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Optimum Design of Impact Resistance of Laminated Glass Plate

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1. Abstract

Laminated glass is widely used to enhance the structural functions such as safety, security and automobile applications. Generally, the laminated glass plate consists of two or more layers of float glass sheets combined by adhesive interlayer of tough Polyvinyl Butyral (PVB) under heat treatment. The impact resistance of this laminated glass plate is higher than that of a single glass plate of same thickness in total. After the glass plies are broken, the fragments of the glass are kept together by the PVB interlayer. The impact fracture behavior of the laminated glass is more complicated than that of the single glass, because of the combined influence of the large deformation and delamination strength so that two float glasses and the interlayer constitute the brittleness and hyper-elasticity, simultaneously. By using a finite element analysis code for the continuum, it is difficult to simulate the impact fracture behavior from a crack growth of the glass to the failure of the interlayer for the laminated plate.

In this study, the impact fracture behavior of a laminated glass plate for the outside surface of modern buildings has been studied by the numerical simulations and the experiments. The 3-D Discrete Element Method (DEM) was adopted in this numerical simulation. From the results of the simulation, the entire failure processes are predicted in detail. The impact loads and deformations of laminated plates were observed to be in good agreement between the experiments and the DEM simulations. Additionally, the laminated glass structures have been optimized for attaining the maximum durability against the impact fracture based on the response surface approach. The tensile strength of the interlayer and the adhesive strength between glasses and interlayer are taken as the design variables. From the results of optimization it has been observed that the laminated glass was hard to be broken in the case that the tensile strength was tough and that the adhesive strength was a little light. The optimum structure in high resistance of the impact fracture has been obtained for the PVB interlayer. The validity of the optimum design was confirmed by the verification analysis.

2. Keywords: Optimum design, Laminated glass, Impact fracture behavior, Discrete element method, Response surface methodology

3. Introduction

Laminated glass is widely used to enhance the structural functions such as safety, security and automobile applications. The laminated glass plate consists of two or more layers of float glass sheets combined by an adhesive interlayer of tough Polyvinyl Butyral (PVB) under heat treatment. It was noticed that several types of the laminated glass are manufactured, using different types of the float glass and interlayer. The aim of the laminated glass is to prevent the splinters from flying away and from injuring people.

The impact resistance of the laminated glass plate is higher than that of a single glass plate of the same total thickness. After the glass plies are broken, the fragments of the glass are kept together by the PVB interlayer. The impact fracture behavior of the laminated glass is more complicated than that of the single glass, because of the combined influence of the large deformation and delamination strength so that two float glasses and the interlayer constitute the brittleness and hyper-elasticity, simultaneously. For examples, the experiments and calculations with the failure criteria of conventional glass have been shown by previous authors [1-4]. Also, many scholars have tried to solve on the failure of the simple and laminated glasses by using numerical methods [5-8]. However, it is difficult to simulate the impact fracture behavior in detail from a crack growth of the glass to the failure of the interlayer for the laminated plate using a finite element analysis (FEA) for the continuum.

In this study, the impact fracture behavior of a laminated glass plate for the outside surface of modern buildings has been studied by the numerical simulations and the experiments, where a Discrete Element Method (DEM) based on non-continuum mechanics was adopted as the numerical simulation. Because the dynamic behavior and failure processes in the laminated glass occurred in the material formation and structure from the continuum to non-continuum. DEM is one of a numerical simulation method to propose firstly by Cundall [9], it was used to solve the non- continuum problems such as geology engineering. Since over 40 years ago, the application region of

DEM has expanded the field of continuum mechanics. At present, there are many successful papers to simulate the transition process from the continuum to non-continuum [10-12]. In this study, the entire failure processes in laminated glass are predicted in detail by DEM. The results in DEM were confirmed by the experiments.
In addition, the laminated glass structures have been optimized for attaining the maximum durability against the impact fracture based on the response surface approach. In this optimum problem, the tensile strength of the interlayer and the adhesive strength between two glasses and the interlayer are taken as the design variables. From the results of optimization it has been observed that the laminated glass was hard to be broken in the case that the tensile strength was tough and that the adhesive strength was a little light. The optimum structure in the high resistance of the impact fracture has been obtained for the PVB interlayer. The validity of the laminated glass in the optimum design was confirmed by the verification analysis.

4. Impact fracture by DEM simulation

4.1. DEM theory

In DEM simulation, there are more rigid sphere elements. As shown in Fig.1, the external force F and rotational moment M acting between both elements (m is element mass) are given by Newton's second law. The translational displacement of the center in element is u and the angular displacement in element is ϕ at a time $t,$ the values are expressed in the following formulas:

$$m[\ddot{u}]_t + \eta[\dot{u}]_t + K[u]_t = f(t) \tag{1}$$

$$I[\ddot{\phi}]_t + \eta r^2[\dot{\phi}]_t + Kr^2[\phi]_t = g(t) \tag{2}$$

where η, K, I and r are the viscosity coefficient, elastic modulus, moment of inertia and radius in the element, respectively. In the formulas, h and K compose a dashpot and spring between all contact elements. The dynamic behavior of an analysis object can be expressed by untying the simultaneous equation of all elements.

4.2. Analysis condition in DEM simulation

In DEM simulation, the analysis model is a quarter for a symmetric geometry as shown in Fig. 2. The radius of the glass elements in the top and bottom layers is 0.84mm. The radius of the PVB elements in the interlayer is 0.42mm. The ratio of the element diameter of the glass and PVB is 2:1. The density of the impact striker is 1.028kg, because it is a quarter of the symmetric geometry in the analysis model. The initial velocity 7.27m/s (26.2km/h) (to fall over the height: 3m) was given in the impact striker with laminated glass. After the collision to the striker and glass, the

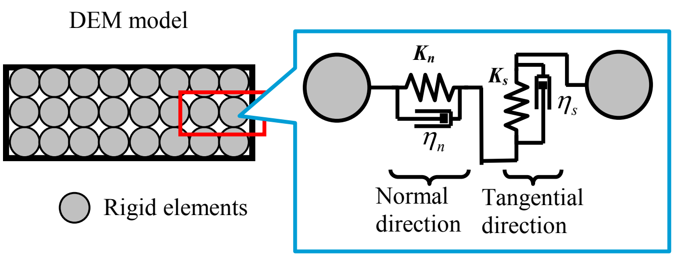


Figure 1: Geometry of two particles in DEM model

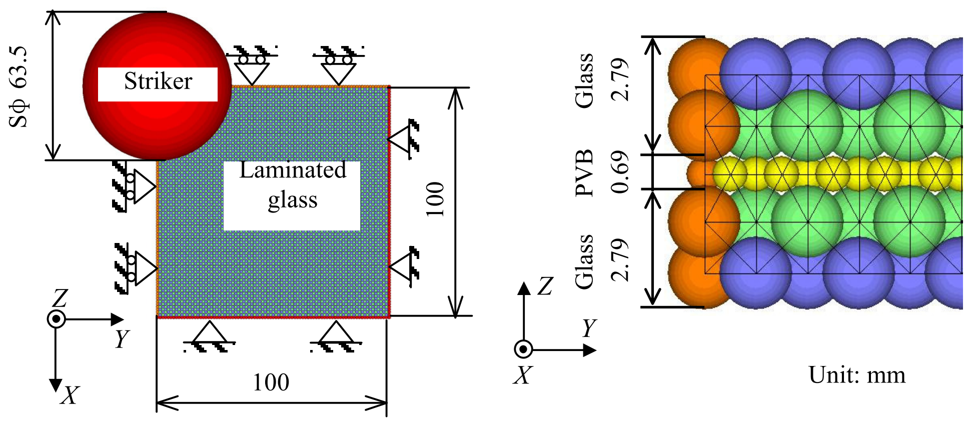


Figure 2: DEM model of laminated glass and impact striker

Table 1:Material properties of glass, interlayer (PVB) and impact striker

	Glass	Interlayer (PVB)	Impact striker
Density, ρ [kg/m^3]	2500	928	1950
Young's modulus, E [GPa]	61.35	0.5	210.8
Poisson's ratio, ν [-]	0.225	0.42	0.29
Compressive strength, σ_c [MPa]	430	-	-
Tensile strength, σ_t [MPa]	43.05	4 - 110	-
Shearing strength, σ_s [MPa]	(129.2)	(2.3 - 63.5)	-
Adhesive strength, σ_a [MPa]	-	1.2 - 60	-
Coefficient of friction, μ [-]		0.1	

dynamic behavior was simulated at a period from 0 to 10ms.
The material properties of the glass, interlayer and impact striker are shown in Table 1, respectively. In this DEM, the tensile strength σ_t and adhesive strength σ_a were simulated as a variable, where the value of the shearing strength σ_s is calculated by the tensile strength in the following equation:

$$\sigma_s = \frac{\sigma_t}{\sqrt{3}} \tag{3}$$

Generally, Young's modulus of the PVB is related to the strain velocity. In the simulation, it is used the value of Young's modulus is used considering the strain velocity of the PVB in the previous study [12]. Also, the coefficient of friction in the glass and the impact striker was fixed at 0.1 in the previous study [11].

4.3. DEM simulation results
One of the results of the fracture simulation in the laminated glass by DEM, it was simulated that the tensile strength σ_t of the interlayer (PVB) and the adhesive strength σ_a (between glass and PVB) were changed, because the effect of both value is big in the fracture destruction of laminated glass.
The adhesive strength is σ_a =90MPa under the constant condition, and it was simulated to change only the tensile strength σ_t. The result of the time-history in the striker velocity is shown in Fig. 3. It was understood that the tensile strength of the interlayer (PVB) is easy to be penetrated with little strength.
Next, the tensile strength is σ_t =90MPa under the constant condition, and it was simulated to change only the adhesive strength σ_a. Figure 4 shows the result of the time-history in the striker velocity. In cases of more than σ_a =60MPa, the fracture behavior did not obtain change. In the result, it was understood that the adhesive strength of the glass and PVB is hard to be penetrated with little strength. However, it was observed that the glass and PVB peeled in the cases when the adhesive strength was too little.

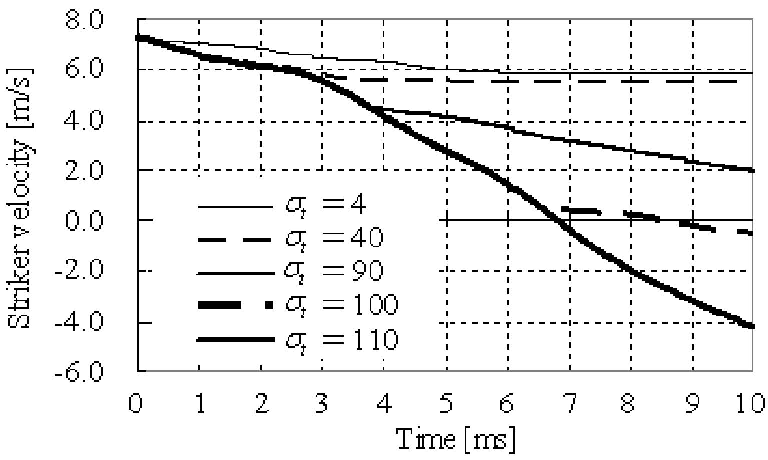


Figure 3: Time-history of impact striker velocity under constant adhesive strength σ_a =90MPa

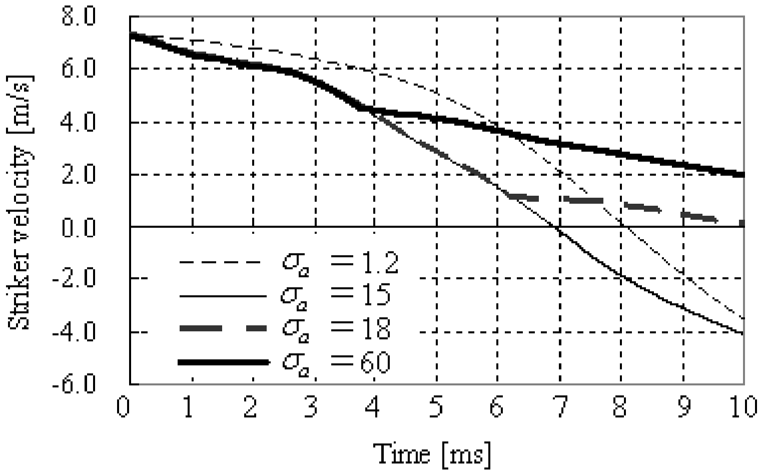
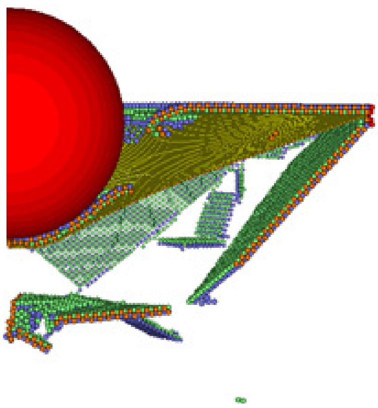
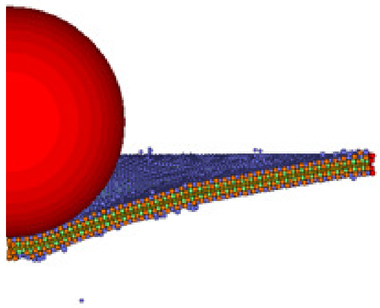


Figure 4: Time-history of impact striker velocity under constant tensile strength is σ_t=90MPa



(a) σ_t=90MPa, σ_a=1.2MPa



(b) σ_t=90MPa, σ_a=15MPa

Figure 5: Dynamic fracture state of laminated glass by DEM simulation at 10ms

From a collision in the impact striker at 10ms, the result of the fracture simulation in the laminated glass by DEM is shown in Fig. 5. Figure 5(a) shows the tensile strength and adhesive strength are resulted in σ_t=90MPa and σ_a =1.2MPa, respectively. Also, Fig. 5(b) shows the result in σ_t=90MPa and σ_a=15MPa.
In Fig. 5(a), it is supposed that the glass and interlayer (PVB) are delaminated when the adhesive strength is little (σ_a=1.2MPa). So, the adhesive strength σ_a is required to be more than this value.
From results in DEM simulation, a laminated glass performs high penetration resistance when the tensile strength of PVB is high. However, the adhesive strength between the glass and PVB is needed to have a value in some degree. Thus, it is thought that there is optimal value in the adhesive strength.

5. Impact fracture test and comparison with DEM simulation

The impact fracture experiment was tried under the same conditions as in DEM simulation. Figure 6 shows the test equipment. The fracture behavior of the laminated glass was recorded by a high speed camera (made by Nac Co., Ltd.). One of the images which filmed the dynamic fracture of the laminated glass is shown in Fig. 7. It was understood that the laminated glass is not able to be penetrated, because the impact striker is reflected by the glass. In the impact fracture experiment and DEM simulation, the time-history of the reaction force which occurred on the fixed edge in the laminated glass is shown in Fig. 8.
In comparison with the experiment and DEM, the experiment values are occurred a delay in progress time of the peak force. Also, the peak force of both values is different a little. The force-time diagram is almost similar. It is thought that DEM simulation can almost reproduce the dynamic fracture behavior of the laminated glass.

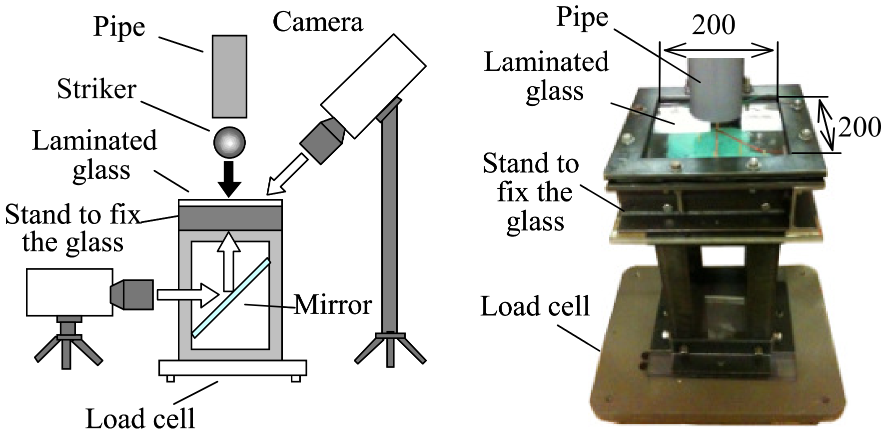


Figure 6: Impact fracture equipment of laminated glass

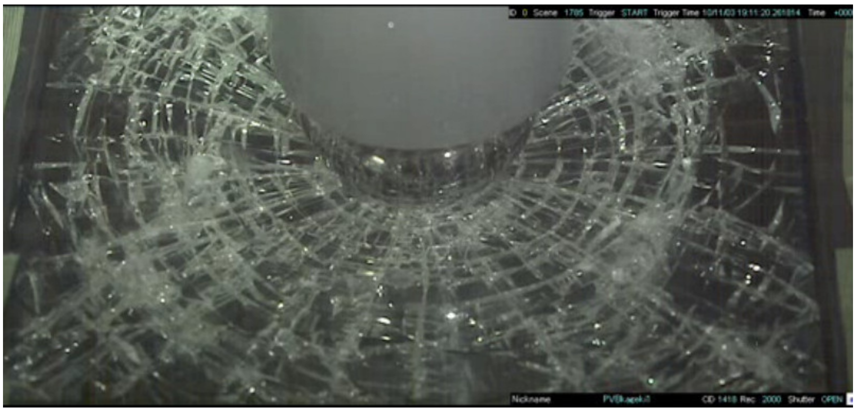


Figure 7: Dynamic fracture state of laminated glass by a high speed camera at 10ms

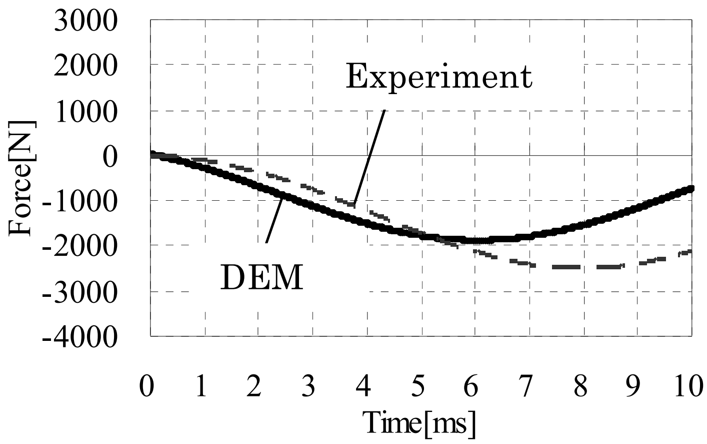


Figure 8: Time-history of reaction force of fixed edge by DEM and experiment

6. Optimum design of laminated glass

6.1. Objective function and design variable

In this study, the optimum design of the laminated glass has been attempted using a response surface method [13-14]. A response surface method created to calculate an approximation function using the response values in some sample points (experiment and simulation data). An optimum solution is created an approximation optimum value in which it can be optimized for the response surface. In this study, the response surface is expressed as quadratic polynomials using a least squares method in order to treat two variables (x_1 and x_2) in the next equation.

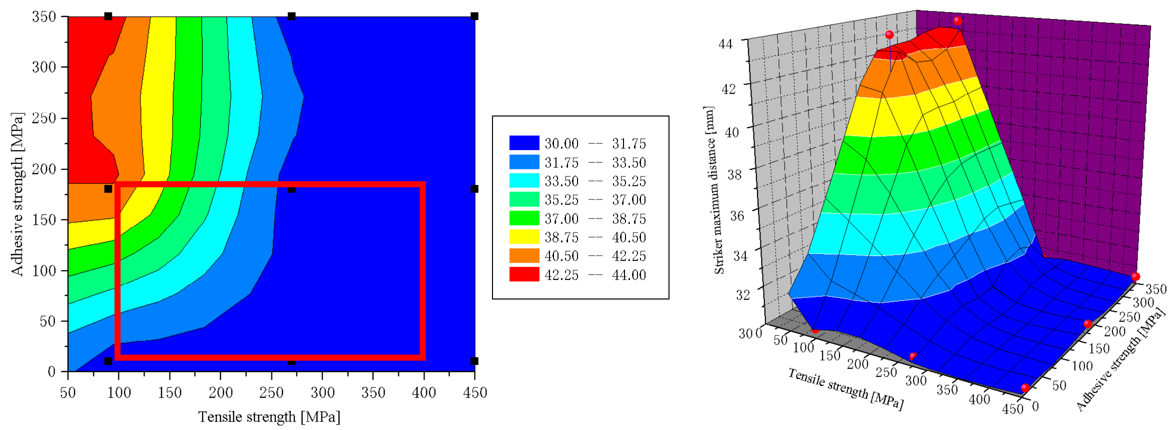
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^{\,2} + \beta_4 x_2^{\,2} + \beta_5 x_1 x_2 \tag{4}$$

where β_i (i=0, 1, ,5) are unknown coefficients, x_i (i=1, 2) are variables and y is a response value. In the optimum design problem, the tensile strength σ_t of the interlayer (PVB) and the adhesive strength σ_a (between the glass and PVB) were the design variables, because the effect of both values is big in the fracture destruction of the laminated glass. Also, an objective function was the maximum displacement of the impact striker (δ). Both formulas are given by:

$$Minimize \qquad \delta = \sum_{i=0}^{n} \frac{(V_{i+1} + V_i)}{2} \Delta t \tag{5}$$

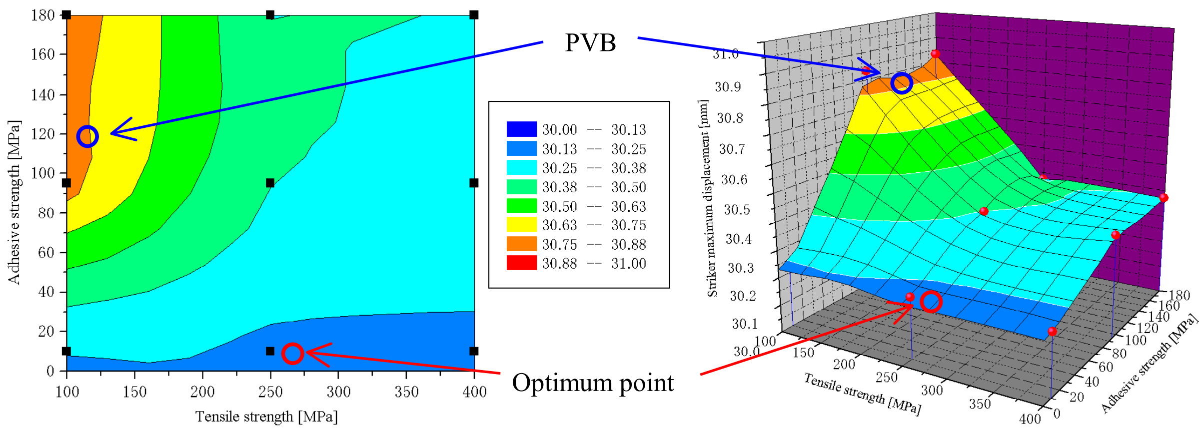
$$Subject\ to \qquad 90 \le \sigma_t \le 450 \quad and \quad 10 \le \sigma_a \le 350 \tag{6}$$

where i is the number of the time step, V is the velocity of the impact striker and Δt is the time step in DEM simulation. In this optimization problem, the objective function (δ) expressed in Eq. (5) is minimized.
The response surface method was used for the optimization. The response surface was constructed as the response values in the maximum displacement of the impact striker provided from the results of DEM simulation. Also, "RSMaker for Excel" was used for construction of the response surface [15].



(a) Counter plots (b) Three dimensions view

Figure 9: Response surface of maximum displacement δ by interpolation calculation



(a) Counter plots (b) Three dimensions view

Figure 10: Zoomed response surface of maximum displacement δ by RSMaker

6.2. Results in optimization of δ

Figure 9 shows the response surface of δ by the interpolation calculation in which is constructed the nine sample points on 3*3 in the case of σ_t= (90, 270, 450) and σ_a= (10, 180, 350). The δ value became high when σ_a was smaller than 250MPa. Conversely, δ became a low value in wide region when σ_a is bigger than 250MPa. That is, the expansion of the interlayer (PVB) is lost under the adhesive strength with a big value, and it was thought that the laminated glass was easy to destroy. With a constructed response surface, it was optimized. As a result, it was predicted that the smallest response value was in the conditions of σ_t=292.5MPa and σ_a=10.5MPa.

Next, it was observed once again in the surrounding region of the value that an enlarged response surface was constructed, and the optimum point was found in the region (quadrilateral inside red lines) within nine points of σ_t = (100, 250, 400) and σ_a= (10, 95, 180).

The zoomed response surface demanded by the quadratic polynomials approximation is shown in Fig. 10 (seal ○ is the optimum point). Also, the seal of PVB laminated glass is shows in the same figure. Afterwards, it was optimized by "RSMaker for Excel" and using this figure, the optimum response value (δ=30.1mm) under the optimum condition in the design variables (σ_t=268.8MPa and σ_a=10.0MPa) was pursued. The regression formula within the response surface coefficients is shown in the following:

$$y = 30.5 + (-3.57\times10^{-3})\sigma_t + (8.11\times10^{-3})\sigma_a + (6.81\times10^{-6})\sigma_t^{\,2} + (-9.02\times10^{-6})\sigma_a^{\,2} + (-2.13\times10^{-5})\sigma_t\sigma_a \qquad (7)$$

In the optimum result, the penetration performance of optimal laminated glass was improved about 3% in comparison with the laminated glass using the PVB interlayer, because the maximum displacement value δ is reduced about 0.7mm. Also, the adjusted coefficient of multiple determination became $R_{ad}^{\,2}$ =0.884. Generally, $R_{ad}^{\,2}$ is considered to be a good approximation more if it is than 0.8. In the calculated regression formula, it is evaluated at a higher approximation accuracy.

Additionally, the response values around the optimum value indicated that there was small variation in a large design range. The pursued optimum value can be said to have a higher robustness.

Finally, the verification analysis by DEM simulation was carried out using the optimum design variables once again because the validity of the optimum value was examined. As a result of the verification analysis, the value of the maximum displacement became δ' =30.2mm. It was thought that the pursued optimum value was almost a valid value, because the difference of both values (δ and δ') is very small (about 0.1mm).

7. Conclusion

In this study, the impact fracture behavior of a laminated glass plate using a PVB interlayer for the outside surface of modern buildings have been studied by the numerical simulation (DEM) and the experiments. Additionally, the laminated glass structures were optimized for attaining the maximum durability against the impact fracture based on the response surface approach. The provided results are summarized as follows:

(1) From results in DEM simulation, a laminated glass performed high penetration resistance when the tensile strength in a PVB interlayer was high. However, the adhesive strength between the glass and interlayer was needed to have a value in some degree.

(2) As the optimum results, the penetration performance of optimal laminated glass improved about 3% in comparison with the PVB laminated glass in general.

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