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Structural Optimization of Motion Structure using Constraint Force Design Method

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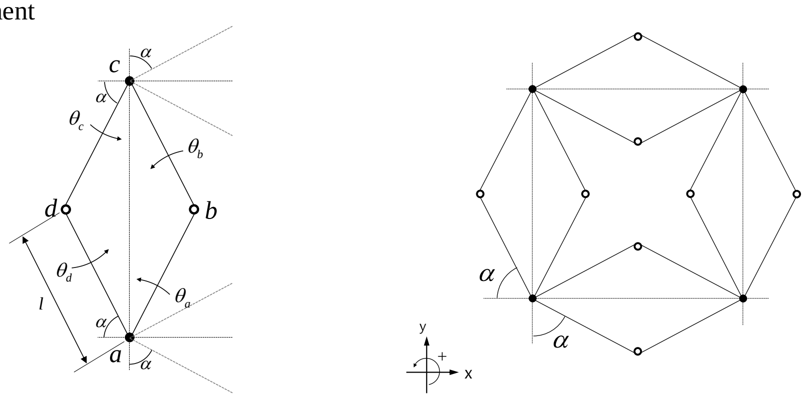
1. Abstract
This research develops a new structural optimization method for foldable motion structure with repeated rigid-body mechanism cell. Compared with conventional rigid-body mechanism, there are many kinematic advantages of a motion structure as it can achieve large shape transformation for real applications such as satellite solar panels, space antennas to shelters, and swimming pool cover. Until now, many researches related with the design of foldable motion structure are focused on how to design an internal spatial unit structure to dictate the nature of structural deformability. However, only designing an internal spatial unit structure has limitations to design an arbitrary shaped structure. Therefore, this research proposes a new motion structure design approach which determines the rotational direction of joints and setting angles between unit cells by using a genetic algorithm to design an arbitrary shaped structure. The straight rigid bars connected by pivot joints are used as an internal spatial unit and an appropriate formulation of objective function is proposed which gives a desired motion structure after an optimization. Through the proposed motion structure design method, the rotational direction of joints and setting angles of several unit cells can be determined automatically and this leads to obtain a desired arbitrary shaped structure in stable.
2. Keywords: foldable motion structure, design optimization, genetic algorithm.

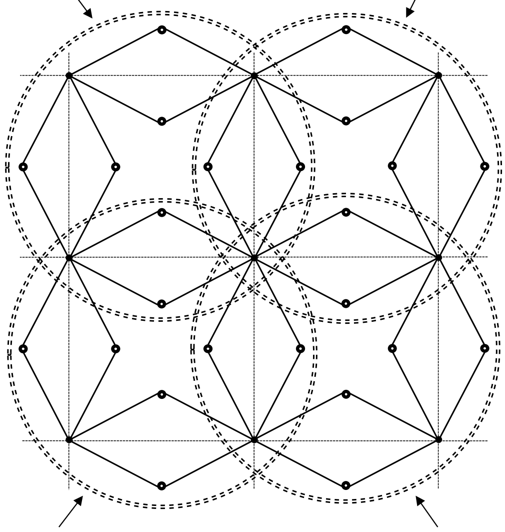
3. Introduction
Foldable motion structure is comprised of repeated rigid-body mechanism which has internal mobility and allows large shape transformation to satisfy actual requirements. Unlike conventional structures, it is used for various fields such as space antennas, exhibition stands and medical implants. To the best of our knowledge, a number of studies have been carried out to discover deformation mobility of motion structure by developing a new topology of rigid body mechanisms [1, 2] or by studying general repeated rigid body mechanism[3, 4]. However, there is few motion structure design approach which determine the connectivity and setting angles between several repeated unit cells to achieve an arbitrary shaped structure for various purpose of installation place. If we can determine the connectivity and setting angles between unit cells for the arbitrary shaped structure, a required time for installation of motion structure can be reduced which gives benefit for an emergency situation. Therefore this research proposes a new motion structure design approach by using the genetic algorithm for an arbitrary shaped structure consisting of the 4RE linkage of figure 1(b).

4. Structural optimization of motion structure
The kinematic approach with repeated building blocks is employed to design a motion structure consists of straight rods, pivots, and scissor hinge. To determine planar mobility modes of motion structure, the motion structural analysis approach with rhombic element which is suggested by Tanaka et al. is considered [5].

4.1. Rhombic element



(a) (b)



(c)

Figure 1: The motion structure, (a) rhombic element, (b) 4RE linkage(repeated building block with rhombic element) and, (c) a motion structure.

Figure 1(a) shows the rhombic element model of 4 rigid straight links connected by several joints. To build repeated building block which is called as 4RE linkage of figure 1(b), rhombic element is used as unit cell. For design of motion structure, 4RE linkage which consists of 16 rigid straight links, 8 two bar joints and 4 four bar joints is also considered as unit cell. Figure 1(c) shows the motion structure model of 4 4RE linkages connected by 8-bar pivot joints. In this research, to propose a new motion structure design approach, we consider this motion structure.

4.2. The constraint force design method

The constraint force design method[6, 7] is used for topology and size optimization of rigid-body mechanism. To optimal design of rigid-body mechanism, general existing synthesis methods are based on the kinematic analysis of an initial mechanism with a fixed number of links and joints. In other words, graphical or analytical techniques are commonly used to express the trajectory of a given mechanism. Unlike general synthesis method, this method use kinetic analysis rather than kinematic analysis. The key ideas of the present constraint force design method are that unit masses are represented as revolute or prismatic joints depending on displacement constraints, and that the artificial forces maintaining the relative lengths among the unit masses are represented as rigid links as shown in figure. 2. The set of dynamic equation of masses with forces imposing the relative lengths among masses can be described using Lagrange equation as follows :

$$L = \frac{1}{2}\sum_i m_i \dot{\mathbf{r}}_i^2 - U(\mathbf{r}_i) \qquad (\mathbf{r}_i^2 = \mathbf{r}_i \cdot \mathbf{r}_i) \tag{1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{r}}_i}\right) - \frac{\partial L}{\partial \mathbf{r}_i} = \sum_{k=1}^{N_i^{RL}} \lambda_k (\mathbf{r}_{ij}^2 - d_{ij}^2) \quad (i = 1, 2, ..., n) \quad (\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j,\ \mathbf{r}_{ij}^2 = \mathbf{r}_{ij} \cdot \mathbf{r}_{ij}) \tag{2}$$

Where L is the Lagrangian (the summation of kinetic energy and potential energy). The position and velocity of the i -th amsses are denoted by $\mathbf{r}_i$ and $\dot{\mathbf{r}}_i$, respectively. The i -th masses m_i has constant mass. The total number of unit masses is n, and the number of length constraint of the i -th mass is N_i^{RN}. The Lagrangian multiplier for the k -th length constraint is λ_k. The current distance vectors and the imposed relative distance between the i -th and j -th masses are denoted by $\mathbf{r}_{ij}$ and d_{ij}. We use Eq. (1,2). It is possible to derive Newton's second law :

$$m_i\ddot{\mathbf{r}}_i = \mathbf{f}_i + \mathbf{g}_i \quad (\mathbf{f}_i \equiv 0) \quad (i = 1, 2, ..., n) \tag{3}$$

where the acceleration of the i -th masses are denoted by $\ddot{\mathbf{r}}_i$. The forces applied to the i -th masses is denoted by $\mathbf{f}_i$, and the force acting on the i -th masses from the length constraint is $\mathbf{g}_i$. The constraint force is defined by using length constraint which composed between i -th and j -th masses and length constraint is defined:

$$\mathbf{g}_i = -\sum_{k=C_i}^{N_i^{RL}} \lambda_k \nabla_i \sigma_k \quad \text{and} \quad \nabla_i \sigma_k = 2\mathbf{r}_{ij} \tag{4}$$

$$\sigma_k \equiv \mathbf{r}_{ij}^2 - d_{ij}^2 \tag{5}$$

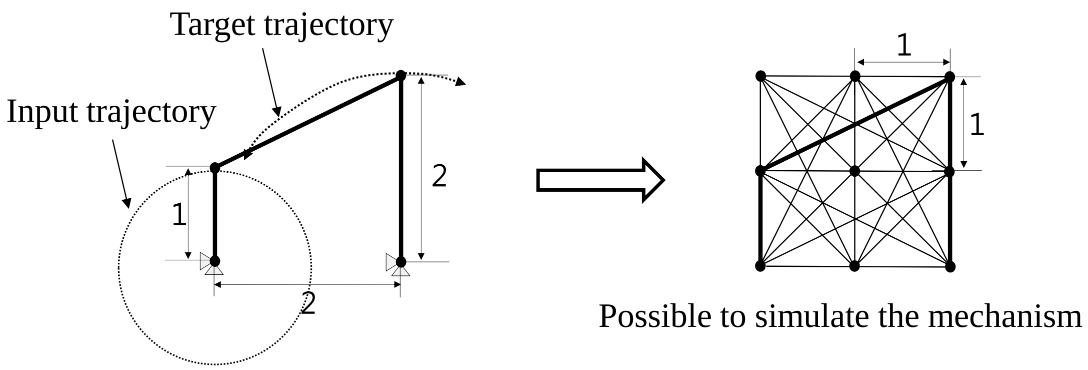


Figure 2: The concept of the constraint force design method applied to a four-bar linkage.

4.3. Parameterization of the rotational direction of joints and values of rotary angle

For the motion structure design approach, it is important to devise a proper parameterization method for the rotational direction of joints and step size of angle. For this purpose, we find that deformation mobility of motion structure can be determined by controlling the rotational direction of joints and step size of angle of repeated building block. To parameterize there values, The first, a rotational direction of joints is assigned binary designs variable (Figure 3(a)) :

$$\gamma_i = \begin{cases} 0 & : \text{Clockwise motoin} \\ 1 & : \text{Counter Clockwise motoin} \end{cases} \quad (i = 1, 2, ..., 8) \tag{6}$$

The second, a step size of angle is also assigned integer design variable as show in figure 3(b). Therefore, to find out parameterization method, we use mixed design variables :

$$\mathbf{x} = [\ \underbrace{\gamma_1 \cdot \cdot \cdot \gamma_8}_{\text{Rotary direction of joint}}\ ,\ \underbrace{\beta}_{\text{Angle}}\ ] \qquad (0 \le \gamma_i \le 1) \quad (0 \le \beta \le 90) \tag{7}$$

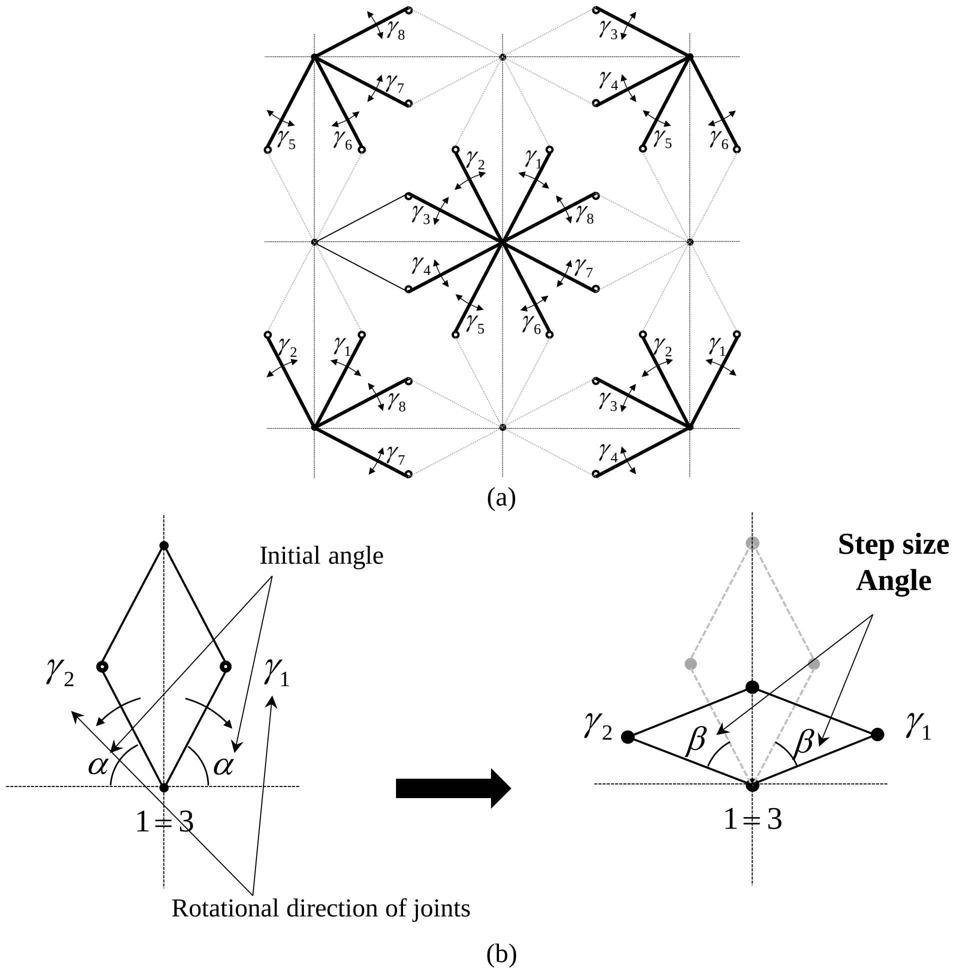


Figure 3: Parameterization of the rotational direction of joints and values of rotary angle, (a) a rotational direction of joints of repeated building block and, (b) a step size angle of joints.

4.4. Optimization formulation

Given that the parameterization method uses binary and integer design variables, a genetic algorithm (GA) [8] as show in the figure 4 is most suitable algorithms for the structural optimization of motion structure. To implement the genetic algorithm into the structural optimization problem, a fitness function must be devised. In this research, to find out general structural optimization, we consider the objective functions :

$$l_{ij} = \sqrt{(x_{ref,i} - x_{current,j})^2 + (y_{ref,i} - y_{current,j})^2} \quad (i = 1,2,...,n) \ (j = 1,2,...,m) \tag{8}$$

$$\text{Min} \quad \sum_{i=1}^{n} \min(l_{i,1},\ l_{i,1},\ l_{i,1},...,\ l_{i,m}) \tag{9}$$

where the reference edge point position and the current edge point position are denoted by $x_{ref,i}$, $y_{ref,i}$, $x_{current,i}$ and, $y_{current,i}$, respectively. The total number of reference edge point and total number of current edge point are denoted by n and m, respectively. The distance vector between reference edge point and current edge point is denoted by l_{ij}.

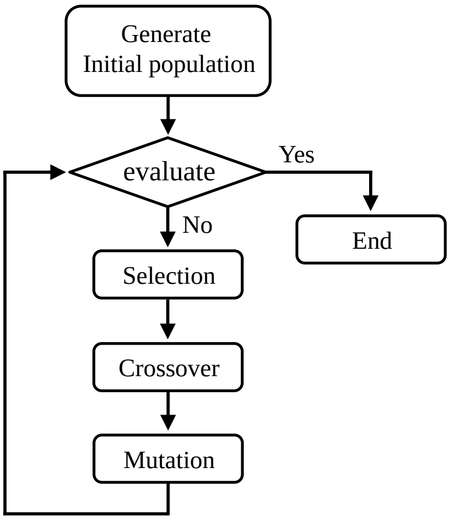
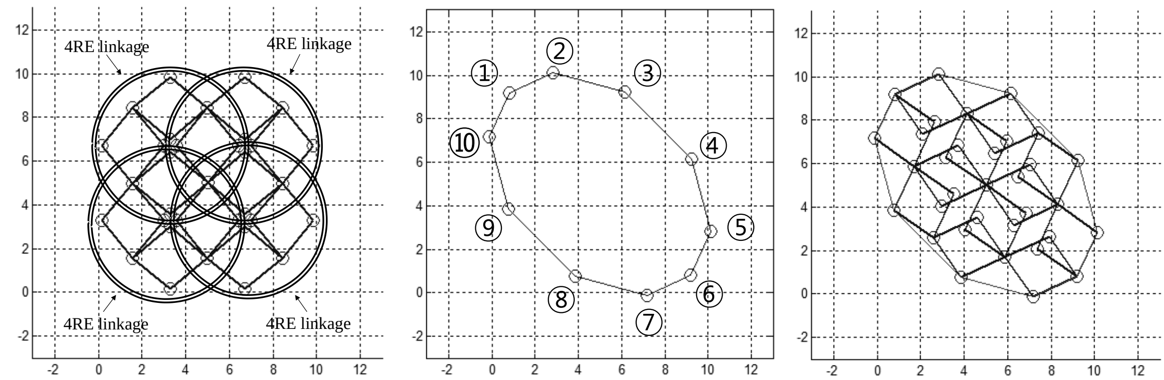


Figure 4: The flow chart of the genetic algorithm

4.5. Numerical example

To validate the performance of the present method, the numerical example of structure optimization of motion structure is considered in figure 5. Before finding optimal deformation mode, it is possible to consider initial condition of motion structure which consisting of 4 4RE linkage, as shown in figure 5(a). Figure 5(b) show reference edge point of motion structure and actual point value is denoted by table 1. By using the new method, the optimal deformation mode is found, as shown in figure 5(c).



(a) (b) (c)

Figure 5: Numerical example, (a) initial mode of motion structure, (b) reference deformation mode of motion structure and, (c) optimal deformation mode using objective function.

Table 1: Reference point position value

Position	Point 1	Point 2	Point 3	Point 4	Point 5
x	0.8042	2.8308	6.1399	9.2541	10.1408
y	9.1958	10.1408	9.2541	6.1399	2.8308
Position	Point 6	Point 7	Point 8	Point 9	Point 10
x	9.1958	7.1692	3.8601	0.7459	-0.1408
y	0.8042	-0.1408	0.7459	3.8601	7.1692

5. Conclusion

In this paper, we introduce the structural optimization method for motion structure. Unlike other synthesis method, this newly developed method use kinetic analysis rather than kinematic analysis for deformation mobility of motion structure. This synthesis method make possible to overcome several limitations of conventional synthesis method.

6. Acknowledgements
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