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Application of Group Based Sorting Method to Multiple-Constrained Optimization Problems

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1. Abstract

The purpose of this study is to develop a practicable method for multiple-constrained optimization problems. In real-world optimization problems, the complexity of design space depends on their formulation. Thus, in order to search useful solutions within feasible computational time, the simplicity of applications is necessary for the technique of constraint satisfaction. The proposed method of this paper attempts to treat multi-constraint conditions with simple rules by sorting in the order of satisfactory degree to each constraint. In this method, a criterion to satisfy each constraint can be defined respectively. Then, solution candidates are divided into groups based on their violated constraint conditions. In a group, candidates are compared by the criterion appropriate to corresponding constraints. In addition, the relation among constraints is represented as the comparison of groups. Therefore, the optimization method can obtain practicable solutions efficiently with the simple formulation of constraint conditions. Furthermore, it is expected that the proposed technique is useful for the evaluation of multi-attributes. In this paper, by applying to Genetic Algorithm and Particle Swarm Optimization in several problems, numerical examples are presented to demonstrate the practical utility of the proposed method.

2. Keywords: Optimization, Constraint satisfaction, Sorting rule, Probabilistic search algorithm

3. Introduction

The optimization of real-world problems is very difficult due to various factors, such as constraint conditions, different objectives and uncertainties. Hence, the design space of many problems involves complicated structure that cannot be handled by the mathematical approach. Therefore, these optimizations are generally defined as the combinatorial problems. The probabilistic search algorithm, such as Genetic Algorithm (GA) [1] and Particle Swarm Optimization (PSO) [2], is used as the useful method for the optimization problems described above [3]. This method can obtain optimal solution efficiently by the intensive and global searches. However, the effectiveness of many methods has been demonstrated in the test problems. Due to the circumstances, it is necessary to develop a practicable method for real-world problems.

In this paper, an attempt is made to propose an effective method for the multiple-constrained optimizations. In the existing researches [4], many approaches transform the problem to a non-constrained problem, such as the penalty function method. However, the transformation of multiple constraint conditions is likely to cause the complication of design space. On the other hand, the probabilistic search method can search feasible solutions directly in the design space with constraints. This approach significantly depends on the search efficiency of optimization method. Therefore, the improvements of both the search method and the constraint satisfaction technique are required for optimization problems. In addition, to keep the simplicity of the method is important for the practical utility. This is because the application is performed by trial and error based on the mechanism of search and the analysis of obtained results.

This paper proposes Group Based Sorting (GBS) as a method that can handle multiple constraint conditions by the simple sorting rule. In the proposed technique, solution candidates are divided into groups based on their states to the constraint conditions. Then, divided candidates are compared and sorted in each group. In the group for the violation of certain constraint condition, this operation is performed by using the rule with a criterion appropriate to satisfy the corresponding constraint, such as the violation quantity of given limitation. In addition, the relationship of multiple constraints is handled by the comparison of groups. Because these rules are described by the simple comparison of solution candidates, it is expected that GBS technique enables optimization methods to search useful solutions efficiently with the simple design space. This paper applies this technique to GA and PSO. And, numerical experiments using several problems are presented to demonstrate the applicability of the proposed technique for real-world problems.

4. Group Based Sorting Technique

This paper aims to propose a new technique useful to any optimizations. In order to achieve this purpose, the proposed technique was developed by taking into account the general versatility, simplicity and extensibility.

Therefore, the approach using Group Based Sorting technique is expected to be effective for the application to not tests but real-world problems.

4.1. Outline

In general, optimizations of real-world problems have multiple constraint conditions. These constraints include various elements, which partially or holistically have the correlation or are independent. For this reason, it is very difficult to transform these constraint conditions and the objective function to one criterion. Therefore, GBS technique divides solution candidates into groups based on their states according to constraints. Then, each constraint condition is handled respectively in the corresponding group. For example, a problem with two constraints is treated by the proposed technique, as shown in Figures 1 and 2.

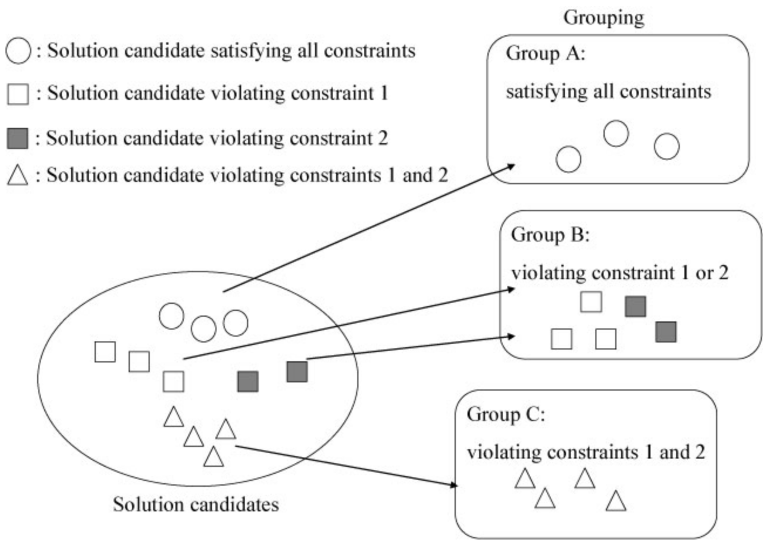

Figure 1: Grouping of solution candidates

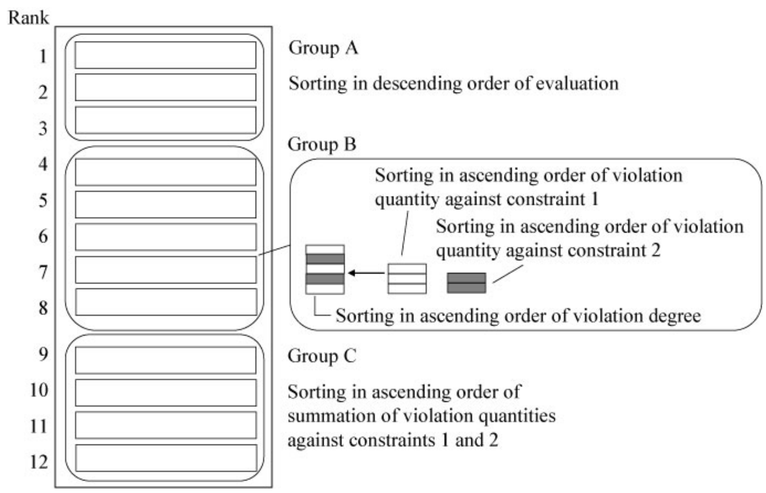

Figure 2: Sorting of solution candidates

Figure 1 shows the grouping of solution candidates by the method with GBS. In this paper, solution candidates are grouped based on the number of constraint violation. In this example, Group A is a set of candidates satisfying constraints. A solution candidate violating one constraint is divided in Group B. And, Group C is composed of candidates violating both constraints 1 and 2. Then, solution candidates are sorted based on the relationship among groups and the criterion according to each group, as shown in Figure 2. In Group A, solution candidates satisfying constraints are sorted in descending order of evaluation in order to search for optimal solutions. On the other hand, candidates of Groups B and C are sorted in ascending order of the criterion, such as the violation quantity, in response to each constraint condition. In this way, the constraint satisfaction is performed by the simple rule. In addition, GBS technique ranks the current solutions based on the priority of each group. This ranking enables a method to treat the relations of various elements, such as objectives, demands and constraints. In the example of Figure 2, the priority of each group is decided by the number of violating constraints. Because this value is a criterion measuring the degree of constraint violation, its reduction promotes the constraint satisfaction. Especially, if constraints have the same property, it is expected that the criteria of this example are effective for the constraint satisfaction.

In the proposed technique, the criterion for the constraint satisfaction does not need to be transformed from the violation quantity. This is because the criterion of sort is decided in response to the corresponding group. Furthermore, this sorting is considered as the comparison of solution candidates. Thus, GBS technique can adapt to various methods such as the local search, the multipoint search, the evolutional algorithm and the swarm intelligence. Moreover, the coding of this technique can be realized easily by using the sorting function and the comparative operator like function or class for the comparison, which have been implemented preliminarily in many programming languages. Therefore, it is expected that GBS method is useful for multi-constrained optimizations.

4.2. Approach of Group Based Sorting

Various optimization methods can easily introduce GBS technique to the comparison of solution candidates. In the application to Genetic Algorithm [1], this technique is applied to the setting of individuals' fitness as shown in Figure 3. This GA can control the evolution of population. Firstly, all individuals are sorted by the procedures described in section 4.1. Secondly, their fitness is calculated based on their ranks. In this calculation, the linear normalization is useful even if equivalent individuals are included. This is because the replacement of individuals per generation is performed stochastically in GA. Finally, the natural selection is performed by using the fitness decided from these processes. In this way, individual of each group evolves based on the different criterion. On the other hand, all individuals are evaluated relatively by their fitness to the optimal solution. In addition, even if an individual satisfying constraints generates children violating constraints, their gene information promotes the constraint satisfaction of others because they have similar genetic array to their parents. Therefore, the satisfaction of constraint and the optimization are realized simply by the GA with GBS.

If the difference between solution candidates cannot be evaluated, it is difficult for the probabilistic search algorithm to search better solutions. Hence, the comparison of candidates significantly influences the search performance. In the search process, while the comparison is workable, solution candidates improve their evaluations. This implies that an appropriate rule for the comparison can prevent the search from being trapped by the initial convergence and the stagnation. Through the description of simple comparison rule, the proposed technique attempts to improve the practical utility of optimization methods.

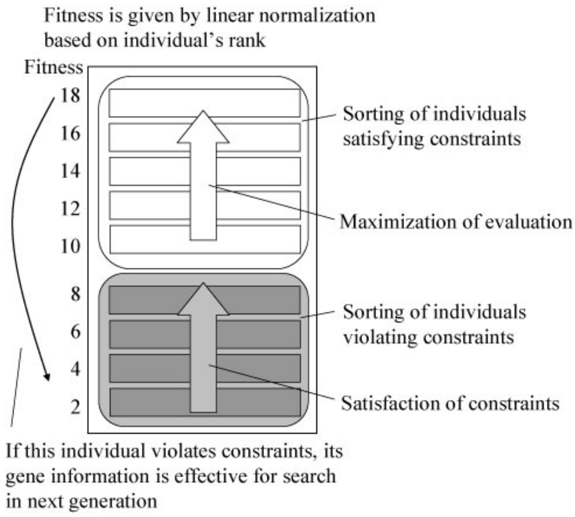

Figure 3: Control of evolution by GBS technique

5. Application to Test Functions

The applicability of Group Based Sorting technique is demonstrated by the application to 5 functions proposed by Michalewicz [5]. These functions G_1 to G_5 have multiple constraint conditions. This paper compares the proposed technique with the alpha constrained method [6, 7]. This is because the constraint satisfaction of that method is based on the ranking and comparison like GBS technique.

In these numerical experiments, this paper uses Particle Swarm Optimization [2] as the useful method to the optimization designed by the continuous value. In PSO, GBS technique is applied to the comparison for the updating of the best particle. In order to investigate the influence of searching ability of optimization method, the *gbest* model and the *lbest* model PSOs (GBS-*g*-PSO and GBS-*l*-PSO) are applied to test functions respectively. Here, the *lbest* model PSO is more effective for complicated problems than the *gbest*. On the other hand, α -PSO, which is PSO with the alpha constrained method [6], is applied as a target of comparison. In the configuration of common parameters for each method, the number of particles is 70, the number of iterations is 5,000, the weight parameters c_1 and c_2 are 2 and the limit of velocity v^{max} is the difference between the upper and lower limits of domain in each function. In the parameters of inertia weight, the initial value w^0 is 1 and the terminal value w^T is 0.2. In the constraint satisfaction of GBS method, the summation of quantity violating each constraint condition and the number of violating are used as the criteria. A particle with fewer violations against constraints is superior to another. If the number of violations is equal, a particle that has lower violating quantity is better. In the α -PSO, the violating quantity of each condition is used as the degree of constraint satisfaction. The parameters b_i and b_j for this degree are set the maximum violating quantity dynamically in the search. The control of alpha level is applied to functions G_2, G_4 and G_5. On the other hand, the alpha level used for G_1 and G_3 is fixed to 1. Using these parameters and executing each method 50 times, obtained results are shown in Table 1. In addition, the optimal solution of each function is shown in Table 2.

Table 1: Result of each function

Function	Method	Average	Best	Worst	Standard deviation	Constraint satisfaction
G_1	GBS-*g*-PSO	-14.180	-15.000	-12.000	1.335	50 / 50
	GBS-*l*-PSO	-15.000	-15.000	-15.000	0.000	50 / 50
	α -PSO	-14.600	-15.000	-12.000	1.010	50 / 50
G_2	GBS-*g*-PSO	24,134.850	11,000.000	30,000.000	5,306.675	42 / 50
	GBS-*l*-PSO	20,941.326	11,100.000	30,000.000	5,811.385	50 / 50
	α -PSO	12,221.148	7,657.447	30,000.000	5,578.120	19 / 50
G_3	GBS-*g*-PSO	1,074.435	680.631	10,525.491	1,948.775	50 / 50
	GBS-*l*-PSO	680.647	680.632	680.683	0.009	50 / 50
	α -PSO	680.642	680.632	680.666	0.009	45 / 50
G_4	GBS-*g*-PSO	1.021	0.459	4.197	0.711	39 / 50
	GBS-*l*-PSO	0.810	0.409	1.066	0.217	30 / 50
	α -PSO	-	-	-	-	0 / 50
G_5	GBS-*g*-PSO	797.475	143.810	3,869.402	1,062.093	50 / 50
	GBS-*l*-PSO	173.600	143.703	632.159	99.059	50 / 50
	α -PSO	-	-	-	-	0 / 50

Table 2: Optimal solution of each function

G_1	G_2	G_3	G_4	G_5
-15.000	7,049.331	680.630	0.054	24.306

Table 1 shows the values calculated with solutions satisfying all constraint conditions. Here, the constraint satisfaction in Table 1 represents the number of constraint satisfaction in 50 trials. By the comparison of GBS-*g*-PSO with α -PSO, it is found that the proposed method could satisfy the constraint condition but was more likely to be trapped by the initial convergence. The GBS technique can handle various criteria by using simple rules of sorting and grouping. Thus, it is guessed that the stagnation of particles is not likely to occur until satisfying all constraints because particles can be compared easily by the proposed method. However, the rules and criteria used in these applications were very simple. For this reason, GBS-*g*-PSO was trapped by the initial convergence after the constraint satisfaction in the complicated design space such as functions G_2, G_4 and G_5. On the other hand, α -PSO introduced the control of alpha level in order to avoid this convergence. Thus, if particles could satisfy all constraint conditions, this method can obtain better solutions than the proposed method. This difference of results depends on the search performance of the original PSO. The results of GBS-*l*-PSO was better than GBS-*l*-PSO. This difference is guessed to be derived from the ability of global searching.

Through these experiments, it is expected that GBS technique can satisfy complicated constraint conditions by simple rules. In addition, by improving the efficiency, the proposed method enables the optimization method to obtain practicable solutions with keeping its simplicity. On the other hand, the optimization with the alpha constrained method is useful for multi-constrained problems by setting the appropriate control of alpha level. However, it is predicted that this method is difficult to handle the more complicated constraint conditions because of this constraint satisfaction involving the optimization of objective function.

6. Application to Bridge Maintenance Planning

In order to demonstrate the practical utility of GBS technique, this paper applies the planning problem for bridge management [8]. In this problem, the plan is formulated with ensuring the safety of bridges during the service period. In addition, the actual maintenance has to consider the annual budget constraint. By satisfying these conditions, it is important to reduce the maintenance cost in a long term in order to establish the sustainable bridge management.

In this experiment, problems with two patterns of constraint condition are applied. Firstly, the optimization aims to minimize the maintenance cost in consideration of ensuring only the safety level of bridges. In the problem defined in [8], there are two types of degradation model for components of bridge. One is the type of performance deterioration and another is the durable period type. The part of the former reduces its health degree over time. Hence, it is necessary to maintain the performance of at least the given safety level by the reinforcement and repair. On the other hand, the latter needs to replace the part after the passage of its durable period. By handling the deterioration of bridge described above, the maintenance plan can guarantee the safety during the service period. The problem of the second pattern takes into account the annual budget constraint along with the constraint conditions of the first. In order to satisfy this constraint, this paper sets the limit of annual cost for the maintenance. Because of this limitation, it becomes necessary to use the annual budget appropriately in consideration of the maintenance method and the executing period of work. Therefore, this paper treats three constraint conditions, the performance, the durable year and the annual budget. Firstly, the violation quantity of performance is calculated by the summation of annual lacking performance from the given safety level during the service period. Secondly, the total number of years when end-of-life components are used is defined as the violation quantity of the durable year. Finally, the violation of annual budget constraint is quantified by the total excess of annual maintenance cost during the service period.

In order to optimize these problems, GBS technique is applied to Genetic Algorithm [1, 8]. Then, the comparison with α -GA [7] is presented to demonstrate the effectiveness of the proposed method. In this application, the population is divided into two groups, based on the state of individual. Then, individuals satisfying all constraints are sorted in ascending order of the maintenance cost. Individuals violated constraint conditions are sorted in ascending order of the number of violation at first. Then, individuals with the same state are sorted in ascending order of violation quantity of each constraint condition in order to satisfy constraints. Here, this paper defines that an individual that satisfies the constraints of performance and durable year is superior to another that satisfies the annual budget constraint with violating any constraints. In this way, the planning can preferentially satisfy the constraints about the safety. In α -GA, the violation quantity of each constraint is minimized by the alpha constrained method. In the configuration of common parameters for each method, the number of population is 1,000, the crossover rate is 60%, the mutation rate is 0.05% and the number of executing generations is 5,000. The proposed method, which is represented as GBS-GA, applies the elite saving roulette selection as the method of natural selection. The fitness of each individual is given based on the linear normalization after sorting by the GBS technique described above. On the other hand, α -GA uses the linear ranking selection applied in [7]. And, α -GA uses the control of alpha level with the same parameters as the application of Chapter 5. The application results, executing 5 times, are shown in Tables 3 and 4.

Table 3: Cost of obtained plan (thousand yen)

Method	Average	Best	Worst	Constraint satisfaction
GBS-GA	3,507,901	3,413,882	3,551,302	4 / 5
α -GA	3,355,233	3,342,245	3,376,235	3 / 5

Table 4: Cost of obtained plan with annual budget constraint (thousand yen)

Method	Average	Best	Worst	Constraint satisfaction
GBS-GA	3,915,523	3,825,446	3,996,855	4 / 5
α -GA	-	-	-	0 / 5

Values in Tables 3 and 4 were calculated by using solutions that satisfied all constraints. In the results of Table 3, the both methods could obtain useful solutions satisfying the constraints of safety. On the other hand, it is found that α -GA was difficult to satisfy three constraints simultaneously as shown in Table 4. In the case that considered the constraints of performance and durable year, α -GA could obtain better solutions than the proposed method. This is because α -GA searches for solutions in consideration of the optimization of evaluation and the minimization of violation quantity, using the control of alpha level. Thus, the maintenance cost of obtained plan was more reduced but the accuracy of constraint satisfaction also decreased.

In the case with the annual budget constraint, α -GA could not satisfy the constraint conditions. The reduction of excess from the annual budget limitation is mainly performed by the cancellation of maintenance work. However, this operation involves the decrease of safety. Due to the trade-off between the cost saving and the safety keeping, α -GA was trapped by the stagnation of search. On the other hand, the proposed method could satisfy all constraints in the both cases, using the simple sorting rules. By satisfying the constraints of performance and durable year previously, the satisfaction of annual budget constraint is considered as the optimization for the cost reduction. Such as the rule to overcome this trade-off, GBS technique can handle the multiple-constrained conditions easily. Therefore, the proposed method is practically useful for the bridge maintenance planning.

7. Conclusions

In order to develop a practically useful method for optimizations of real-world problems, this paper attempted to propose a technique effective for multiple-constrained problems. By sorting solution candidates based on the simple comparison rule like grouping, the proposed method can handle various constraint conditions. The effectiveness of Group Based Sorting technique was demonstrated by the applications to the test functions and the bridge maintenance planning.

Many existing techniques for the multiple-constrained optimization are performed by the transformation to non-constrained optimization by merging constraint conditions into the objective function. This approach is more likely to involve the complication of design space. On the other hand, Group Based Sorting technique satisfies the constraint conditions by defining the comparison rules and evaluation criteria in response to the problem. The satisfaction to each constraint can be described in consideration of only the corresponding elements because the candidates are optimized in each group respectively. In addition, the relations among constraint conditions are treated by the comparison of groups. Therefore, by using the proposed technique, it is expected that the optimization method can search for useful solutions efficiently with keeping the simplicity of design space.

Through numerical experiments, it was shown that the proposed technique is effective for the constraint satisfaction. In this paper, GBS technique was applied to GA and PSO. In the applications of both, the constraint satisfaction was performed easily even if the problem involved constraint conditions with different properties such as a trade-off. However, it was found that the search accuracy and calculation cost depend on the applied optimization method. Therefore, for the application to real-world problems, the proposed technique requires the improvement in response to the problem.

In the optimization problem, the search is continued whenever the difference between solution candidates exists. This feature implies that the comparison of candidates significantly influences the search performance because their difference is determined by the comparison. GBS technique is based on the comparison of solution candidates. Therefore, it is expected that the definition of comparison rule becomes more simple and powerful by using the proposed method. In addition, the description of rule is required for the consideration of multiple-evaluation criteria, such as the multi-objective optimization. The applications of GBS technique has been attempted in authors researches [8, 9, 10]. For example, in the bridge maintenance planning, by maximizing the health degree among solution candidate with the same cost, the method can obtain an optimal plan with the preventive maintenance [8]. In the method, GBS technique was applied to the comparison rule for solutions that satisfied constraint conditions. This application was realized without the complication of design space and the transformation of objective function. In this way, GBS technique is practically useful for the optimization problem.

8. References

[1] J. H. Holland, Adaption in Natural and Artificla Systems, *University of Michigan Press*, 1975.

[2] J. Kennedy and R. Eberhart, Particle Swarm Optimization, *Proceedings of IEEE International Conference on Neural Networks IV*, 1942-1948, 1995.

[3] E. Aiyoshi and K. Yasuda, “Metaheuristics”, *The Institute of Electrical Engineering of Japan*, *Ohmsha*, 2007 (in Japanese).

[4] A. V. Fiacco and G. P. McCormick, “Nonlinear programming: Sequential unconstrained minimization techniques”, *Society for Industrial and Applied Mathematics*, 1990.

[5] Z. Michalewicz, Genetic algorithm + data structures = evolution programs 3rd ed., *Springer-Verlag*, Berlin, 1996.

[6] T. Takahama and S. Sakai, Constrained Optimization by the alpha Constrained Particle Swarm Optimizer, *Journal of Advanced Computational Intelligence and Intelligent Informatics*, 9 (3), 282-289, 2005.

[7] T. Takahama and S. Sakai, Constrained Optimization by alpha Constrained Genetic Algorithm (alphaGA), *Systems and Computers in Japan*, 35 (5), 11-22, 2004.

[8] K. Nakatsu, H. Furuta, Y. Nomura, N. Miyoshi, K. Takahashi and K. Ishibashi, Application of genetic algorithm to long term bridge management under uncertain environment, *6th International ASRANet Conference for Integrating Structural Analysis, Risk and Reliability*, United Kingdom, 2012.
[9] K. Nakatsu, H. Furuta, K. Takahashi, K. Ishibashi and K. Ando, Multi-Agent Optimization Method Focused on Individual Evolution, *9th World Congress on Structural and Multidisciplinary Optimization* (*WCSMO-9*), CD-ROM, Shizuoka, Japan, 2011.
[10] K. Nakatsu, H. Furuta, K. Takahashi, K. Ishibashi and K. Ando, Reliability analysis of structure using metaheuristic method, *Reliability and Optimization of Structural Systems*, *IFIP WG 7.5 Working Conference*, Armenia, 2012.