Chance constrained business case of a three-engines hybrid aircraft

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1. Abstract

The purpose of this article is to present a Chance Constrained Optimization of an unconventional configuration of hybrid-powered-aircraft. The Chance Constrained Methodology is applied using a method of uncertainty propagation based on error distribution moments. The hybrid configuration is compared to a conventional, pure thermodynamic one, both being designed according to the same set of requirements. Models are briefly described in the article, more details can be found in literature. We present the method of uncertainty propagation through the well-known Rosenbrock function and validate it in comparison to a Monte-Carlo method. Concerning the aircraft and engine models, the uncertainty brought by each design and simulation module has been assessed individually, according to its predictive performance versus existing database. The results show that under model uncertainties we would be able to reach an economically viable hybrid-powered-aircraft with a probability of 0.95 when energy and power management technologies become between 2 to 3 times better compared to today values. In addition, the efficiency of uncertainty propagation by the way of moments is really interesting in term of computation time. Improvements are currently done to make this method even more precise especially when the amount of uncertainty increases.

2. Keywords: Uncertainty propagation, Optimization, Overall aircraft design, Hybrid aircraft.

3. Introduction

Fuel price recent evolution and the will for air transportation to become even greener result in a race for fuel saving. In this context, as technologies and configurations of conventional turbofan powered aircrafts to reduce fuel consumption are reaching their limits, electricity appears to be one of the high potential source of propulsion. Actually the concept of using electricity instead of kerosene is recurrent but the way to combine them is not often obvious. Some prefers all electrics, others are more familiar with hybrid engines [7]. In this study we present a new concept of hybrid aircraft with three engines: 2 wing-mounted thermodynamic conventional turbofans and one top-mounted (rear fuselage) electric turbofan with variable pitch.

Figure 1: Hybrid Aircraft Configuration

Before being able to compare performances of airplanes it is necessary to bring them to an equivalent level of maturity. The minimization of an objective related to the cost of the aircraft taking into account operational constraints is a classical way to optimize a first configuration guess [3]. In the case of new aircraft concepts the optimization becomes quite tricky considering the high level of uncertainty of available models. To avoid modeling issues, we propose to use derivative-free optimization methods: an adapted Nelder and Mead method [13] [8] and a Differential Evolution Algorithm [1], both modified to deal with constraints. Although it is important to design an optimal aircraft, it is also mandatory to have an idea of the sensitivity and the robustness of the result. This is why we also focus on studying the uncertainty along this optimization process, by catching the uncertainty coming from models and processes, propagating it and getting the uncertainty on the different outputs. It is also well-known that a Monte-Carlo analysis is highly time-consuming and even more when using black-box model functions. Therefore we propose an analytical method of propagating uncertainty via moments of distribution law, with an approach different from what is usually done (see e.g. [12],[14]). Our uncertainty analysis is compared to a classical Monte-Carlo method applied to a chance constrained optimization [2] of the Rosenbrock function with constraints. Then we run chance constrained optimization for the hybrid-powered-aircraft design problem using the moment propagation analysis and the two considered optimization methods. As a result we compare the deterministic and the robust optimum.

Section **4.** is an introduction of the uncertainty analysis method using error distribution moments propagation. We present here a test case of the Rosenbrock function optimization under constraints where we add uncertainty. The next section explains in a first part the two optimization methods used for this study and how they have been modified to take constraints into account. Section **6.** is an overall presentation of the aircraft design models and an introduction to the optimization problem. In section **7.** the optimization results are presented.

4. Uncertainty Propagation and Chance Constrained Optimization

When dealing with future project studies, where assumptions are made around evolution of technologies, uncertainty management is a key factor in order to be aware of the study validity and also to be the most competitive. In this article we face an unconventional aircraft configuration with an additional electrically powered engine, thus it is important to run the design optimization of the configuration taking into account the uncertainties around electrical technologies evolution. Doing so, we can determine with a certain percentage of probability when these technologies will allow the hybrid configuration to reach the performance level of current conventional configurations. Actually, the goal of uncertainty management is to obtain the uncertainty behavior of outputs of a problem given the inputs uncertainties. For that, one of the most known propagation method is Monte-Carlo. It uses samples of the input random variables with respect to their distribution and runs the design configuration process for each sample. We then get as a result the distribution of the uncertainty for each output. Monte-Carlo is also well-known as a very time-consuming method so we look for a new method of propagating error through models and processes. Based on [11] we look for a way of dealing with uncertainty using moments propagation.

4.1. Introduction of the new error propagation method by the way of moments

Assumptions on the models are that the error distribution is unimodal and that the model is not too noisy. Using a homemade probability distribution [4] with 4 parameters defining the shape (how tall and skinny or short and squat it is) and the support, we are able to represent a wide range of distributions and more particularly those we are dealing with. In order to introduce the uncertainty propagation through a function of two variables we consider $f \in C^1(\mathbb{R}^2, \mathbb{R})$, $(\varepsilon_1, \varepsilon_2)$ the uncertainty on input (x_1, x_2), and ε_y the uncertainty on output $y = f(x_1, x_2)$. With a first order Taylor development of f we obtain:

$$\varepsilon_y = \partial_{x_1} f(x_1) \cdot \varepsilon_1 + \partial_{x_2} f(x_2) \cdot \varepsilon_2 + o(\|(\varepsilon_1, \varepsilon_2)\|). \tag{1}$$

Now let $(a_1, ..., a_i, ..., a_n)$ denote the input random variables of the model with corresponding uncertainty $(e_1, ..., e_i, ..., e_n)$. Thus as x_1 and x_2 can be seen as a function of model inputs, using a first order Taylor development of x_1 and x_2 (seen as functions) of $(a_1, ..., a_i, ..., a_n)$, we have:

$$\varepsilon_j = \sum_{i=1}^{n} \partial_{a_i} x_j \cdot e_i + o(\|(e_1, \ldots, e_n)\|) \text{ , for j=1,2.} \tag{2}$$

Let (μ, ν, s, k) denote respectively the mean, variance, skewness and kurtosis, the last two being standardized moments of order 3 and 4:

$$\mu_y = E\left[\varepsilon_y\right], \tag{3}$$

$$v_y = E\left[(\varepsilon_y - \mu_y)^2\right], \tag{4}$$

$$s_y = \frac{E\left[(\varepsilon_y - \mu_y)^3\right]}{v_y^{3/2}}, \tag{5}$$

$$k_y = \frac{E\left[(\varepsilon_y - \mu_y)^4\right]}{v_y^2} - 3. \tag{6}$$

Here we denote $X = (x_1, x_2)$ and $y = f(X)$ and after calculations, we obtain μ_y, ν_y, s_y, k_y as a function of $(\mu x_1,\, \nu x_1,\, s x_1, k x_1,\, \mu x_2,\, \nu x_2,\, s x_2,\, k x_2,\, \mu_{a_i},\, \nu_{a_i},\, s_{a_i}, k_{a_i},\, \partial_{a_i} x_1,\, \partial_{a_i} x_2,\, \ldots,\, \nu x_2,\, s x_2,\, k x_2,\, \mu_{a_i},\, \nu_{a_i},\, s_{a_i}, k_{a_i},\, \partial_{a_i} x_1,\, \partial_{a_i} x_2)$, such that:

$$\mu_y = \partial x_1 f(X) \cdot \mu x_1 + \partial x_2 f(x_1, x_2) \cdot \mu x_2 + o(\|\varepsilon_X\|), \tag{7}$$

$$\nu_y = (\partial x_1 f(X))^2 \cdot \nu x_1 + (\partial x_2 f(X))^2 \cdot \nu x_2 + 2 \cdot \partial x_1 f(X) \cdot \partial x_2 f(X) \cdot \sum_i \partial_{a_i} x_1 \cdot \partial_{a_i} x_2 \cdot \nu_{a_i} + o(\|\varepsilon_X\|^2), \tag{8}$$

$$s_y = \frac{1}{\nu_y^{3/2}} \cdot \Big((\partial x_1 f(X))^3 \cdot s x_1 \cdot \nu_{x_1}^{3/2} + (\partial x_2 f(X))^3 \cdot s x_2 \cdot \nu_{x_2}^{3/2} + 3 \cdot \big((\partial x_1 f(X))^2 \cdot \partial x_2 f(X) + ... \tag{9}$$

$$... + \partial x_1 f(X) \cdot (\partial x_2 f(X))^2 \big) \cdot \sum_i \big((\partial_{a_i} x_1)^2 \cdot \partial_{a_i} x_2 + \partial_{a_i} x_1 \cdot (\partial_{a_i} x_2)^2 \big) \cdot s_{a_i} \cdot \nu_{a_i}^{3/2} \Big) + o(\|\varepsilon_X\|^3),$$

$$k_y = \frac{1}{\nu_y^2} \cdot \Big((\partial x_1 f(X))^4 \cdot k x_1 \cdot \nu_{x_1}^2 + (\partial x_2 f(X))^4 \cdot k x_2 \cdot \nu_{x_2}^2 + ... \tag{10}$$

$$... + 4 \cdot (\partial x_1 f(X))^3 \cdot \partial x_2 f(X) \cdot \sum_i \big((\partial_{a_i} x_1)^3 \cdot \partial_{a_i} x_2 \cdot k_{a_i} \cdot \nu_{a_i}^2 \big) + ...$$

$$... + 4 \cdot \partial x_1 f(X) \cdot (\partial x_2 f(X))^3 \cdot \sum_i \big(\partial_{a_i} x_1 \cdot (\partial_{a_i} x_2)^3 \cdot k_{a_i} \cdot \nu_{a_i}^2 \big) + ...$$

$$... + 6 \cdot (\partial x_1 f(X))^2 \cdot (\partial x_2 f(X))^2 \cdot \sum_i \big((\partial_{a_i} x_1)^2 \cdot (\partial_{a_i} x_2)^2 \cdot k_{a_i} \cdot \nu_{a_i}^2 \big) \Big) + o(\|\varepsilon_X\|^4).$$

We implement these propagation formulas in the models and processes such that at each step of the calculation we get with the current calculated value its mean, variance, kurtosis and skewness. We so save a lot of time compared to Monte-Carlo analysis: in only one run we are now able to get the uncertainty distribution of all outputs.

4.2. Introduction of Chance constrained optimization

Now we have a fast uncertainty propagation method, we are able to run a chance constrained optimization in reasonable time. It is a very powerful way of doing robust optimization [2]. It consists in solving an optimization problem with stochastic objective or/and stochastic constraint functions and to add as a new constraint some probabilities of the constraints function to reach a given value. To sum up, the deterministic problem is transformed into a probabilistic problem as follows:

$$\left\{ \begin{array}{l} \min\limits_{X \in \mathbb{R}^n} f(X), \\ \text{s.t. } G(X) \leq 0. \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \min\limits_{X \in \mathbb{R}^n} E[f(X)], \\ \text{s.t. } \text{Prob}\big(G(X) \leq 0\big) > p,\, p \in [0,1]. \end{array} \right. \tag{11}$$

When uncertainties are known for input variables of a model or functions this way of optimizing is very powerful because it takes into account the robustness of the optimum. The moment propagation method adapted to this kind of problem is tested in the next section on the Rosenbrock function.

4.3. Chance constrained optimization of the Rosenbrock function: Monte-Carlo and Moments methods

The Rosenbrock function is a non-convex function defined from $\mathbb{R}^2$ to $\mathbb{R}$ by:

$$f(x_1, x_2) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2. \tag{12}$$

It has a global minimum at $(x_1, x_2) = (1, 1)$, where $f(x_1, x_2) = 0$. Now we add the following arbitrary constraints:

$$\left\{ \begin{array}{l} G_1(x_1, x_2) \leq 1, \\ G_2(x_1, x_2) \leq 0. \end{array} \right. \tag{13}$$

$$\text{Where: } \left\{ \begin{array}{l} G_1(x_1, x_2) = (x_1 + 1)^2 + (x_2 - 0.3)^2, \\ G_2(x_1, x_2) = -0.05 - 2 \cdot x_1 - 0.8 \cdot x_2 - 0.01 \cdot f(x_1, x_2). \end{array} \right. \tag{14}$$

The basic optimization problem is defined by:

$$\left\{ \begin{array}{l} \min\limits_{(x_1, x_2) \in \mathbb{R}^2} f(x_1, x_2); \\ \text{s.t. } \left\{ \begin{array}{l} G_1(x_1, x_2) \leq 1; \\ G_2(x_1, x_2) \leq 0. \end{array} \right. \end{array} \right. \tag{15}$$

We solve it using a Sequential-Quadratic-Programming method [10]. Now let us introduce the uncertainty $(\varepsilon_1, \varepsilon_2)$ on the Rosenbrock function and $(\varepsilon_3,\varepsilon_4)$ on the constraints. The shape of uncertainty is given to be closed to a Normal distribution and the error is set to be 20% on each value. The functions become:

$$\begin{cases} f_{unc}(x_1, x_2) = (1 \cdot \varepsilon_1 - x_1)^2 + 100 \cdot \varepsilon_2 \cdot (x_2 - x_1^2)^2, \\ G_{1_{unc}}(x_1, x_2) = (x_1 + 1 \cdot \varepsilon_3)^2 + (x_2 - 0.3 \cdot \varepsilon_3)^2, \\ G_{2_{unc}}(x_1, x_2) = -0.05 - 2 \cdot x_1 - \varepsilon_4 \cdot 0.8 \cdot x_2 - \varepsilon_4 \cdot 0.01 \cdot f(x_1, x_2). \end{cases} \tag{16}$$

And so we have the following chance constrained optimization problem:

$$\begin{cases} \min\limits_{(x_1, x_2) \in \mathbb{R}^2} E[f_{unc}(x_1, x_2)] \\ \text{s.t.} \begin{cases} \text{Prob}\big(G_{1_{unc}}(x_1, x_2) \leq 1\big) > 0.95 \\ \text{Prob}\big(G_{2_{unc}}(x_1, x_2) \leq 0\big) > 0.95 \end{cases} \end{cases} \tag{17}$$

We solve problem (17) using Nelder-Mead method and obtain the results presented in Table 1. Optima are drawn on Figure 2.

Table 1: Results of Chance constrained optimization on the Rosenbrock function using moments propagation method and Monte-Carlo (MC) method for two samples of different sizes

	Deterministic Optimization	**Moments Propagation**	**MC ($N = 500$) Propagation**	**MC ($N = 5000$) Propagation**
$x_{1_{opt}}$	-0.416	-0.68	-0.70	-0.68
$x_{2_{opt}}$	-1.112	-1	-0.98	-0.99
Rosen. Opt. Value	167	224	226	224
$\text{P}\big(G_{1_{unc}}(x_1, x_2) \leq 1\big)$	0.013	0.95	0.95	0.95
$\text{P}\big(G_{2_{unc}}(x_1, x_2) \leq 1\big)$	0.12	0.95	0.95	0.95
Computation time (s)	0.8	1.5	5	100

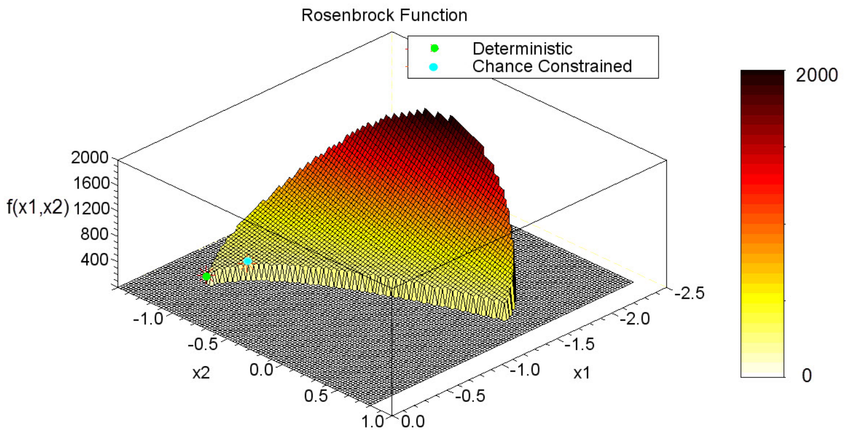

Figure 2: The Rosenbrock constrained function optimization: Comparison of deterministic & Chance constrained Optimum

As we could expect, we observe a difference between the deterministic and the chance constrained optimum: the introduction of uncertainty leads to a new optimum where the objective function value is degraded compared to the deterministic one. When we look at the deterministic optimization in Table 1 we observe a value of 1.3% probability of satisfying the first constraint when the uncertainty is introduced. This reflects the fact that when the error of a model can be determine, it is very important to run chance-constrained optimization instead of deterministic one. The objective function will certainly be degraded but the optimum will be robust. For example in aeronautic industry if models errors during the first steps of the aircraft conception are not taken into account, it will lead after some steps to a configuration that does not satisfy the requirements anymore. So the conception will have to start again from the beginning. To avoid this costing kind of inconvenience, it is primary to ensure with a high probability that the conception is robust and that the given constraints are satisfied. Chance-constrained optimization is thus very adapted to conception problem where the robustness of the solution is necessary. Moreover, this test- case shows the efficiency of this new method using moments propagation. Whereas using a Monte-Carlo method it takes around 100 seconds to run consistent optimization, it only takes less around 1.5 second to this method. The next section presents the models of the hybrid aircraft design chance-constrained optimization and the application of the moments propagation method to this case.

5. Tools and Methods for optimization

5.1. The Nelder-Mead method managing constraints

The Nelder-Mead method [13] is a heuristic search method for non-linear optimization problems. The method uses the concept of a simplex, which is a convex hull of $N+1$ vertices in a N-dimensional space, e.g a line segment in 1 dimension, a triangle in dimension 2. The algorithm detailed in [8], performs geometrical operations (reflection, expansion, contraction) on some candidate vertices of the convex hull in order to move this convex hull in descent directions. Basically for one iteration of the algorithm, the chosen vertex is first reflected to obtain a new point, if this one improves the objective function value, an expansion is attempted. Otherwise the polytope is reduced. This is done until the polytope size reaches a given precision. In order to deal with constraints, we define an additional function evaluating the distance to the constraints in order to compare two unfeasible points: this function gives a lowest value to the point the closer to the constraints boundary. We introduce the following order relation between points, considering that A is better than B in the following cases:

- if constraint function value of A is lower than B's one, when they both are not satisfying constraints,
- when constraints are satisfied for A and not for B,
- when constraints are satisfied for both points: we compare them via the objective function value.

Drawbacks of this algorithm is that it can sometimes be stuck in a local optimum. Moreover, convergence properties of the Nelder-Mead algorithm [9] are only proved for some strictly convex class of functions in dimensions 1 and 2.

5.2. Differential Evolution Algorithm managing constraints

Differential Evolution algorithms are a search heuristic method that belongs to the evolutionary algorithms class [6]. It generates solutions to optimization problems using techniques such as mutation, selection and crossover. Each step creates a new generation of individuals using the aforementioned processes, that ultimately result in individuals with better performances in term of a given objective function. However, even if it is known to be a very efficient method, it does not guarantee the optimality of the solution and it is a time consuming algorithm. The basic algorithm is not specified for constrained optimization problems. In order to deal with constraints and based on the basic algorithm [6] we implemented the same comparison process as for the Nelder-Mead algorithm.

6. Aircraft design models and processes

This section is devoted to the presentation of the models and processes used to build the objective and constraints functions of the optimization problem. It is important to notice that we present models and processes from a global point of view and we do not go into details (see [3] for more details). The problem to be solved is presented at the end of the section.

6.1. Processes and Models

Aircraft preliminary design is a multidisciplinary process where different physics interact, whose most important are Geometry, Aerodynamics, Weights and Propulsion. Each field is composed of intrinsic integer or real parameters, numerous non-linear and non-convex functions of all parameters, and even with non linear equations system solvers. To sum up all combined models count around 180 parameters and 50 functions. A complete description of these models can be found in [3].

Then, we have to choose some variable parameters that will allow to define an aircraft configuration and that will then be optimized. Engineers practice and know-how lead us to choose the Wing Area that controls the dimension of the wing and the Sea Level Static Thrust ($SLSThrust$) that controls the engine size. At the same time we need to fix some requirements, e.g. the range of the aircraft that corresponds to some customer demands (Table 2).

Finally the future aircraft configuration has to be optimal given a selected objective. It bring us

Table 2: Description of Requirements

Name	Value
Number of Passengers (Npax)	180
Design Range	2000 NM
Cruise Mach number	0.76
Wing Aspect Ratio	9
Number of Electrical Engine	0 or 1
Number of Thermal Engine	2
By Pass Ratio	10
Reference Altitude (ZpRef)	35000 ft
Overall Pressure Ratio	40

about minimizing for example the Maximum Take-Off Weight (MTOW), the fuel consumption or the cash operating cost (COC). Moreover, the aircraft configuration optimization has to be computed with respect to some constraints corresponding to real needs for safety or operations (Table 3).

Table 3: Description of Constraints

Name	Value
Approach Speed (LdSpeed)	< 130 kt
Climb Vz Ceiling (ClbVz)	> 500 ft/min
Cruise Vz Ceiling (CrzVz)	> 300 ft/min
Take-Off Field Length 1 (at Sea Level) (Tofl1)	< 2000 m
Take-Off Field Length 2 (in High & Hot conditions) (Tofl2)	< 2500 m

Models and functions that drive the aircraft configuration computation are nested into each other and we try to present it from a global point of view in diagram of Figure 3. It shows the way the different physics interact, how an aircraft is computed from requirements and input variables and how objectives and constraints can be calculated.

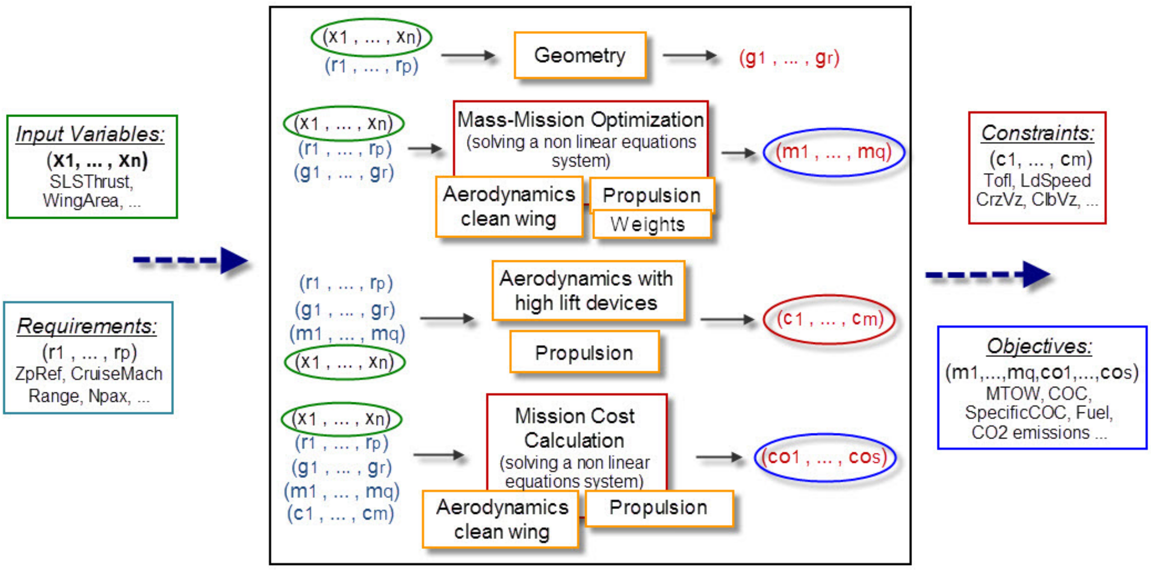

Figure 3: Aircraft Simple Design Process Diagram

Once the core of models have been introduced, we now focus on the additional degrees of freedom provided by the hybrid aircraft. First of all we decide to keep the conventional configuration and to add the electric engine on the rear fuselage of the aircraft (Figure 1). In order to power the electric engine we first add accumulators, in the airplane cargo hold for example. Secondly electric generators are placed in the turbofan engines so that we can get additional energy. An electric ratio parameter is introduced allowing to control the amount of energy given by the electric generators (as a part of the total thermal energy produced by the turbofan) to the electric engine. Then we make assumptions about how the electric powered aircraft differs from the conventional aircraft taking into account the new possibilities brought by hybridization. Clearly, it is not possible to envisage accumulators as a long term source of energy during the flight but only as an additional source that can be used temporarily to avoid thermal engine oversizing. The following strategy has been selected:

- We choose a variable pitch electric fan in order to get the thrust-reverser off from the two thermodynamic turbofans. Then we place the electric fan so that the carter of the fan could contribute to longitudinal and lateral stability of the airplane.
- We use electrical engine during critical phases (active design constraints) of the flight in order to relieve thermal engines performances and so decrease their dimensions. Notice that we assume that accumulators are assumed fully loaded when the aircraft takes-off: it allows them to power the electrical engine during this phase.
- During descent electrical engine is used to refill accumulators. Then during taxi-in thermal engines can be off and accumulators power the electrical engine to drive the aircraft to the parking. It is a way ofl reducing fuel consumption, but also noise and emissions at the airport.

Now we can build several models: first the new propulsion model of the electric turbofan, second the different mass penalty functions due to electric engine and accumulators and finally we modify the impacted functions from the processes in order to take into account the changes brought by the new items of the hybrid configuration. At last we are able to control the electrical part of the hybrid aircraft thanks to parameters listed in Table 4. Given values for Energy and Power density are fixed here according to last technologies values. We select as new degrees of freedom the Fan Power, the Generator Electric Ratio and the Fan Diameter for the optimization.

Table 4: Description of the electrical engines parameters

Name	Value
Fan Power	$1.5MW$
Energy Density of Accumulators	350 Wh/kg
Generator Electric Ratio	0.013
Power Density of Electrical Engine	3.5 kW/kg
Power Density of Electrical Generator	3.5 kW/kg

Running the processes for the hybrid aircraft we logically observe that current technologies performances do not allow this configuration to be as efficient as the conventional configuration. The idea proposed is to add to the models some prediction functions of the technology improvement as a function of the year for the power density of electrical engine and the power density of electrical generator (EPD), and for the energy density of accumulators (EED). Thanks to some assumptions found in literature ([15],[5]) we draw these functions with their corresponding uncertainty (Figure 4).

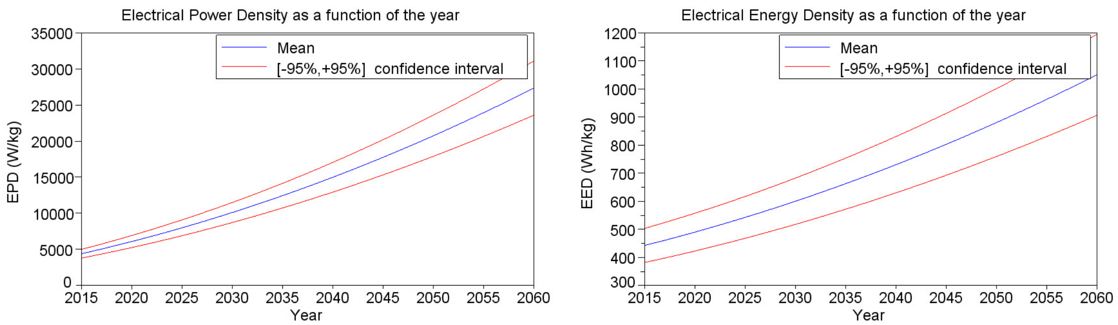

Figure 4: Prediction functions of electrical technologies evolution with their uncertainty (normal distribution with 50% of error)

6.2. Problem to be solved

To sum up, we previously describe the models and the way of calculating objectives and constraints. Now that this step is done we are able to run deterministic optimization of the conventional aircraft configuration and thanks to uncertainty introduced in prediction function of energy technologies evolution we can also run the chance constrained optimization. The goal is to find the closest year from now when the hybrid aircraft will have the same value of the given objective as the conventional configuration with a 0.95 probability. We then deal with two optimization problems for the conventional and the hybrid configurations, still sharing the same requirements. We denote:

- x_1 the Wing Area (in m^2),
- x_2 the SLSThrust (in N),
- x_3 the Generator Electric Ratio (no dimension),
- x_4 the Fan Power (in W),
- x_5 the year (driving the technology maturity).

Let also $F : \Omega \to \mathbb{R}$ denote the black box type function computing the objective and $G : \Omega \to \mathbb{R}$ the black box type function computing the constraints of Table 3. We have:

- For the conventional configuration, the non-linear, non-convex and constrained deterministic optimization problem (with constraints of Table 3) is of the form:

$$\left\{\begin{array}{l} \min\limits_{x \in \Omega_c} \ F_c(x), \\ \text{s.t.} \ \left\{\begin{array}{l} G_{C_1}(x) \leq 500, \\ G_{C_2}(x) \leq 300, \\ G_{C_3}(x) \geq 2000, \\ G_{C_4}(x) \geq 130, \\ G_{C_5}(x) \geq 2500, \end{array}\right. \\ \text{where: } x = (x_1, x_2),\ \Omega_c = ([100, 600] \times [100000, 350000]) \subset \mathbb{R}^2. \end{array}\right. \tag{18}$$

  Let $F_{C_{min}}$ denote the value $F_c(x_{opt})$ where x_{opt} is the solution of problem defined by Equation (18).

- For the hybrid configuration, we add uncertainty around the battery technologies and we solve the non-linear, non-convex and constrained chance-constrained optimization problem is of the form:

$$\left\{\begin{array}{l} \min\limits_{x \in \Omega_h} \ x_5, \\ \text{s.t.} \ \left\{\begin{array}{l} \text{Prob}\big(G_{C_1}(x) \leq 500\big) > 0.95, \\ \text{Prob}\big(G_{C_2}(x) \leq 300\big) > 0.95, \\ \text{Prob}\big(G_{C_3}(x) \geq 2000\big) > 0.95, \\ \text{Prob}\big(G_{C_4}(x) \geq 130\big) > 0.95, \\ \text{Prob}\big(G_{C_5}(x) \geq 2500\big) > 0.95, \\ \text{Prob}\big(F_h(x) \leq F_{C_{min}}\big) > 0.95, \end{array}\right. \\ \text{where: } x = (x_1, x_2, x_3, x_4, x_5),\ \Omega_h = \Omega_c \times [0.01, 0.02] \times [10^6, 3 \cdot 10^6] \times [2012, 2060] \subset \mathbb{R}^5. \end{array}\right. \tag{19}$$

Since F and G are both black box functions (see Figure 3), a well adapted tool for solving problem (19) is derivative-free optimization (DFO). We then select among the whole set of DFO's methods the Nelder-Mead algorithm and a differential evolution algorithm both modified to deal with constraints.

7. Results

Once both thermal and hybrid propelled aircraft optimization problems are now defined we are able to run deterministic and chance-constrained optimization. We choose as objective the Maximum Take-Off Weight of the aircraft (MTOW), a recurring parameter to be optimized in preliminary design phases. We first run a deterministic optimization of a conventional configuration (without hybridization) (cf problem (18)) and we obtain the results of Table 5. Active constraints are Climb Ceiling and cruise ceiling.

Table 5: Conventional Configuration Optimization results

x_1 SLSThrust (daN)	x_2 WingArea (m^2)	$F_{C_{min}}$ MTOW (kg)
10575	141.7	72672

Then we run the chance constrained problem (19) with $F_{C_{min}} = 72672$. In a first time we use differential evolution algorithm to find a solution satisfying the constraints in order to have a good start to the Nelder-Mead optimizer. We obtain a first result running a deterministic optimization (around $3h$) and then we ran the chance-constrained problem with two different errors, first with a normal distribution with 20% of error and then with 50% of error (around 8h). Results are shown in Table 6. Active constraints are in all cases the probabilities on Climb Ceiling, Cruise ceiling and on MTOW.

Table 6: Hybrid Configuration Chance-constrained and Deterministic Optimization results

	x_1 SLSThrust (daN)	x_2 WingArea (m^2)	x_3 ElectricRatio	x_4 FanPower (MW)	x_5 (=obj.) Year	MTOW (kg)
Determ.	10014	142	0.0166	1.0252	2022	72672
20% error	10014	142	0.0168	1.0247	2023	$P(. \leq 72672) = 0.95$
100% error	10015	142	0.015	1.0282	2025	$P(. \leq 72672) = 0.95$

The figures presented in Table 6 show that the hybrid configuration optimization results in a decrease of the engine size x_1 in comparison to the conventional configuration (Table 5). The uncertainty brought by technologies evolution leads to some changes in electric ratio and fan power in order to ensure the stochastic constraints and the objective minimization. Logically we also observe that the objective value is degraded as far as uncertainty is greater. However with the widest error, we obtain an encouraging result: the hybrid configuration should be equivalent to the conventional configuration with a 0.95 probability in year 2025.

8. Conclusion

This paper presented a method of uncertainty propagation by the way of moments that could be very powerful for robust optimization. The method is validated via a chance constrained optimization test case using the Rosenbrock function. The results show a significant improvement in term of computational time in comparison with the Monte-Carlo method without degrading the precision. In section **7.** an application to an industrial problem is presented: the chance-constrained optimization of an unconventional hybrid aircraft configuration. The results seem to be consistent. As a matter of fact, the hybrid aircraft configuration chance-constrained optimization suggests that the technologies evolution needed to reach the equilibrium with current conventional aircraft will occur in around 12 years. This bodes well for future. Moreover, through these results, it is important to stress the impact of a good adequateness between the technologies, the way to operate it and how it is implemented within the general arrangement. In other terms, an important work of integration needs to be done to take the maximum benefit of a new technology.

The next step of the study is to further develop the moment propagation method and to adapt it to widest and noisiest problems. A comparison with already existing uncertainty propagation methods has to be also considered. For industrial applications, optimization of the hybrid aircraft configuration can be extended adding degrees of freedom from engine geometry, airframe and flight trajectory. We also plan to introduce new objectives such as climatic impact. For sure, mixing multidisciplinary optimization with uncertainty propagation is a big challenge for future studies.

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