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Combining the best of both worlds - combination of homogenization and sensitivity based methods for shape optimization

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1. Abstract

The current paper describes a new method of combining sensitivity based shape optimization [16, 5, 8] with a homogenization method, the controller based method [19, 18, 9, 13].
The sensitivity or gradient based method is a classical mathematical approach which uses at least the first derivative of functions to minimize, maximize or constrain certain structural responses. The controller based method is a gradient-less homogenization method that uses the physics of a large class of structural problems where increasing material in highly stressed areas and decreasing material in other areas leads to a homogeneous stress distribution.
The combination of the two methods is rooted in the assumption that the sensitivity of an objective function can be approximated with the controller method's nodal values. This combines the best of both worlds and opens up the possibility to solve a large amount of problems, which for different reasons could not be realized within the single framework of either the sensitivity or homogenization approach. The new method is implemented in an industrial framework and an example from an automotive application is shown. The shape optimization is non-parametric in the sense that no CAD parameters are used. Instead, the nodal positions are the shape changing design variables.

Keywords: Shape optimization, industrial applications, large scale optimization

2. Introduction

Structural shape optimization is an important tool in todays industry especially for avoiding stress concentrations. Shape optimization is a great option to avoid tedious trial-and-error changes to achieve a lower stress or other goals. The dream of many engineers is to have a simple and fast solution where stress concentrations and fatigue issues can be solved without a massive gain of weight or other unwanted effects. This work is a contribution to get a step closer to a commercially available solution for these kinds of problems.
Since the ground braking paper from Zienkiewicz and Campbell [22] from 1973 there have been suggested many methods for structural shape optimization methods using the finite element method (FEM). Sensitivity based methods in shape optimization have evolved the last 40 years, for overview see e.g. Choi et al.[5] or Pedersen [16] and the references herein. An easy and efficient implementation of shape sensitivities uses semi-analytical sensitivities which have an unfortunate error discovered by Barthelemy and Haftka [2]. Various corrections for this problem has been proposed [17, 11, 4] where the latter by Bletzinger et al [4] is used in this current work. To avoid too many mesh-irregularities the sensitivities are filtered according to Bletzinger et al. [3]. Using the semi-analytic method sensitivities can be made available in the commercial optimization system *TOSCA Structure* [7] which relies on solving the primary structural FEM-equations with commercial finite element solvers (e.g. *Abaqus, Ansys, Nastran*, etc. [6, 1, 14, 20]). In non-parametric shape optimization we usually have many design variables (design nodes) and a few (< 100) constraints which makes the adjoint method the adequate choice. This work is restricted to linear elasticity and sensitivity calculations of displacements and volume. Extending shape sensitivity calculation with commercial solvers to non-linear analysis and other design responses e.g. stresses, will be done in other related research.
The great advantage of sensitivity based methods is that complex optimization problems with many objectives and constraints can be solved using a robust non-linear constrained optimizer. In this work Svanbergs MMA [21] is used. A major draw back is that only the design responses for which it is

possible to determine sensitivities can be used. Therefore, some interesting problems can not be solved e.g. minimizing fatigue using a commercial fatigue code (e.g. *Femfat* [12]). The reason for missing sensitivities can either be that functions are non-differentiable, but in our case the main issue is typically that the necessary information for sensitivity analysis in not readily available when using commercial products for calculating these results.

In 1979 Schack [19] proposed a non-parametric gradient-less shape optimization process for stress concentration problems using FEM. The homogenization method is based on Neuber's Fade-away Law [15]. Sauter [18] adopted this method in a modified form in the 1990's which became the commercial optimization product *TOSCA Structure* [7]. This homogenization method will in the following be referred to as the *controller method.*

Simply put, it can be described as adding material where high stresses are present and removing material in low stressed areas which (mostly) leads to a homogenization of the stresses and mostly a decrease of the maximum stress. One of the main advantages of controller method is that it can easily be integrated in commercial finite element solvers ([6, 1, 14, 20]) which makes shape optimization in industrial CAE-work-flow straight forward [7, 18]. Furthermore, the some key-features made the method successful:

- easy set-up; simply choose nodes in design area
- fast convergence; 5-15 iterations are often sufficient
- extension to support of highly non-linear analysis [13], even fatigue [9]

The major draw back of this gradient-less method is that it is very difficult, if not impossible, to implement a way to handle constraints. The current version of *TOSCA Structure* can only handle a single volume constraint. Constraints are essential in concurrent industrial design because of the increasing amount of specifications for the structural characteristics of almost any industrial component. Typically, these specifications include minimal stiffness requirements, maximal allowed stresses, dynamic properties like minimum eigenfrequencies.

This work will only focus on non-parametric shape optimization [13, 10, 8, 7]. Of course, the optimization changes geometric parameters, in our case nodal positions, but these are not parameters in the sense of a CAD-geometry. Discussion of advantages and disadvantages of non-parametric shape optimization can be found in the aforementioned references.

This paper describes a new shape optimization method combining the controller and sensitivity methods. This has to the author knowledge never been done before. It opens up for new range of shape optimization problems that can be solved in an efficient way. The problems are of industrial interest because it solves one of main draw backs of the controller method which is already widely used in the industry: The missing capability of including restrictions to the optimization problem. This is done without loosing the advantage of doing shape optimization with highly non-linear analysis types, e.g. optimizing for better fatigue behaviour. The new method is given the name: *controller-sensitivity method.* The new method is tested using an industrial example and the results are compared to the existing controller based method. Finally, the paper concludes and provides the reader an outlook.

3. Sensitivity analysis

In this work we use the semi-analytical adjoint method [11, 8] which is used to minimize, maximize or constrain a design response $\Psi(\mathbf{u}(\mathbf{a}), \mathbf{a})$ where $\mathbf{a}$ is the vector of design variables and $\mathbf{u}(\mathbf{a})$ are the primary variables in the linear static equilibrium:

$$\mathbf{K}(\mathbf{a})\mathbf{u}(\mathbf{a}) = \mathbf{f}(\mathbf{a}) \tag{1}$$

The stiffness matrix $\mathbf{K}(\mathbf{a})$, the displacements $\mathbf{u}(\mathbf{a})$ and the forces $\mathbf{f}(\mathbf{a})$ are dependent on the shape design variables. In the following the dependency on design variables $(\mathbf{a})$ is left out of the notation.

A typical optimization problem can be stated as:

$$\min\left(f\left(\mathbf{u}, \mathbf{a}\right)\right) \tag{2}$$

$$\text{subject to: } \mathbf{Ku} = \mathbf{f}$$

$$g_i\left(\mathbf{u}, \mathbf{a}\right) \leq 0 \quad i \in [1, N]$$

$$\mathbf{a}_{\min} \leq \mathbf{a} \leq \mathbf{a}_{\max}$$

The objective f is minimized subject to N constraints as well as the static equilibrium Eq.(1) and the side constraints $\mathbf{a}_{\min}$ and $\mathbf{a}_{\max}$. The objective f and the constraints g_i are given by one of the supported design responses Ψ. To solve the optimization problem Eq.(2) the gradients of the objective f and the constraints g_i are needed. Derivative of the design response is calculated using the adjoint method. Requiring symmetry of the stiffness matrix ($\mathbf{K} = \mathbf{K}^T$) the adjoint equation becomes:

$$\mathbf{K}\boldsymbol{\lambda} = \frac{\partial \Psi}{\partial \mathbf{u}} \tag{3}$$

Solving the above, the derivative is evaluated by:

$$\frac{d\Psi}{d\mathbf{a}} = \frac{\partial \Psi}{\partial \mathbf{a}} - \boldsymbol{\lambda}^T \left(\frac{d\mathbf{K}}{d\mathbf{a}} \mathbf{u} - \frac{d\mathbf{f}}{d\mathbf{a}} \right) \tag{4}$$

TOSCA Structure uses a commercial FEM-solver to solve the equations Eq.(1) and Eq.(3) which means we must evaluate $\frac{\partial \Psi}{\partial a}$ and $\frac{\partial \Psi}{\partial u}$, which is straight forward in the case of displacements:

$$\frac{\partial \Psi}{\partial a} = 0$$

$$\frac{\partial \Psi}{\partial u} = 1$$

Further, to evaluate the second term of Eq.(4) we assume that the external forces $\mathbf{f}$ do not change with the design variables ($\frac{df}{da} = 0$). The derivative of the stiffness matrix is approximated with the finite difference

$$\frac{d\mathbf{K}}{d\mathbf{a}} \approx \frac{\Delta \mathbf{K}^*}{\Delta \mathbf{a}}$$

Where $\frac{\Delta \mathbf{K}^*}{\Delta \mathbf{a}}$ is corrected with the correction term from Bletzinger et al. [4].
Volume sensitivities are easily obtained using geometric considerations and these are not dependent on the finite element equation system as such.

4. Combining the best of both worlds

Adding constraints to the controller method seems to a near impossible task. Instead we assume that the controller values are good enough to minimize the objective and we use this information as a pseudo-sensitivity for the objective. The controller values are the stress, strain or fatigue values which are present for all design nodes. For the constraints we use the correct sensitivities from Eq.(4). Applying a sufficiently robust optimizer that can handle the inaccurate pseudo-sensitivities in the objective can now solve the optimization problem.
The new method for shape optimization is called; *controller-sensitivity method,* were we state the new optimization problem:

$$\min \left(f^*(\mathbf{u}, \mathbf{a}) \right) \tag{5}$$

$$\text{subject to: } \mathbf{K}\mathbf{u} = \mathbf{f}$$

$$g_i\left(\mathbf{u}, \mathbf{a}\right) \leq 0 \quad i \in [1, N]$$

$$\mathbf{a}_{\min} \leq \mathbf{a} \leq \mathbf{a}_{\max}$$

where f^* is a function supported by the controller approach (stress, strain, fatigue, etc. see *TOSCA Structure* Manual [7]). Our trick is to set:

$$\frac{df^*}{da} = -\alpha\, \mathbf{c} \tag{6}$$

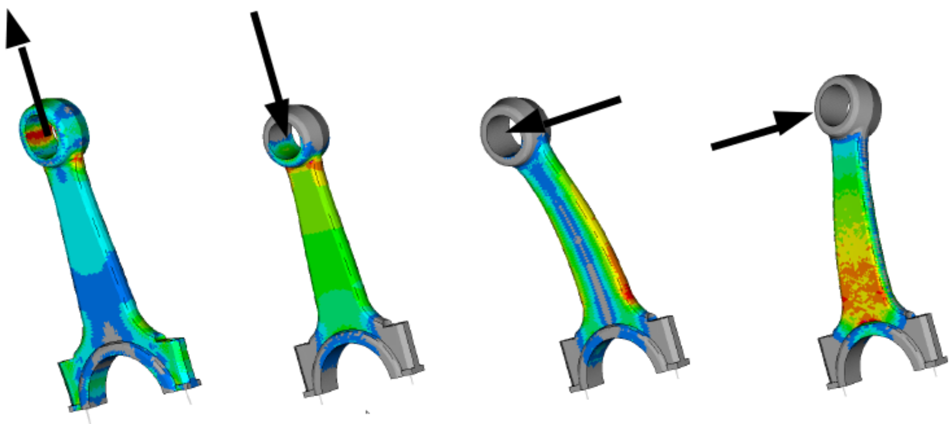

Figure 1: Load cases for a conrod where the arrows show the applied force, the colour plot are the stresses of each load case.

where $\mathbf{c}$ are the nodal controller values and α is a scale-factor. This calls for some comments. First of all we assume that the controller method is really minimizing the objective thus the negative sign in Eq.(6). That the objective is really minimized is not always given and may be quite problem-dependent. The assumption may not hold rigorous mathematical proof, but the experience of the author and many *TOSCA Structure* users show that the minimizing of the objective is very often given by doing homogenization provided by the controller method. Secondly, we encounter a problem of scale factor α. We assume that the pseudo sensitivities must be in the same value range as the sensitivities of the constraints. This is solved by scaling the controller input to the range of the volume sensitivities.

4.1 Controller-sensitivity method example

This example shows a connection rod (conrod) with 4 load cases, see figure 1. The model consists of 68068 linear tetrahedral elements, 15102 nodes (45306 DOFs) and some rigid bodies, bar-elements which compromises a typical industrial application. The material is linear isotropic and the solving the primary and adjoint system is done *NX Nastran* [20]. The fatigue analysis is done with *Femfat* [12].
The load cases are tension (LC1), compression (LC2) and two stiffness loads; one in the mid-plane of the conrod (LC3) and one out of this plane (LC4). The optimization problem is minimization/homogenization of fatigue combined from the load cases LC1 and LC2 (f^*_{damage}) with constraints on the maximum displacements in LC3 and LC4 ($\bar{u}_{\mathrm{LC1}}$ and $\bar{u}_{\mathrm{LC2}}$) as well as a volume constraint ($\bar{V}$). The constraint values are chosen to be 99% of the original. The optimization problem is thus posed as follows:

$$\min\left(f^*_{damage}(\mathbf{u},\mathbf{a})\right) \tag{7}$$

$$\text{subject to: } \mathbf{K}\mathbf{u}_{\mathrm{LC}j} = \mathbf{f}_{\mathrm{LC}j} \quad j \in [1,4]$$

$$g_1(\mathbf{u},\mathbf{a}) = u_{\mathrm{LC1}} \leq \bar{u}_{\mathrm{LC1}} = 0.99\, u^{ORG}_{\mathrm{LC1}}$$

$$g_2(\mathbf{u},\mathbf{a}) = u_{\mathrm{LC2}} \leq \bar{u}_{\mathrm{LC2}} = 0.99\, u^{ORG}_{\mathrm{LC2}}$$

$$g_3(\mathbf{a}) = V \leq \bar{V} = 0.99\, V^{ORG}$$

$$\mathbf{a}_{\min} \leq \mathbf{a} \leq \mathbf{a}_{\max}$$

The figure 2 shows the convergence of the new method (*CTRL/SENS*) and for comparison the pure controller method without displacement constraints, but inclusive the volume constraint, is plotted as well (*CTRL*). Both solutions have a final volume of 99% of the original.

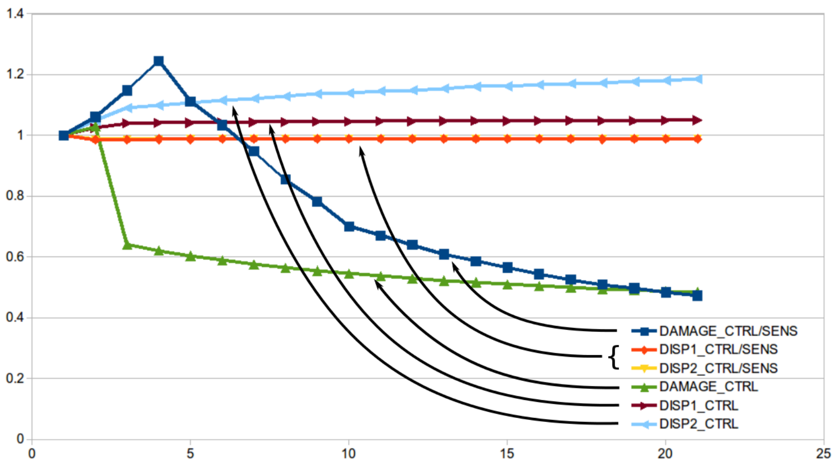

Figure 2: Convergence of normalized values. *CTRL/SENS*: Controller-sensitivity method. *CTRL*: Pure controller method. *DAMAGE*: The maximum damage calculated by *Femfat*. *DISP1* and *DISP2*: Displacements of load cases LC3 and LC4

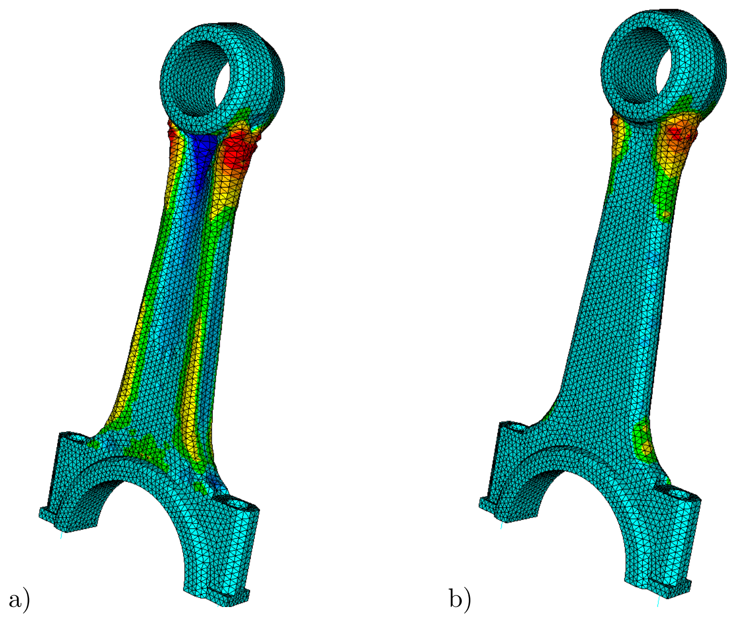

Figure 3: Optimization displacements. Red colour: +3mm. Blue colour: -3mm. a) New combined controller-sensitivity method. b) Pure controller based method.

The convergence of the controller method is very rapid, but it is not possible to constrain the displacements of LC3 and LC4 (see figure 2). The combined controller-sensitivity method has a much slower convergence, but it keeps the constraints on the displacements. The displaced nodes from the shape optimization are shown in figure 3.

5. Conclusion

The controller-sensitivity method shows a large potential for industrial relevant applications. Using highly non-linear responses like fatigue is a typical industrial application for shape optimization with *TOSCA Structure*.shape. The ability to use extra constraints will get a warm welcome by the industrial users. Although, we were able to show a successful application here there is still some work to be done to make the method robust enough for to be used in a standard industrial work flow.
Another major issue is the extension of possible constraints of the method. The possibility to support linear stress constraints is a currently being pursued in another research project (ShapeOpt2CAD[1]). Constraints based on non-linear analysis using commercial finite element solvers will be investigated further in a near future.

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