10^{th} **World Congress on Structural and Multidisciplinary Optimization**
May 19 - 24, 2013, Orlando, Florida, USA

A case study of multicriteria shape optimization of thin structures

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1. Abstract
Aerosol cans are usually made of thin high performance steel and are filled with fluid at high pressure. For these two reasons, and considering usage and packaging requirements, the structural stability of their ends, top and bottom is then delicate to maintain. In the present work, we address the problem of shape optimization of the bottom of a can, in order to control the *dome growth* DG (e.g. displacement of can base) at a proof pressure as well as the *dome reversal pressure* DRP, a critical pressure at which the can bottom looses stability (e.g. initiates buckling). We first implemented and validated an RBF-like metamodel to have at hand cheap criteria surrogates (DG,DRP) using a 2D spline representation in an axi-symmetric setting. Then, we implemented a Normal Boundary Intersection -NBI- with filtering formulation in order to capture the -approximate- Pareto Front, using an FSQP method for the NBI-related sub-optimizations.
The obtained approximate Pareto Fronts corroborate the antagonistic behavior of the DG and DRP criteria, and are successfully compared to the projection on the exact cost evaluations. We also identify Pareto-optimal solutions which lie in a restrictive industrial-prescribed acceptable interval for the DG-DRP costs.
We then consider the problem of selection of solutions among the Pareto Front. We model the selection problem as a Nash game played by the two costs DG and DRP, and show that an arbitrary splitting of the shape parameters among the two players may lead to inefficient solutions (strictly dominated by Pareto-optimal ones).

2. Keywords: Shape optimization, Multiobjective optimization problem, Metamodel, Radial Basis Function, Normal Boundary Intersection, Nash equilibria.

3. Introduction
Structural multidisciplinary shape optimization -MDO- is known to demand costly computational resources, notably when one seeks to identify the Pareto Front, one of the most relevant MDO tools. To overcome this obstacle, it is classical to couple methods for the Pareto capture with metamodels aimed at cheap costs evaluation [1] [2] [3]. There are two possible couplings between methods to identify the set of Pareto optimal solutions, and metamodels : The first idea is to lead optimization with the dedicated algorithms (NBI or others) and use an updated metamodel for a certain number of evaluations until finding the solutions (strong coupling). The second idea is to lead optimization with the metamodel and only do the exact calculations of the metamodel-obtained solutions (weak coupling).
In our work, the normal boundary intersection (NBI) method [4] and the radial basis function (RBF) metamodel [5] are used to build our algorithm (NBI RBF) using a weak coupling. The implemented algorithm is validated against mathematical test-cases, and then used to perform a multicriteria shape optimization of structures which undergo highly nonlinear deformations. We compare the results obtained for different a priori discretizations of the Pareto fronts. We also address the problem of selecting solutions among the Pareto optimal ones, by using a Nash game approach.

4. Normal Boundary Intersection
In this section, we present the methodology and background used throughout the paper.

A multiobjective optimization problem is given as follows:

$$\min_{x \in D} \quad F(x) = (f_1(x), f_2(x), ..., f_m(x))^T$$

$$s.t. \begin{cases} g_j(x) \geq 0 & \text{j = 1, ..., J} \\ h_k(x) = 0 & \text{k = 1, ..., K} \\ x^{lower} \leq x \leq x^{upper} & \end{cases} \tag{1}$$

The Pareto Front is defined as the set of non-dominated designs, in the objective space. A design point, $x^* \in D$ is non-dominated if there is no other point, $x \in D$, such that:

$$f_i(x) \leq f_i(x^*) \qquad i = 1, .., m$$

with strict inequality for at least one index.

Normal Boundary Intersection method NBI is a solution methodology developed by Das and Dennis (1996) for the approximation of Pareto surfaces [4].
The method is based on the intersection of the so-called CHIM 's (convex hull of individual minima) normal and the objective space border.
We summarize it as follows:
Let x_i^* be the respective global minimizers of $f_i(x)$, i=1,..,m over $x \in (D)$.
Let $F_i^* = F(x_i^*)$, i=1,..,m.
Let $F^* = [f_1(x_1^*), f_2(x_2^*), ..., f_m(x_m^*)]^T$.
Let ϕ be the $m \times m$ matrix whose ith column is $F(x_i^*) - F^*$ known as the pay-off matrix.
Then the set of points in R^m that are convex combinations of $F(x_i^*) - F^*$ is referred to as the CHIM, i.e., $CHIM = \{\phi\beta, \ \beta \in R^m \ avec \ \sum_{i=1}^{m} \beta_i = 1, \ \beta_i \geq 0\}$. The set of attainable objective vectors $\{F(x) : x \in (D)\}$ is denoted by $\boldsymbol{F}$ and is usually referred to as the objective space. Let denote the boundary of $\boldsymbol{F}$ by $\partial\boldsymbol{F}$.
NBI method determines the portion of $\partial\boldsymbol{F}$ which contains the Pareto optimal points. The principal idea behind this approach is that the intersection point between the boundary $\partial\boldsymbol{F}$ and the normal pointing towards the origin emanating from any point in the CHIM is a point on the portion of $\partial\boldsymbol{F}$ containing the efficient points. This point is guaranteed to be a Pareto optimal point if the trade-off surface in the objective space is convex. This is the algebraic idea behind NBI's approach, Das and Dennis have shown that this approach can be written mathematically and also that the point of intersection of the normal and the boundary of $\boldsymbol{F}$ closest to the origin is the global solution of the following single problem:

$$\max_{x,t} \quad t$$

$$s.t. \ \ (D_{NBI}) \begin{cases} \phi.\beta + t.\mathbf{n} = F(x) - F^* & \\ g_j(X) \geq 0 & \text{j = 1, ..., J} \\ h_k(X) = 0 & \text{k = 1, ..., K} \\ x^{lower} \leq x \leq x^{upper} & \end{cases} \tag{2}$$

The advantage of the NBI method is that it gives a set of well distributed solutions over the Pareto Front. One may need however to postprocess the results with a filter, to eliminate non-pareto or local pareto points (if the trade-off surface in the objective space is not convex).

The basic idea of RBF metamodeling is to construct a function approximations using function values at some sampling points, which are typically determined using experimental design methods such as Latin hypercube, uniform distribution of the search space [6].
Let f(x) be the true objective or response function and $\tilde{f}(x)$ its approximation obtained from a classical RBF with the general form :

$$\tilde{f}(x) = \sum_{i=0}^{n} \omega_i \phi(||x - x_i||) \tag{3}$$

where x is the vector of design variables, x_i is the vector of the ith sampling point, n is the number of sampling points, $||x - x_i||$ is the Euclidean distance, ϕ is a basis function (for example, Gaussien one $\phi(r) = e^{-ar^2}$ where a is the attenuation coefficient $(0 \prec a \leq 1)$) [7], and ω_i is the unknown weighting coefficient which is obtained by solving the linear system :

$$\mathbf{f} = \mathbf{A}.\omega \tag{4}$$

where $\mathbf{f} = [f(x_1), .., f(x_n)]^T$ and $\mathbf{A}_{i,j} = \phi(||x_i - x_j||)$ (i=1,..,n ; j=1,..,n).

5. Coupling NBI and RBF methods

Our goal is to perform a weak coupling between the NBI algorithm and RBF metamodel in order to have a simple tool with a reasonable calculation time to solve multiobjective optimization problems and test its effectiveness for academic cases, and also for an industrial cases.
Throughout our work, we address a special case of MDO (two objective functions):

$$\min_x \quad F(x) = (f_1(x), f_2(x))^T$$
$$s.t.\ (D) \quad \begin{cases} g_j(x) \geq 0 & \text{j} = 1, ..., \text{J} \\ h_k(x) = 0 & \text{k} = 1, ..., \text{K} \\ x^{lower} \leq x \leq x^{upper} & \end{cases} \tag{5}$$

The NBI algorithm shows the need for an optimizer in each stage in the sense that the NBI method requires optimization of each objective function, as well as the objective function NBI, so it is necessary to perform a large number of evaluations of cost functions, which can be very costly in terms of computation time. In these conditions, we replaced all the objective functions approximated by functions built with the RBF metamodel. Let $\tilde{f}_1(x)$ and $\tilde{f}_2(x)$ the approximations obtained from a classical RBF for f_1 and f_2, respectively. For RBF metamodel used, there are two parameteres to determine: The attenuation factor will be determined using the technique of Rippa, and we chose a uniform distribution of the search space to select the sampling points.

The NBI RBF coupling algorithm is the following:

Algorithm 1 NBI RBF coupling algorithm

1: **for** $i = 1:2$ **do** ▷ Minimize each objective function subject to constraints
2: $\quad x_i^* = \min_x \tilde{f}_i(x)$
3: **end for**
4: $\tilde{F}_i^* = [\tilde{f}_1(x_i^*), \tilde{f}_2(x_i^*)]^T$ ▷ Individual minima for each objective
5: $\tilde{F}^* = [\tilde{f}_1(x_1^*), \tilde{f}_2(x_2^*)]^T$ ▷ Shadow Minima matrix
6: ϕ ▷ Pay-off matrix $(m \times m)$
7: **for** $i = 1:2$ **do**
8: $\quad \phi_i = \tilde{F}(x_i^*) - \tilde{F}^*$ ▷ Pay-off matrix ith column
9: **end for**
10: $\mathbf{n} = -\phi.e$, $e = (1,1)^T$ ▷ Normal vector
11: $\beta = (\beta_1, \beta_2)^T$, $\beta_i \geq 0$, $\sum_{i=1}^{2} \beta_i = 1$ ▷ weights vector
12: **for** each $\beta = (\beta_1, \beta_2)^T$ **do**
13: $\quad$ Solving the problem:

$$\max_{x,t} \quad t$$

$$s.t.\ \ (D_{NBI}) \begin{cases} \phi.\beta + t.\mathbf{n} = \tilde{F}(x) - \tilde{F}^* & \\ g_j(X) \geq 0 & \text{j} = 1, ..., \text{J} \\ h_k(X) = 0 & \text{k} = 1, ..., \text{K} \\ x^{lower} \leq x \leq x^{upper} & \end{cases}$$

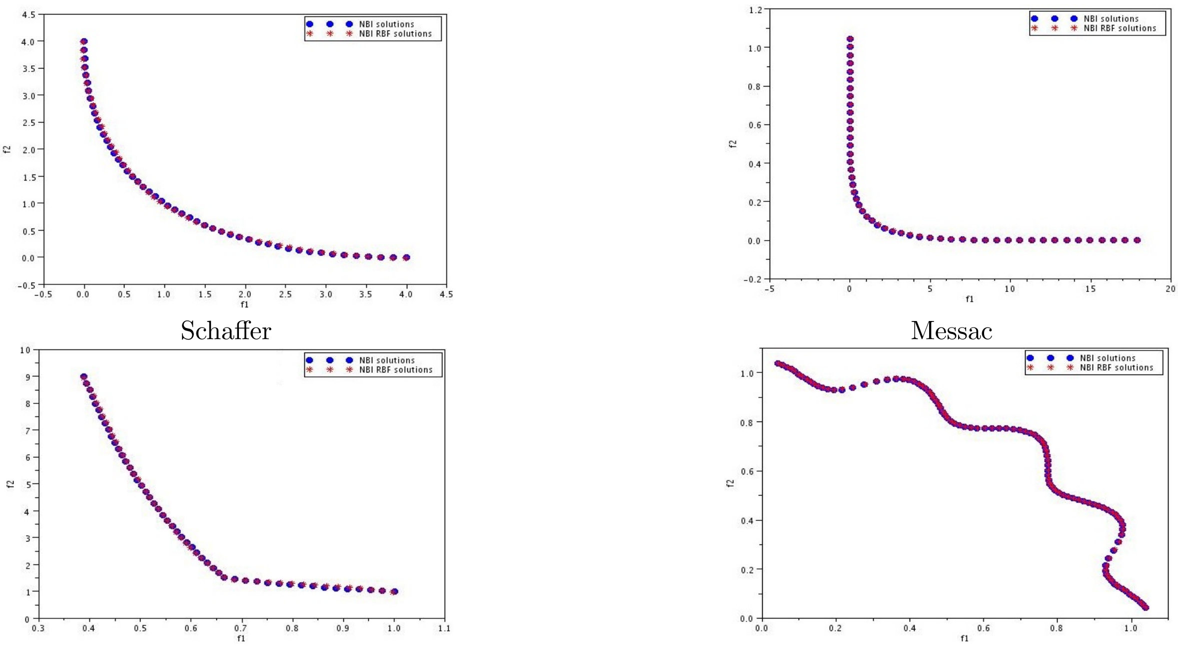
14: **end for**

The coupling approach is first tested for several optimization problem known as test problems, which are mathematical explicit functions (Schaffer, Messac, Constraints minimization and Tanaka problem) [10]. The results, Figure 1 on page 4 and Table 1, show that on the coupling NBI RBF converges to the

Pareto frontier with an approximately 80%, 95%, 99% and 97% fewer number of objective functions calls compared to a conventional NBI for Schaffer, Messac, Constraints minimization and Tanaka problem, respectively.

Table 1: Functions call number required by NBI and NBI RBF methods

Problem	Method used	Prescribed Pareto points	Functions calls
Schaffer	NBI	50	208
	NBI RBF	50	40
Messac	NBI	50	701
	NBI RBF	50	32
Constraints minimization	NBI	50	2315
	NBI RBF	50	18
Tanaka	NBI	100	1884
	NBI RBF	100	50



Schaffer

Messac

Constraints minimization

Tanaka

Figure 1: Comparison between the results obtained by NBI RBF approach, and the exact Solutions NBI

After the validation of algorithm versus academic test cases, we present an application for industrial test case which is the shape optimization of the bottom of two kinds of aerosol cans.

6. Shape optimization of the bottom of aerosol cans

6.1. Motivation

Aerosol cans are usually made of thin high performance steel and are filled with fluid at high pressure. For these two reasons, and considering usage and packaging requirements, the structural stability of their ends, top and bottom is then delicate to maintain. In the present work, we address the problem of shape optimization of the bottom of a can, in order to control the dome growth DG (e.g. displacement of can base) at a proof pressure as well as the dome reversal pressure DRP , a critical pressure at which the can bottom looses stability (e.g. initiates buckling).

Dome growth Dome reversal pressure

Figure 2: Two criteria to be optimized

6.2 Types of bottom's cans shape and steel characteristics
For our work, we use two types of bottom's cans, Figure 4 on page 6, aerosol can N1 and aerosol can N2 are made of thin high performance steel which have the following characteristics:
* Steel thickness: e=0.46 mm
* Strain hardening exponent: n= 0.2
* Yield strength: Re = 270 MPa
* Ultimate strength: Rm = 380 MPa
* Strength coefficient : $K = e^{(Rm.(n.(1-ln(n))))}$
* Hollomon law : $\sigma = K\epsilon^n$

Aerosol bottom N1 Aerosol bottom N2

Figure 3: Two different purpose shapes of the bottom

6.3. Presentation of the MDO framework
The goal is to figure out a design of bottom of aerosol cans, which satisfies a DRP value bigger than DRP of initial shapes and a DG value smaller than 1 mm. Our initial aerosol can N1 design has 19.1 bar and 0.89 mm for DRP and DG values, respectively, and our initial aerosol can N2 design has 15.2 bar and 0.48 mm for DRP and DG values.
The test of this industrial study case of optimization would require at first to use LS-DYNA software which performs the calculations of deformed elasto-plastic in order to determine the objective criterions DRP and DG. Then we will use our developed approach of optimization which allows to set the parameters of an axisymmetric shape by using cubic splines and then optimizing in the space of splines under border constraints.
As a first step, we present the characteristics related to shape optimization (design variable, constraints, metamodel database and multiobjectif optimization formula).

6.3.1. Design variable
For our cases study, the bottom of the can is divided into two parts, a fixed non modifiable one, and a variable part, to be optimally designed.

Aerosol bottom N1 Aerosol bottom N2

Figure 4: Design variable

6.3.2. Design constraints and metamodel database

Aerosol bottom N1 Aerosol bottom N2

Figure 5: Four design variable and for each point three different positions, so we have a set of 81 points

We selected a set of 81 points (3^4), each point representing a given shape of the bottom of the can. Let us mention that in our case, the above uniform sampling turned out to be more efficient than the Latin hypercube sampling. These points will be considered as sampling points for the RBF metamodel. Then, for each point, we calculate the exact value of two criteria DRP and DG. We collect these values to set a database allowing us to build the RBF metamodel for each criterion, and the optimization problem will be studied using these metamodels.

6.3.3. Optimization formula

Our aim is to solve the problem with our developed approach (NBI RBP coupling), and exact cost evaluations are performed for the final Pareto optimal designs, in order to assess the efficiency of our approach to solve this industrial optimization problem.

Let φ denote a cubic spline shape of the bottom of the can, or equivalently the ordinates (abscissae are fixed) of that cubic spline. Then, our original problem is stated as the following:

$$\begin{aligned} \max_{\varphi=(\varphi_1,\varphi_2,\varphi_3,\varphi_4)} \quad DRP(\varphi) \ \ / \ \min_{\varphi=(\varphi_1,\varphi_2,\varphi_3,\varphi_4)} \quad DG(\varphi) \\ s.t. \ \ (D\ can) \left\{ \ \varphi^{lower} \leq \varphi \leq \varphi^{upper} \right. \end{aligned} \tag{6}$$

With our approach, we will solve the problem equivalent to the original one, replacing the criteria with their metamodels:

$$\begin{aligned} \max_{\varphi=(\varphi_1,\varphi_2,\varphi_3,\varphi_4)} \quad D\tilde{R}P(\varphi) \ \ / \ \min_{\varphi=(\varphi_1,\varphi_2,\varphi_3,\varphi_4)} \quad \tilde{DG}(\varphi) \\ s.t. \ \ (D\ can) \left\{ \ \varphi^{lower} \leq \varphi \leq \varphi^{upper} \right. \end{aligned} \tag{7}$$

Let be $\varphi_0 = (\varphi_{01}, \varphi_{02}, \varphi_{03}, \varphi_{04})$ the initial shape of the bottom of the can, and α a small positive offset. Then, we choose NBI constraints as follows:
* $\varphi^{lower} = (\varphi_{01} - \alpha, \varphi_{02} - \alpha, \varphi_{03} - \alpha, \varphi_{04} - \alpha)$
* $\varphi^{upper} = (\varphi_{01} + \alpha, \varphi_{02} + \alpha, \varphi_{03} + \alpha, \varphi_{04} + \alpha)$

6.3.4. Optimization results

For $\alpha = 0.5mm$, we computed an approximate Pareto front for the DG/DRP costs using the NBI+RBF coupling. For different prescribed number of Pareto points N, we show the overall time and total number of exact or surrogate evaluations used for the Aerosol can N1 case in Table 2, and for the Aerosol can N2 case in Table 3.

Table 2: Time required for the different functions call - Beverage can - (***)=(*)+(**)

N	Total $time^{***}$	Objective function		Approximated function	
		Total calls	Time required**	Total calls	Time required*
6	3h 46 min 10 s	87	3h 21 min 24 s	86835	24 min 46 s
12	4h 07 min 28 s	93	3h 37 min 06 s	87444	30 min 22 s
24	4h 27 min 39 s	105	4h 00 min 13 s	85193	27 min 26 s
50	5h 43 min 23 s	131	5h 06 min 55 s	90883	36 min 28 s

Table 3: Time required for the different functions call - Spray can - ((***)=(*)+(**))

N	Total $time^{***}$	Objective function		Approximated function	
		Total calls	Time required**	Total calls	Time required*
6	4h 21 min 07 s	87	3h 57 min 34 s	76524	23 min 33 s
12	4h 38 min 53 s	93	4h 11 min 00 s	78246	27 min 53 s
24	5h 11 min 31 s	105	4h 38 min 45 s	80784	32 min 46 s
50	6h 19 min 40 s	131	5h 47 min 20 s	80405	32 min 20 s

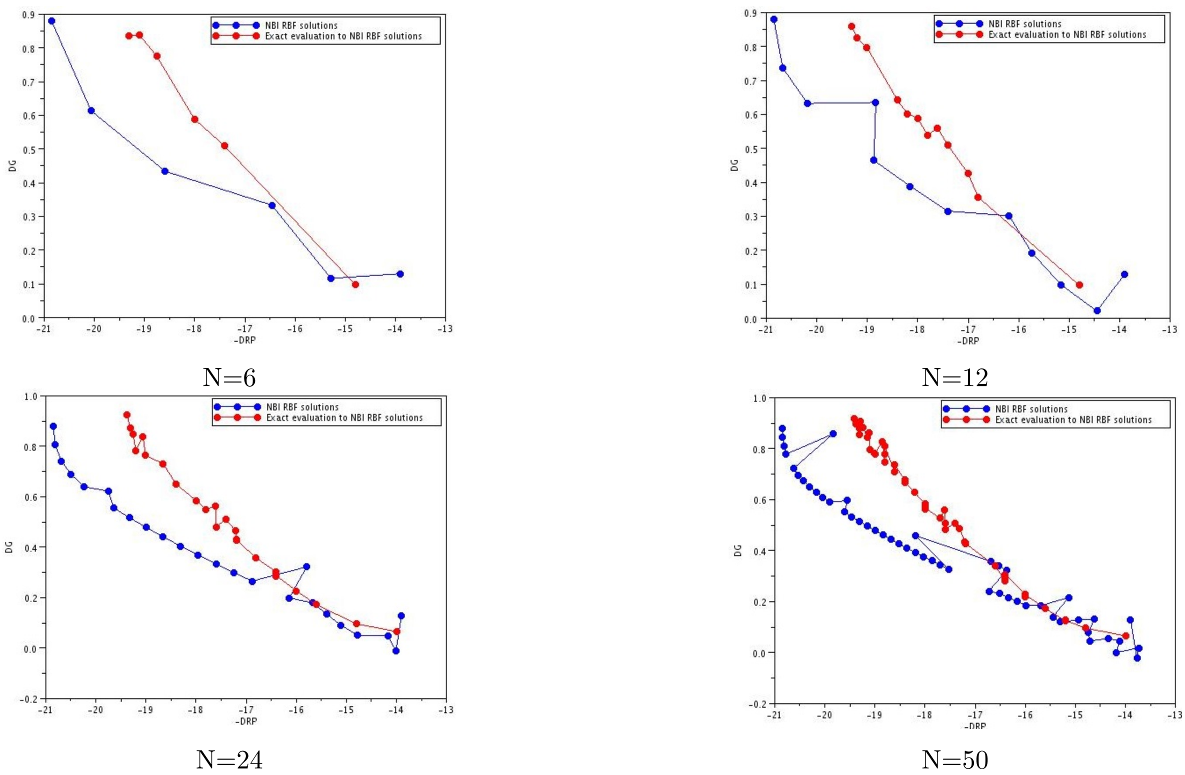
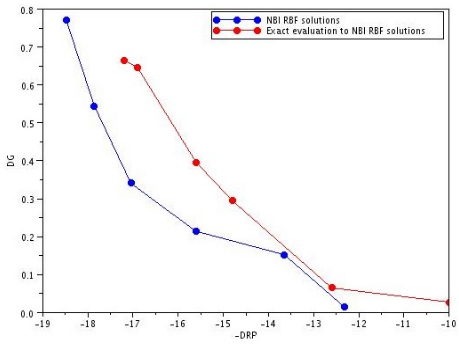
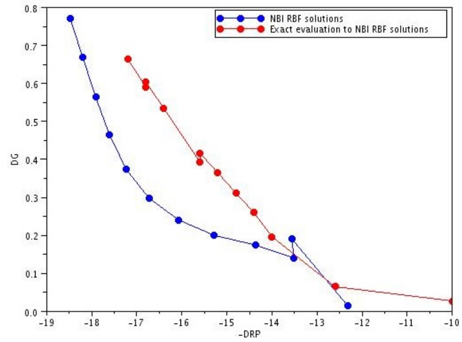


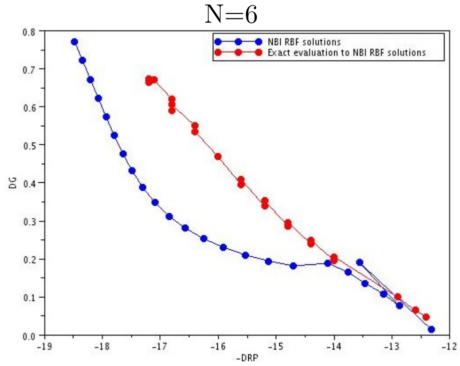
Figure 6: Comparison between the results obtained by NBI RBF approach, and the exact cost evaluation of these results for several cases - Aerosol can N1 -



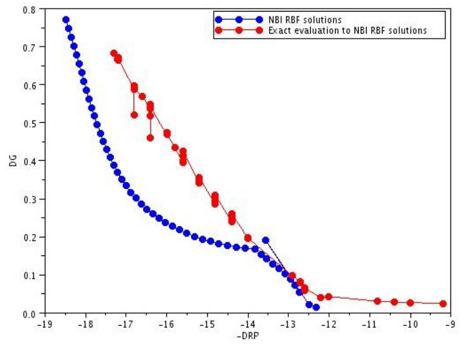
N=6



N=12



N=24



N=50

Figure 7: Comparison between the results obtained by NBI RBF approach, and the exact cost evaluation of these results for several cases - Aerosol can N2 -

6.3.5. Results discussion

A simple comparison between the results obtained by our approach and the accurate evaluation of these solutions, Figure 6 Figure 7, allows us to assess that our results remain good ones notwithstanding the complexity of our cases study.

Similarly, it is clear from Table 2 and Table 3 that our approach has allowed us to save a remarkable computational time. For example, if we take the case with N = 50 from Table 2, there are 131 calls of exact function evaluations and 80405 for approximated function, respectively, which represent 0.16 % and 98.24 % of the total function calls used in our approach, but at the same time, we note that only this 0.16% of total calls take 99.85% of the total computing time required. This last remark explains the idea why we chose not to apply roughly the NBI method with exact evaluations to solve the industrial case.

In the beginning of our work, we presented our goal that was to look for new profiles for the bottom of the aerosol cans satisfying some requirements (DRP higher than initial shapes DRP values and DG lower than 1 mm), a goal that we achieve successfully, Figure 9.

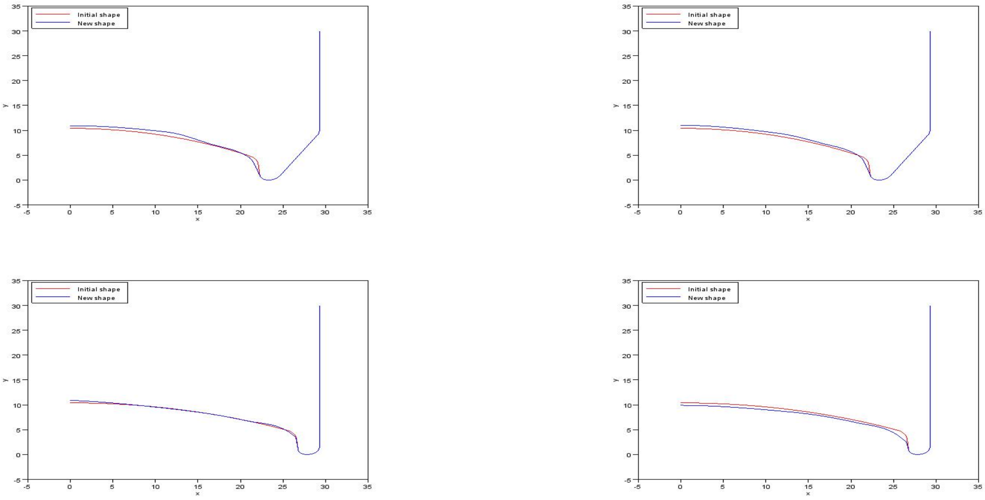
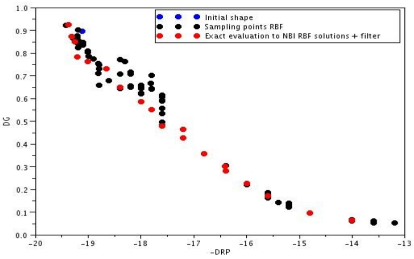
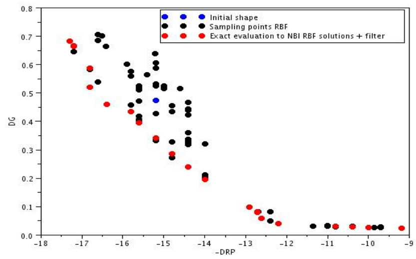


Figure 8: Some profiles which meet the operational industrial requirements

we used a filter to eliminate all dominated points, and we remarked that all remaining solutions are almost located at the boundary of the space formed by the elements of the RBF database Figure 9. Then we can conclude that the solutions obtained are likely non-dominated solutions and our approach is able to solve the industrial problem with a reasonable computation time.



Aerosol can N1 - N=24 -



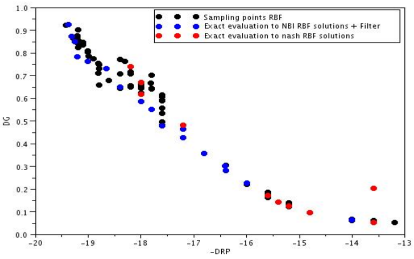
Aerosol can N2 - N=50 -

Figure 9: NBI RBF solutions after filtering with RBF data

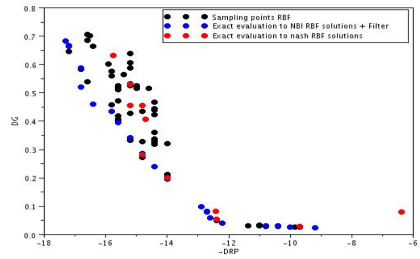
7. Nash equilibrium and RBF coupling approach

We then consider the problem of selection of solutions among the Pareto Front(i.e NBI solutions). We model the selection problem as a Nash game played by the two costs DG and DRP approximated by RBF metamodel.

The results presented in Figure-10 show the Nash equilibria obtained for different splittings of the shape coordinates among the two players GD and DRP. There are remarkable Nash solutions which lie on, or are close to, the Pareto Front. But, unfortunately, in the region of interest for the operational industrial applications (upper-left zone of the Pareto Front), almost all the Nash solutions are inefficient (strictly dominated by Pareto-optimal ones). Thus, arbitrary splitting is not advisable.



Aerosol can N1



Aerosol can N2

Figure 10: Nash equilibria (red) for different arbitrary splittings of the shape parameters

7. Summary and outlook

The NBI RBF coupling results show that the present approach is able to efficiently solve the multicriteria shape optimization problem of structures with nonlinear (elasto-plastic) behavior, that is, identify regions of interest of the Pareto Front. This is achieved not only with a reasonable computation time, but also by yielding Pareto fronts which are consistent with respect to the total number of prescribed points over the front.
The Nash RBF coupling results show that an arbitrary splitting of the shape parameters among the two players may lead to inefficient solutions (strictly dominated by Pareto-optimal ones).

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