along the thread, and the first turn of the thread can take up to 50% of the total load. The final bullet on minimization of stress concentration factor possesses the largest potential for overall maximum stress level reduction.

For most bolts and nuts the central shape at the stress concentration is an arc of a circle which is the general choice in the design of machine elements. This shape is probably selected due to the simple parameterization or due to the ease of manufacturing. From shape optimization in general or specifically in relation to machine elements it is known that the circular arc shape seldom is optimal.

Designing a nut which results in a more evenly distribution of load along the engaged thread has a limited influence on the maximum stress due to the stress concentration at the first thread root. To further reduce the maximum stress the transition from bolt shank to the thread must be optimized. Stress reduction of up to 34% is found, still with the standard ISO thread. The design changes suggested in the paper also has the positive advantage of reducing the joint stiffness factor. The reduction in the bolt shank directly reduce the bolt stiffness and the design change to the bolt head and the nut has the positive indirect effect of increasing the member stiffness, all leading to a smaller joint stiffness factor (see Section 6).

4. Stress concentration
The theoretical (geometric) stress concentration, K_t, is usually defined as

$$K_t = \frac{\sigma_{\max}}{\sigma_{\mathrm{s}}} \tag{1}$$

where $\sigma_{\max}$ is the maximum von Mises stress in the bolt and σ_s is a nominal stress. Typically the area associated with the nominal stress is defined as

$$A_s = \frac{\pi}{4}\left(\frac{d_p + d_c}{2}\right)^2 \tag{2}$$

where d_p is the pitch diameter and d_c is the core diameter. The area A_s is referred to as the stress area of a bolt. The nominal stress is then taken as

$$\sigma_s = \frac{F}{A_s} \tag{3}$$

here F is the axial force on the bolt. The amplitude of this force is controlled by the size of the external force and the joint stiffness factor. The subscript t on the stress concentration factor indicates that it is a theoretical stress concentration factor where only geometry, boundary conditions and loading conditions are taken into account, no reduction due to material sensitivity is included.

Alternatively the stress concentration factor can be defined relative to the gross cross sectional area

$$K_{tg} = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}} \tag{4}$$

$$\sigma_{\text{nom}} = \frac{F}{\frac{\pi}{4} d_{\text{nom}}^2} \tag{5}$$

here d_{nom} is the nominal bolt diameter, i.e., $d = 12$mm for an ISO M12 bolt etc.

The most complete reference to published stress concentration data is probably [1], with respect to the average values for standard bolt-nut connections these can be found in many textbooks, see e.g. [2].

5. T-head and nut optimization

In [3] and [4] different nut designs for improving the load distribution along the thread is presented, inspired from these designs the fillet parameterization shown in Figure 1 is investigated.

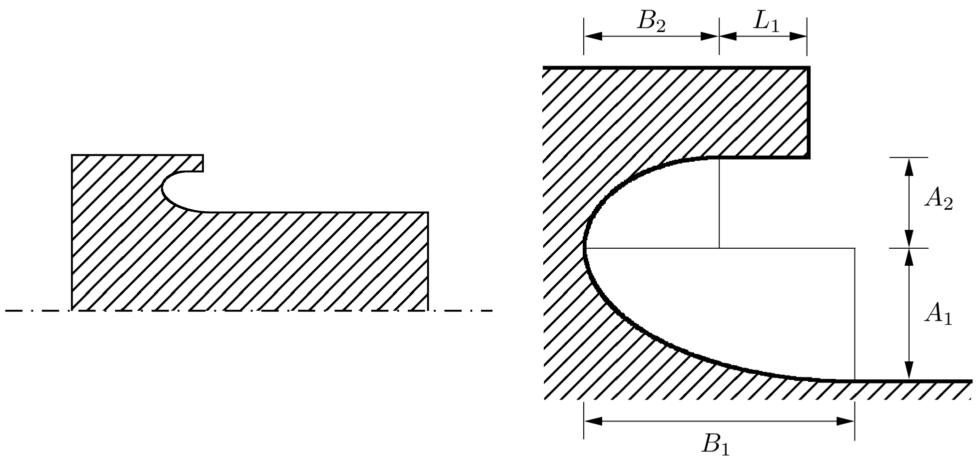

Figure 1: Applied parameterization of bolt head fillet shape.

The shape is parameterized by seven design parameters; four half axis of the two elliptical shapes, A_1, B_1, A_2 and B_2, The straight shoulder length L_1 and finally two super elliptical powers η_1 and η_2 (not visible in Figure 1). The super elliptic shape is in parametric form given as

$$X = A\cos(\theta)^{(2/\eta)}, \qquad \theta \in [0 : \frac{\pi}{2}] \tag{6}$$

$$Y = B\sin(\theta)^{(2/\eta)}, \qquad \theta \in [0 : \frac{\pi}{2}] \tag{7}$$

The shape is optimized by a parametric study. The found optimized T-head fillet shape is shown in Figure 2, where the stress variation is also shown. The shown design corresponds to the design variables; $A_1 = 1.6$mm, $B_1 = 2.0$mm, $\eta_1 = 1.54$ $A_2 = 0.3$mm, $B_2 = 1.8$mm, $\eta_2 = 1.94$ and $L_1 = 0.5$mm.

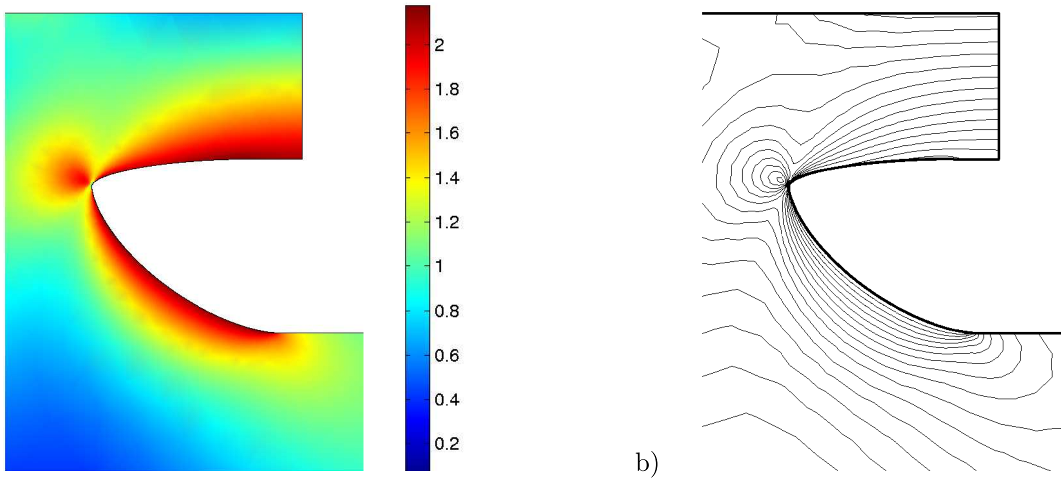

Figure 2: a) Normalized von Mises stress surface colour plot. b) Von Mises stress contour plot.

We expect that the shape is optimized with respect to minimizing maximum stress if the stress is constant along major boundary parts, see [5]. The limitation to the boundary having constant stress is given by the boundary constraints and the possibilities of the parameterization. A parameterization must therefore be sufficiently flexible to be able to return optimal designs. In Figure 3 the von Mises stress along the designed shape is presented. The maximum stress concentration factor $K_{tg} = 2.18$ is shown as the dotted line. It is clearly seen that the stress is overall constant along the shape. The stress has been reduce by 25.3% compared to the best design given by the standard.

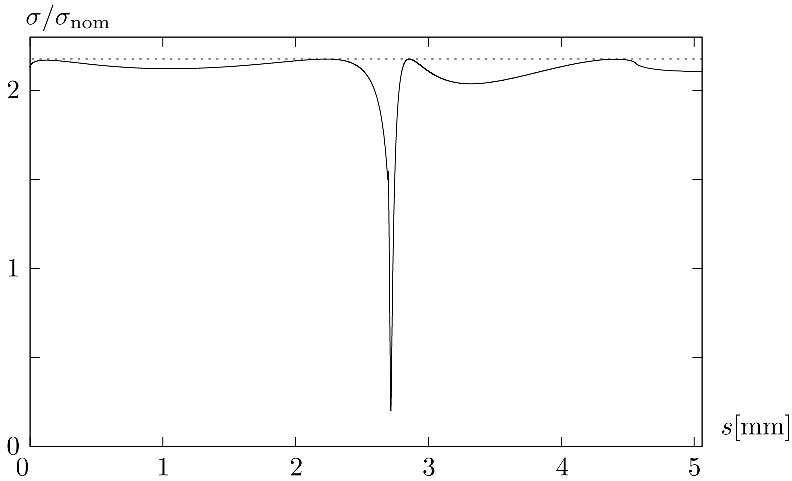

Figure 3: Normalized von Mises stress as a function of the arc length along the designed shape.

6. Overall bolt optimization

Standards exist for bolts with reduced diameter, [6], and with a thread groove, [7]. The primary argument for using bolts with a reduce shank is the bolt stiffness reduction relative to the member stiffness. The ratio of an externally load amplitude that goes into the bolt is controlled by the ratio, Φ, (joint stiffness factor)

$$\Phi = \frac{k_b}{k_b + k_m} \tag{8}$$

where k_b is the bolt stiffness and k_m the member stiffness. As discussed in [8] the bolt stiffness reduction therefore has a positive effect on the fatigue life of bolts. Another positive effect of a thread groove or a reduced shank is the positive effect it has on the bolt/nut connection stress level. To the author's knowledge this has not been given specific attention. In [9] the stress concentration factor for bolts with reduced shank is discussed. The findings here are however too optimistic, they report a stress concentration factor of $K = 1.1$ if the shank diameter is reduced to $d = 0.25d_{\text{nom}}$. In order to compare different designs the same load must be applied and a reduction of the shank diameter to $d = 0.25d_{\text{nom}}$ would give a stress concentration in the shank of $K_{tg} = 16$ based alone on the change in diameter size.

To improve the standard design the bolt shank fillet shape to thread transition is optimized. A simple parameterization with only three design variables is used, facilitating a parametric study. The central shape is the super ellipse.

Applying a super elliptic full quarter shape would not make the bolt outer slope contour continuous. A distortion is applied to achieve design domain continuity. This is done by rotating the line as indicated by the hollow arrows in Figure 4. Rotating the line by an angle of $\pi/6$ the parameterization attains the wanted property.

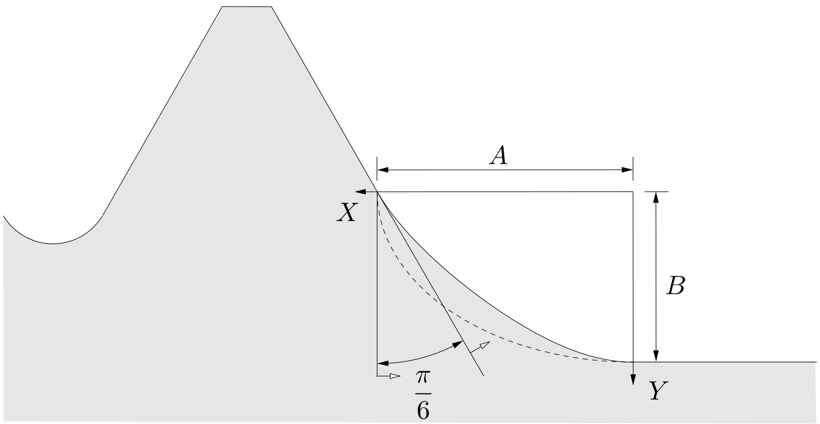

Figure 4: Illustration of fillet design by distortion super ellipse. The original ellipse is shown by the dash line, the distorted is shown by the full line.

The same basic idea of a distorted super ellipse is used in [10] and [11]. The full parameterization is given by

$$X = A\cos(\theta)^{(2/\eta)}(1 - \frac{B}{A}\sin(\theta)^{(2/\eta)}\tan(\frac{\pi}{6})), \qquad \theta \in [0 : \frac{\pi}{2}] \tag{9}$$

$$Y = B\sin(\theta)^{(2/\eta)}, \qquad \theta \in [0 : \frac{\pi}{2}] \tag{10}$$

This is a rather simple analytical root shape parameterization. The only constraint on the design parameters is

$$A \geqq B\tan(\frac{\pi}{6}) \tag{11}$$

The three design variables facilitate a simple parameter study to minimize the maximum stress.

The maximum stress in the bolt by pure tension is rather insensitive to design variations of the fillet because the maximum stress is controled by the thread design and not the fillet design. The stress concentration is as for the DIN2510 design $K_{tg} = 3.43$. The real bolt however has to interact with a nut. The case of pure tension can be seen as the limit to what design improvements to expect.

The final optimization is therefore performed with bolt/nut interaction and thread contact modelling. The design parameters used is the three parameters defining the bolt shank fillet.

The optimal value is found for $A = 2.5$mm, $B = 1.4$mm and $\eta = 2.6$. The stress concentration factor found is $K_{tg} = 3.624$, i.e. 34% lower compared to the maximum stress of the original design. The normalized von Mises stress along the first six thread roots are shown in Figure 5.

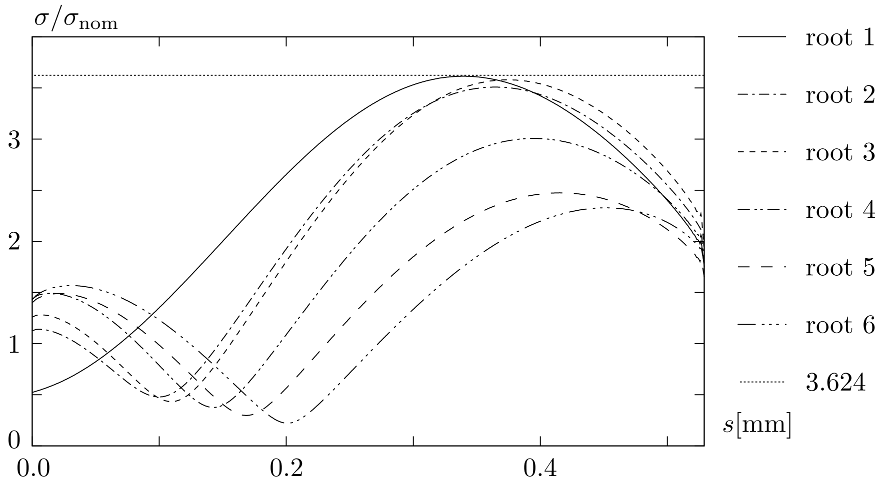

Figure 5: Normalized von Mises stress as a function of the arc length, shown for the first 6 thread roots. The maximum value $K_{tg} = 3.624$ is indicated by the dotted line.

From the stress level it is clear that the individual stress distributions are not optimal because we expect the optimal shape to have constant stress along the shape. For the specific example shown here the optimization has an influences on the stress level for multiple shapes at the same time, and we notice that the maximum stress in the three individual boundaries are almost identical, thereby illustrating that constant stress should be understood in a more general sense. From Figure 5 it is seen that the load is distributed rather evenly especially for the first three thread roots. If further improvements should be achieved then a more involved parameterization should be used, and the three boundary shapes should be designed simultaneously. With the thread root kept constant, the limit to this improvements is a stress minimization of 37.5%, so it may be discussed if further stress optimization is necessary.

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