10th World Congress on Structural and Multidisciplinary Optimization
May 19 -24, 2013, Orlando, Florida, USA

Optimal cornea shape design problem for corneal refractive surgery

Takaki Nakayama, Hisashi Naito, Takeshi Matsumoto, Masao Tanaka

Department of Mechanical Science and Bioengineering, Graduate School of Engineering Science, Osaka University, [takaki.nakayama@msbe., naito@, matsu@, tanaka@]me.es.osaka-u.ac.jp

1. Abstract

Corneal refractive surgery is a promising technique to correct irregularities of cornea shape accompanying various types of aberration in eye ball system, and correction of refractive power for nearsightedness and farsightedness are fundamental application of refractive surgery. However there have been reported some risks such as over-/under-corrections, abnormal sight symptom like halo or glare. Intraocular pressure is always acting in vivo on the posterior surface of cornea, and the secondary deformation is induced by laser ablation. Therefore the cornea shape should be considered by taking the deformation of cornea tissue. In this article, the optimal cornea shape is studied for the correction of higher order aberration, which was not corrected by eye glasses or contact lenses. This optimal shape design of deformed nonlinear elastic body is composed of two sub-problems.

The first is the optimal shape design for correction of higher order aberration. The current cornea shape is regarded as the initial shape in structural optimization and the optimal shape is determined for the anterior surface of cornea so that minimizes the aberration over the optical zone. The design variables are the position of anterior cornea surface within the designated operation zone. Here, the aberration at individual point on cornea surface is calculated by using the reduced eye model and the law of refraction, Snell's law. The solution is the shape in equilibrium to the intraocular pressure, and is regarded as the target shape of cornea for refractive surgery.

The second is the determination of ablation volume. The shape of current cornea in vivo is of the deformed state under the intraocular pressure. The target shape of cornea to be realized after refractive surgery is also the deformed one. The second problem is thus reduced to the determination of the stress-free cornea shapes corresponding to the current and target cornea respectively. The minimization problem is formulated for the integral of squared difference between the surfaces of current/target cornea shapes under the intraocular pressure. These problems determine the stress-free cornea shapes for the current and target cornea shapes respectively by means of the traction method, and the ablation volume is then identified as the difference between these stress-free shapes.

The effectiveness of the proposed optimal shape design approach was examined with a numerical case study of correction of spherical aberration, a kind of higher order aberration, and the sufficient reduction of non-uniform aberration was confirmed with the uniform and sufficiently small aberration within normal range over the optical zone.

2. Keywords: Corneal shape, Optimal shape design, Corneal refractive surgery

3. Introduction

Corneal refractive surgery has been considered as an established technique to correct irregularities of cornea shape accompanying various types of aberration in eye ball system. The fundamental application of refractive surgery includes the correction of refractive power for nearsightedness and farsightedness. Although the correction of higher order aberration is not corrected by using eye glasses or contact lenses, it could be treated by means of refractive surgery. The preoperative planning is carried out based on the geometrical analysis of cornea shape. The cornea is a deformable soft tissue and is always supporting the intraocular pressure. That means the laser ablation of cornea tissue in refractive surgery results reduction of cornea tissue volume and decrease of structural property against the intraocular pressure. The under-/over-correction of refractive power is a typical claim for clinical outcomes and the secondary deformation is recognized as a primal reason. In fact, it was reported that the postsurgical shape of the cornea differs from the predicted shape, and that these differences, which increase as the initial degree of myopia, become significant for visual performance [1].

The material properties and deformation of cornea has been considered as the fundamentals to understand the mechanical behavior of cornea under the presence of intraocular pressure. Many studies have reported the material property of cornea as a nonlinear elastic body [e.g. 2, 3, 4]. Based on these studies for material properties, many researchers have investigated on the cornea deformation under the normal and abnormal intraocular pressures [e.g. 5, 6]. For the refractive surgery, the major attention has been focused on the cornea geometry from the viewpoint of traditional/adaptive optics, and a little attention has been paid for the secondary deformation of cornea due to the refractive surgery [5, 7].

In this study, the optical geometry and mechanical deformation of cornea are considered for preoperative planning of refractive surgery from the structural optimization viewpoint: the optimal shape design for correction of the higher order aberration and the optimal determination of ablation volume considering cornea deformation.

4. Optimal cornea shape design problems

The human eye is a complex optical system, and its three-dimensional structure determines the clarity of vision. Light rays entering the eye are refracted twice at the anterior and posterior surfaces of the cornea and twice at the surfaces of the crystalline lens, and reach the retina. The refraction at cornea accounts for about two-thirds of the total refractive power, and that at crystalline lens mainly functions to adjustment the position of focal point. In emmetropia, incident parallel rays are refracted through the cornea and crystalline lens, and converged to the central part of the retina as shown in Fig.1. Aberration represents the deviation between the refracted ray and the fovea. When the refracted rays do not focus on a single point, it is a higher order aberration those could not be corrected by use of eye glasses or contact lens. Aberration arises even when small geometrical deviation is occurred on corneal surface, and the geometry of corneal surface is crucial for its optical function and quality.

In this section, the shape optimization problems are described for preoperative planning of refractive surgery for optimal optical refraction by taking account of corneal tissue deformation under the intraocular pressure. This optimal shape design of deformed nonlinear elastic body is composed of two problems: one is for the optimal cornea shape for correction of higher order aberration, and the other is for the ablation volume of cornea tissue realizing the optimal shape.

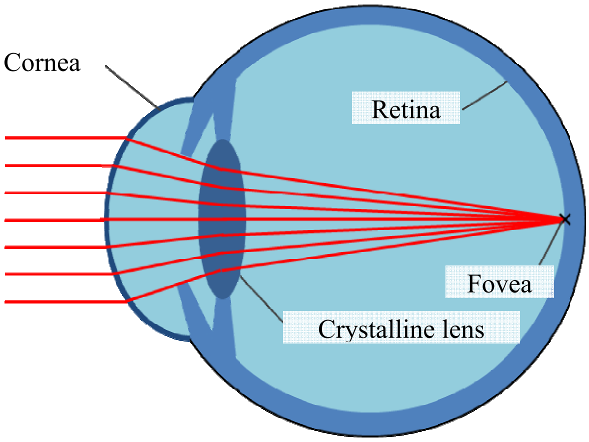


Figure 1: Normal refraction in human eye (Emmetropia)

4.1. Optimal shape design for correction of the higher order aberration

The refraction analysis of real eye system is complex since it has four surfaces of refraction at cornea and crystalline lens. Reduced eye models have developed to calculate refracted ray vector simply. Theoretically, it has only one refractive surface of anterior surface of cornea, and the equivalent axial length of eye ball. In this study, Gullstrand reduced eye model is used for the calculation of aberration by taking the geometrical shape of anterior surface of cornea (Fig.2). The fundamental governing law for refraction is Snell's law, written as

$$|n_1|(\boldsymbol{E}\times\boldsymbol{R}_{in}) = |n_2|(\boldsymbol{E}\times\boldsymbol{R}_{ref}) \tag{1}$$

where n_1 and n_2 are refractive indices, $\boldsymbol{R}_{in}$ and $\boldsymbol{R}_{ref}$ are incident and refracted ray vectors, and $\boldsymbol{E}$ is the surface normal vector.

The shape of Cornea of emmetropia is rotationally symmetric, and the optimal shape of cornea is assumed to be of axisymmetric also. Thus the shape design problem is reduced to a two-dimensional one, and the target of the shape design is focused on the optical zone, that is the central area of cornea. This is coming from the fact that some portion of incident lights is blocked by the pupil and lights never pass through the entire region of the cornea.

As is seen in Eq. (1), the normal vector $\boldsymbol{E}$ of cornea surface is important to represent its optical quality, and it is essential to adjust the normal vector on corneal surface. Although the use of a mathematical function representing the entire corneal shape is straightforward to calculate normal vector on corneal surface, relevant function form for normal vector calculation has not been suggested for the corneal shape [8]. Therefore, in order to calculate normal vector, the local shape approximation of cornea surface is adapted for the optical zone in this study. In the plane of eye axis z and radius r, the curve of cornea surface is discretized by a radial interval and is described as a set of surface points P_i as is illustrated in Fig. 3. The most outer point of designated zone for surgical operation is defined as the prefixed point P_0. The refracted ray vector for a line segment $P_{i-1}P_i$ is calculated for its midpoint Q_i. The axial position of point P_i is adjusted so as to minimize the aberration of the refracted ray vector through Q_i. By the progressive adjustment, the optimal cornea shape is obtained as the set of surface points. The three-dimensional cornea shape is then defined by rotating the two-dimensional curve obtained.

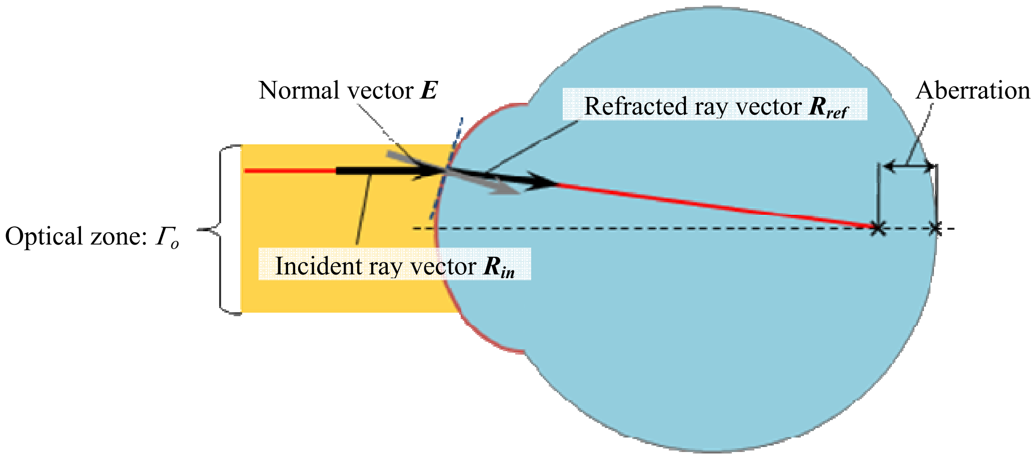


Figure 2: Snell's law for reduced eye model.

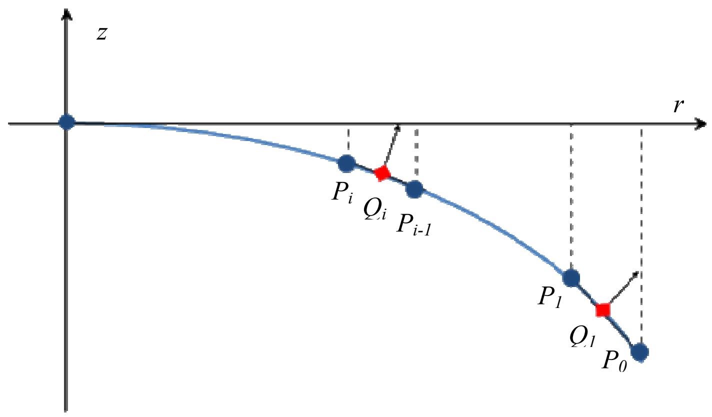


Figure 3: Discretized points P_i of cornea shape and normal vectors at their midpoints Q_i.

4.2. Determination of ablation volume

The optimal cornea shape determined in the previous section is the solution for optical property of cornea, and realizes the correction of aberration including higher order one. Here it should be noted that the cornea is a deformable soft tissue and the shape of cornea in vivo is the deformed one under the presence of intraocular pressure. The difference of the cornea shape before the refractive surgery from the optimal shape determined in the previous section does not gives the cornea tissue volume ablated by refractive surgery. This is coming from the secondary deformation due to the reduction of cornea tissue volume and the intraocular pressure. This section describes the shape design problem realizing the optimal cornea surface as the deformed one and determining the ablation volume. The deformation of cornea is investigated by means of finite element analysis, but the cornea shape measured by clinical device is deformed one due to the intraocular pressure and it is not suitable for finite element modeling of cornea [6]. Therefore it is necessary to identify the stress-free natural shape of cornea from the current shape in vivo and the stress-free natural shape from the optimal shape to be realized in vivo.

The cornea is treated as a deformable nonlinear continuum. Let consider a material point on the cornea surface, and its position be $\boldsymbol{X}$ at the stress-free natural state. The intraocular pressure induces the deformation in the cornea, and the material point at $\boldsymbol{X}$ is moved to the position $\boldsymbol{x}$ with the displacement $\boldsymbol{u}$. When the cornea surface at deformed state is characterized by using an implicit function f, it is written as

$$f(\boldsymbol{x}) = f(\boldsymbol{X}+\boldsymbol{u}) = 0 \tag{2}$$

by taking the relation $\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{u}$ into account. For the case of current cornea shape before surgical operation, the deformed cornea shape $\boldsymbol{x}$ is available as the measured one, and the position $\boldsymbol{X}$ at stress-free natural state is the unknown. For the case of optimal shape to be realized by surgical operation, the deformed cornea shape $\boldsymbol{x}$ is available as the design solution, and the position $\boldsymbol{X}$ at stress-free natural state is again the unknown.

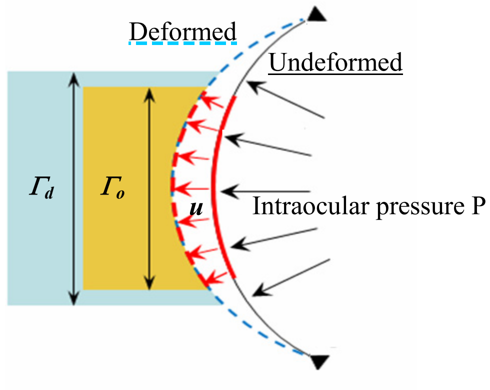


Figure 4: Undeformed(black)/deformed(blue) cornea surfaces and optical(Γ_o)/design(Γ_d) areas.

The shape index $H(\boldsymbol{u})$ is defined as the squared sum over the optical zone Γ_o of the body as

$$H(\boldsymbol{u}) = \int_{\Gamma_o} \{f(\boldsymbol{X}+\boldsymbol{u})\}^2 d\Gamma \tag{3}$$

and the minimization problem

$$\begin{array}{ll} \text{minimize} & H(\boldsymbol{u}) = \int_{\Gamma_o} \{f(\boldsymbol{X}+\boldsymbol{u})\}^2 d\Gamma \\ \text{with respect to} & X \text{ on } \Gamma_d \\ \text{subject to} & a(\boldsymbol{u}, \boldsymbol{w}) = l(\boldsymbol{w}) \end{array} \tag{4}$$

is formulated to determine the unknown $\boldsymbol{X}$, the cornea shape at stress-free state on the surface region Γ_d for surgical operation. The constraint is the equilibrium condition in terms of adjoint/virtual displacement $\boldsymbol{w}$ of the nonlinear continuum in the week form expression. This optimization problem (4) is transformed into an unconstrained problem by Lagrangian multiplier method. The optimality conditions are derived in which the derivative of Lagrange functional is described in terms of the velocity field of the domain variation. This velocity field is evaluated in the context of the traction method [9]. The unknown stress-free natural shape $\boldsymbol{X}$ is obtained in iteration process of domain variation based on the velocity field. This minimization problem works to identify the shape of cornea surface $\boldsymbol{X}_c$ at stress-free state before surgical operation with the current deformed surface $\boldsymbol{x}_c$, and to identify $\boldsymbol{X}_o$ after surgical operation with the optically optimal cornea shape $\boldsymbol{x}_o$ described in the previous section.

The optically optimal shape $\boldsymbol{x}_o$ should be realized by laser ablation, and the possible shape change from the current cornea surface $\boldsymbol{x}_c$ to the target cornea surface $\boldsymbol{x}_o$ is of tissue reduction only. It corresponds to the tissue volume reduction from the current cornea surface $\boldsymbol{X}_c$ to the target cornea surface $\boldsymbol{X}_o$ by referring to the stress-free state. Thus, the feasible velocity of domain variation for the determination of $\boldsymbol{X}_c$ should be limited to the inward change and is written as the negative part of the projection of the velocity for volume deviation on the outer normal vector $\boldsymbol{n}$. That is, the velocity for volume deviation $\boldsymbol{T}$ is expressed as

$$\boldsymbol{T} = -t\boldsymbol{n} \qquad t = \min(\boldsymbol{n}\cdot\boldsymbol{V}, 0) \tag{5}$$

instead of the standard velocity $\boldsymbol{V}$ for volume deviation in the traction method.

5. Numerical case study

The proposed shape optimization process was studied for a numerical case of the correction of spherical aberration. Spherical aberration is one of the higher order aberrations which are not corrected by glasses or contacts.

5.1 Finite element model of cornea

Corneal tissue is modeled as a nearly incompressible nonlinear hyper-elastic continuum. It exhibits clear anisotropy due to the preferential orientation of collagen fibrils distributed around the inferior-superior and nasal-temporal orthogonal meridians in the central human cornea. At the limbus, in contrast, the dominant fibril orientation is reported to be tangential, indicative of a pseudo-annulus of collagen circumscribing the cornea [10]. These collagen fibrils are rarely observed in the anterior part of cornea. These orientation of collagen fibers shown in Fig. 5 are taken into the strain energy function W as an hyper-elastic continuum as

$$W = C_{10}(\tilde{I}_1 - 3) + C_{20}(\tilde{I}_1 - 3)^2 + \frac{1}{\kappa}\sum_{i=4,6} K_i\left(e^{\kappa(\tilde{I}_i - 1)^2} - 1\right) + \frac{\kappa}{2}\left(\sqrt{I_3} - 1\right)^2 \tag{2}$$

where the first and second term of right-hand-side is for the isotropic mode, the third term is for the anisotropic mode, and the last term is for the volumetric expansion with near incompressibility. The constants K_4 and K_6 expresses material property of collagen fibril respectively [11].

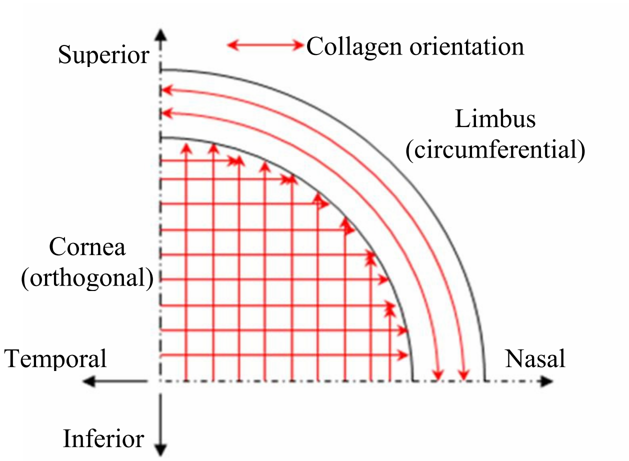


Figure 5: Model of collagen fibril orientation (front view).

Finite element model used for numerical case study has fixed boundary condition on the boundary connecting cornea and sclera. The thickness of central corneal is 0.5mm, the anterior radius of curvature is 7.7mm, and the diameter of cornea is 12mm. Hexahedral elements are arranged in 3 layers, and the total numbers of elements and nodes are 1288 and 516 respectively as is shown in Fig. 6. The optical zone Γ_o and the design area Γ_d are set to be the central area of 7.0mm and 8.0mm in diameter, respectively. The numerical case is aimed at the correction of the spherical aberration, a kind of higher order aberration in optical zone Γ_o by corneal shape adjustment in the design area Γ_d.

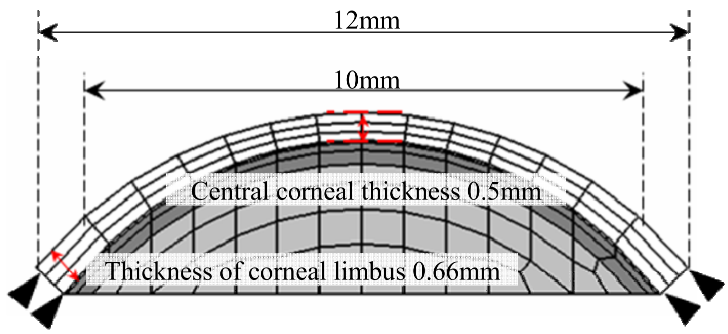
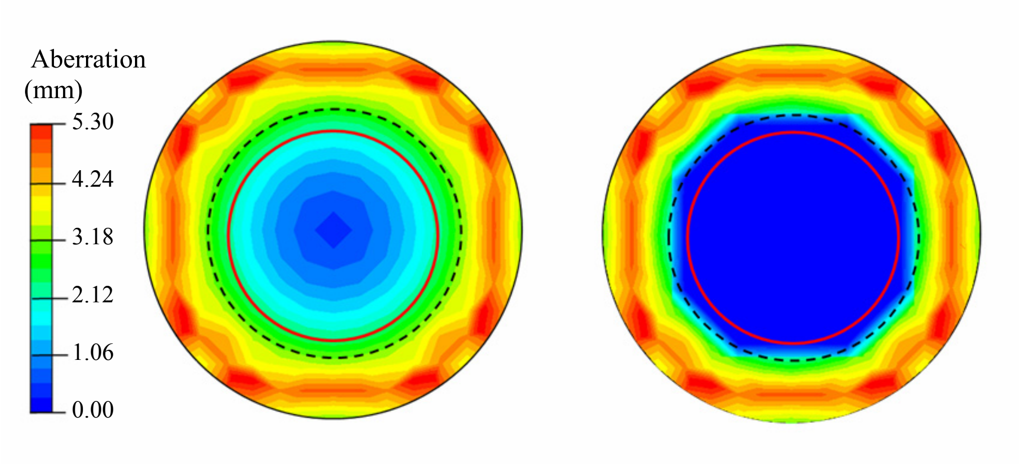


Figure 6: Corneal model under intraocular pressure (sectioned).

5.2 Numerical case for correction of spherical aberration

A cornea with spherical aberration as is described in the previous section has the aberration distribution as shown in Fig. 7(a). The optimal cornea shape obtained as the result of the first problem proposed in section 4.1 enables us to realize the almost zero aberration within the optical zone, the central region of 7 mm in diameter as shown in Fig. 7 (b). In these figures, the red circle represents the optical zone, and the black dotted circle represents the region for surgical operation. The largest aberration in the optical zone was 0.15mm, that is identical to -0.37diopter, in contrast to the maximum aberration 1.92 mm (-4.83diopter) in the optical zone of the initial cornea with spherical aberration.



(a) spherical aberration (b) aberration corrected

Figure 7: Distribution of aberration.

The second problem proposed in the section 4.2 gives us the ablation volume shown in Fig. 8, and the maximum ablation thickness was 56.8 μm at the central cornea. Spherical aberration is classified into a kind of nearsightedness with higher order aberration. Therefore, corneal ablation pattern for spherical aberration was expected similar that for nearsightedness correction. In this study, the ablation pattern for correction of spherical aberration shows a high consistency with that of nearsightedness. The aberration within the central region of 6 mm in diameter was smaller than the threshold of -1 diopter, while the maximum aberration within the optical zone of 7 mm in diameter was 0.85mm (-2.13 diopter) found at the most peripheral region. The aberration of -1 diopter is considered as a threshold for correction in general, and the result was not perfect. However, it was very satisfactory, because the optical zone with normal vision was increased from 2 mm in diameter to 6 mm in diameter by the correction shown in Fig. 8. The maximum aberration was also reduced to 0.85 mm (-2.13 diopter) from 1.92 mm (-4.83 diopter) at the peripheral of optical zone. The normal vision area was increased from 8 % to 73 % of the optical zone, and the refractive power was improved all over the optical zone.

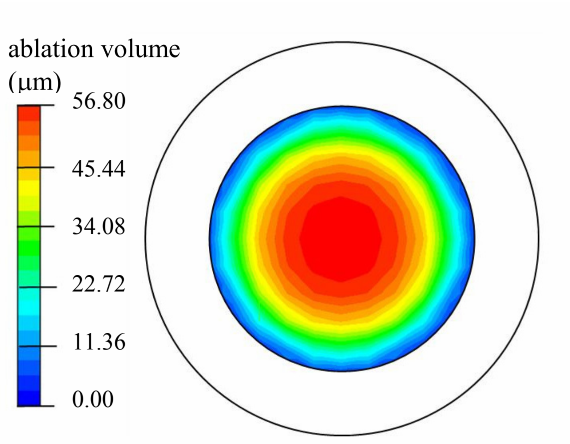


Figure 8: Ablation volume.

6. Conclusion

This article discussed the optimal cornea shape for the correction of higher order aberration. The optimal shape design proposed was composed of two problems in order to consider both optical and mechanical aspects. The first was for the optimal corneal surface design for correction of higher order aberration. This optimal shape design was determined by paying attention to minimize the aberration for light rays entering the eye through the optical zone on cornea surface. This gives us the cornea shape to be realized under the presence of intraocular pressure. The second was the determination of ablation volume by referring to the stress-free natural states before and after correction by ablation. The cornea shape at stress-free state was determined so as to reproduce the cornea shape measured under the presence of intraocular pressure by taking the hyper-elastic deformation of the cornea tissue. The stress-free state for the target cornea shape to be realized by correction was also determined in the same context by paying attention to the fact that the cornea shape change was realized only by ablation reducing the tissue volume. The ablation volume was then determined as the difference between these natural shapes. The effectiveness of the proposed optimization approach was examined and demonstrated through a numerical case study for correction of spherical aberration with satisfactory results.

7. References

[1] R. G. Anera, C. Villa, J. R. Jimenez, R. Gutierrez and L. J. del Barco, Differences between real and predicted corneal shapes after aspherical corneal ablation, *Applied Optics*, 44-21, 4528-4532, 2005.

[2] M. Bryant, P. McDonnell, Constitutive laws for biomechanical modeling of refractive surgery, *Journal of Biomechanical Engineering*, 118, 473–481, 1996.

[3] P. Pinsky, D. van Der Heide and D. Chernyak, Computational modeling of mechanical anisotropy in the cornea and sclera, *Journal of Catarac tand Refractive Surgery*, 31, 136–145, 2005.

[4] A. Pandolfi and G. Holzapfel, Three-dimensional modeling and computational analysis of the human cornea considering distributed collagen fibril orientations, *Journal of Biomechanical Engineering*, 130, 061006, 2008.

[5] H. P. Studer, H. Riedwyl, C. A. Amstutz, J. V. M. Hanson and P. Buchler, Patient-specific finite-element simulation of the human cornea: A clinical validation study on cataract surgery, *Journal of Biomechanics*, 2012.

[6] A. Elsheikh, C. Whitford, R. Hamarashid, W. Kassem, A. Joda and P. Büchler, Stress free configuration of the human eye, *Medical Engineering and Physics*, 35-2, 211-216, 2012.

[7] M. Tanaka, T. Matsumoto, H. Naito and T. Jinno, Shape optimization for corneal refractive surgery planning realizing structural profile by trimming operation, *Proceedings of Seventh World Congress on Structural and Multidisciplinary Optimization (Seoul, Korea)*, 625-630, 2007.

[8] H. T. Kasprzak and D. R. Iskander, Approximating ocular surfaces by generalized conic curves, *Ophthalmic and Physiological Optics*, 26-6, 602-609, 2006.

[9] H. Azegami, Solution to domain optimization problems. *Transactions of Japanese Society for Mechanical Engineers*, 60A, 1479-1485, 1994.

[10] C. Boote, C. S. Kamma-Lorger, S. Hayes, J. Harris, M. Burghammer. J. Hiller, N. J. Terrill and K. M. Meek, Quantification of collagen organization in the peripheral human cornea at micron-scale resolution, *Biophysical Journal*, 101, 22-42, 2011.

[11] G. A. Holzapfel, T. C. Gasser and R. W. Ogden, A new constitutive framework for arterial wall mechanics and a comparative study of material models, *Journal of Elasticity*, 61, 1-48, 2000.