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Optimization of Tow Steered Fiber Orientation Using the Level Set Method

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1. Abstract
Fiber manufacturing machines can lay down composite fibers in curved and continuously varying paths, allowing the fibers to be "steered" into different orientations in different regions of a single or multiple ply structure. This offers potential for the fibers to be better tailored to optimally support the applied loads in complex loading environments and hence may significantly improve the overall structural efficiency. This paper introduces a new method to optimize the orientation of continuously varying angle fiber tow paths in order to create an optimal structure. The fiber paths are defined by an implicit level set function with the fiber tows following the path with constant level set function values. An energy based sensitivity is used to update the level set function and optimize the fiber paths it defines. Test models show that the method optimizes fiber orientation while maintaining continuous fiber paths.

2. Keywords: Fiber, Orientation, Optimization, level set

3. Introduction

It is well known that the strength and stiffness of fibrous composite materials is significantly higher in the direction that the fibers are aligned than other directions. Therefore the orientation of the fibers throughout a composite structure can be optimized to produce a significant increase in structural performance without increasing the structural weight. Experiments have shown that optimal orientation of fibers can significantly increase structural stiffness, failure loading, buckling stress and post buckling performance over the traditional quasi-isotropic fiber construction without increasing the structural weight [1-4]. The potential for very high structural efficiency in these advanced composite structures is attractive for applications where minimization of structural weight is critical, such as aerospace engineering, particularly with the recent increase in the use of fiber composites in these domains [5, 6].

The techniques for manufacturing composite structures with tailored fiber orientations have been developed since the 1980's. The automated fiber placement (AFP) manufacturing technique forms plates by laying down a series of narrow pre-peg tows or slit tapes. Using robotic control AFP machines can be programmed to lay down the fiber tows within the slit tape at any orientation. The fiber tows also can be "steered" as the split tapes are layed down producing curved fiber tow paths, varying the fiber orientation across the structure. The tow paths have to be defined as smooth continuous curves, since sudden sharp changes in fiber orientation during the manufacturing process induces defects. These defects, including fiber buckling, large gaps between neighboring tows or overlapping of tows, can cause a significant reduction in structural performance [5, 6]. Despite this limitation AFP is seeing an increasing use [6]. However, defining the optimal orientation of the fiber tows is a complex task, especially in a structure under multiple loading conditions, which could be achieved by using a form of structural optimization.

3.1 Optimization of Orthotropic Material Orientation

There has been much research into structural optimization of the orientation of orthotropic materials. Optimality criteria methods involve optimizing the orientation of the maximum stiffness direction of an orthotropic material, e.g. the fiber orientation, within each element of a finite element model. The optimality criteria method has been successfully used to improve the stiffness, failure load and buckling performance of composite structures [6, 7, 8]. Usually this is achieved by aligning the fibers with the load path through the element or the elemental principal stress or strain [1, 7]. Since altering the orientation of the material alters both the load paths and the principal stress and strain this process is carried out iteratively [7]. The relationship between fiber orientation and the principal stress or strain complicates the application of the optimality criteria method, as it becomes possible for the method to become trapped at local solutions if the initial fiber orientation is too far from the global optimal solution [7,8]. The risk of this occurrence can be minimized by using the principal stress or strain of a structure made from isotropic material under the same loading conditions to initialize the fiber orientation [7].

The relationship between the optimality criteria methods and improvements in structural performance is causal; a more robust form of optimization is to calculate the fiber orientation that minimizes, or maximizes, an objective function. In this method sensitivity analysis is used to establish direction and magnitude of the change in fiber orientation within each element that would improve the global objective function. This method was originally formulated by Pedersen et al [9] using a strain based sensitivity calculation, solving both compliance minimization and buckling load maximization problems [10]. Diaz & Bendsøe [11] created a similar stress based sensitivity analysis extending the method to statically determinate problems. Both methods have been shown to be able to optimize fiber orientation to minimize compliance or maximize buckling load, producing significant improvements in structural performance over uniform fiber orientations [9, 10, 11]. However the stress or strain based methods assumes uniform strain or stress fields, respectively, within the element with the changing fiber angle [12]. This assumption was removed by the energy based method introduced in Luo and Gea [12], where an implicit method was used to estimate the effect of changing the fiber angle within an element on the stress and strain fields. The energy based method has also been used to solve compliance minimization and buckling load maximization problems, producing solutions that were numerically superior to both the stress and strain based methods [12, 13]. However, the fiber orientation problem is non-convex, making it challenging to find the global optimal solution [8]. Furthermore the solutions often feature large changes in fiber angle between neighboring elements, particularly in bending problems. These discontinuities in the fiber angles make the solutions impossible to manufacture.

Evolutionary optimization methods have also been used to solve fiber orientation problems. The benefit of these methods is that they are robust and able to reliably find a solution close to the global optimum [14]. However the computational cost is considerably higher than other optimization methods and the fiber discontinuities are unavoidable.

An alternative method for solving fiber orientation problems with a focus on creating manufacturable solutions is the Direct Material Optimization (DMO) method, first outlined in Stegman and Lund [15]. The DMO method is an adaptation of the optimal material selection technique defined in Sigmund *et al.* [16]. A gradient based sensitivity analysis is used to optimize the volume ratio of different materials in an element to minimize compliance subject to a constraint on the structural weight. By combining the various volume ratios and penalizing them the problem can be forced to converge to a 1/0 solution so that only one material is left in each element. By replacing the different materials with a single fiber at different orientations the fiber angle can be optimized in each element [15]. This method has been shown to be able to optimize composite structures to minimize compliance and maximize buckling load [15, 17]. The main disadvantage with this method is that each possible fiber angle must be defined by a single material type, so only a finite number of fiber orientations can be used (usually 0°, ±45°, 90°) or else the number of design variables becomes excessive. To reduce the number of design variables a uniform fiber is often applied over multiple elements "patches" [15]. The patch method also has the advantage of creating an optimal solution with several regularly shaped regions of uniform fiber angle that is simple to manufacture. Patch methods have also been used with optimality criteria optimization methods [18]. The DMO method and patch methods do not produce suitable results for optimizing continuous fiber orientation paths due to the limitations of the design variables; however they do demonstrate the advantage of an optimization procedure that creates an optimal structure that is simple to manufacture.

3.2. Level Set Topology Optimization

The level set method is a front-tracking method that has been successfully applied to topology optimization where the structure is defined by a set of signed distance level set functions stored at points, usually for convenience the finite element nodes, spread across the design domain. Local sensitivity values are used to update the level set function values, moving the structural boundary to create a more optimal shape and topology for the structure [19, 20]. The level set topology optimization method is increasing in popularity as it creates discrete solutions whose geometry is clearly defined by a continuous structural boundary [19].

The aim of this paper is to introduce a new method for optimizing the orientation of fibers in a composite plate using the level set method. In this method the fiber path will be defined by the "boundary" described by the level set function. By changing the path of the level set function the orientation of the fiber within the plate can be optimized. Since the boundary described by the level set method is continuous it will in turn define continuous fiber paths across the composite plate. Therefore the level set method will create composites with optimal fiber orientation that can be easily manufactured using a tow steering manufacturing technique.

4. Optimization Method

4.1 Level Set Definition of Fiber Orientation

In this section the level set optimization method of fiber orientation is outlined. The level set function lsf = 0 defines the primary path of a fiber in a composite panel. The paths of the other fiber tows through the structure are defined by constant lsf values. In this way the level set function describes a series of continuous parallel fiber paths throughout the plate, an example of this is shown in figure 1.

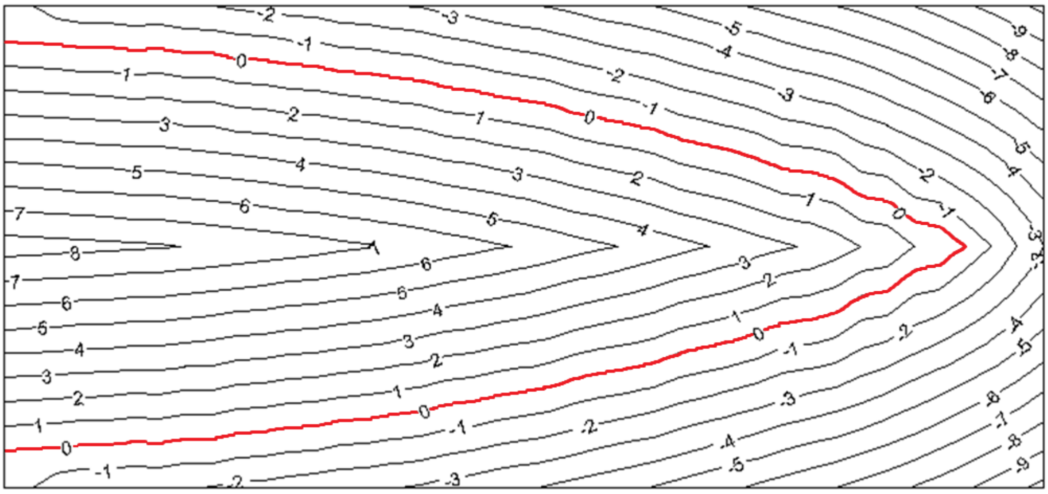

Figure 1: Plot of fiber tow paths through the structure defined by lines with constant integer level set function values. Lsf=0 line highlighted in red.

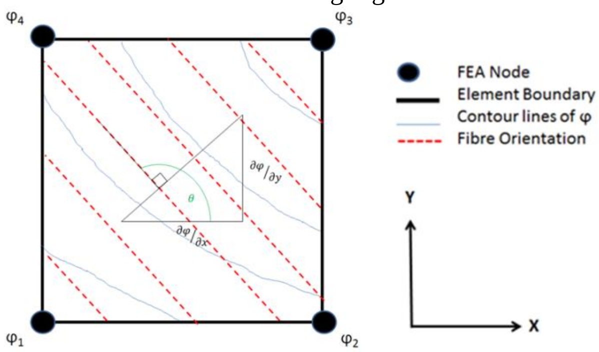

Figure 2: Definition of the fiber orientation within a finite element from the level set function φ.

For the purpose of finite element analysis it is necessary to be able to define the orientation angle of the fiber tow path through each individual element. Since the fibers follow the path of the lsf lines with constant values the orientation of the tows can be defined as perpendicular to the maximum gradient of the level set function over the element, as shown in figure 2. Thus the elemental fiber orientation can be calculated in each element using the following Eq. (1).

$$\theta_e = \frac{\pi}{2} + \arctan\left(\frac{d\varphi/_{dy}}{d\varphi/_{dx}}\right) \tag{1}$$

Where θ_e is the elemental fiber angle orientation, φ is the level set function and x and y are the Cartesian coordinates shown in figure 2.

4.2 Optimality Criteria and Level Set Update Routine.

The objective is to minimize the compliance of the fiber plate (maximize the stiffness) under the applied loads, as defined in Eq. (2). This is achieved by determining the optimum level set function that describes the continuous fiber paths.

$$Min\ E = \sum_{i=1}^{n} \varepsilon_i{}^T C(\theta)\varepsilon_i \tag{2}$$

$$subject\ to: -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

where E is the overall compliance of the structure, C is the element stiffness matrix dependent on the fiber angle θ, ε_i is the strain of element i, and n is the number of elements.

To optimize the fiber orientation local sensitivity analysis is used to update the level set function values around the lsf = 0 line. This local update of the level set function is then extrapolated to the rest of the nodes in the structure. The fast marching method [21] is used to update the fiber orientation throughout the entire structure, while maintaining the evenly spaced continuous tow path definitions.

4.3 Sensitivity Analysis

The orthotropic stiffness matrix at any fiber angle θ, can be calculated in terms of the unrotated stiffness matrix C_0 and the transformation matrix T(θ). So the compliance in a structure of uniform fiber angle θ is.

$$E = \varepsilon^T C\varepsilon = \varepsilon^T\, T^T(\theta) C_0 T(\theta)\varepsilon \tag{3}$$

$$C_0 = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{26} & 0 & 0 & 0 \\ Q_{16} & Q_{26} & Q_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} & 0 \\ 0 & 0 & 0 & Q_{45} & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \\ R_X \\ R_Y \\ R_Z \end{bmatrix} \tag{5}$$

$$T(\theta) = \begin{bmatrix} \begin{matrix} cos^2\theta & sin^2\theta & cos\theta sin\theta \\ sin^2\theta & cos^2\theta & -cos\theta sin\theta \\ -2cos\theta sin\theta & -cos\theta sin\theta & cos^2\theta - sin^2\theta \end{matrix} & [0] \\ [0] & \begin{matrix} cos\theta & sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 0 \end{matrix} \end{bmatrix} \tag{6}$$

The Q terms are calculated from the material properties, longitudinal Young's modulus, E_L, transverse Young's modulus, E_T, the major and minor Poisson's ratios, ν_{LT} and ν_{TL}, and the in plane and out of plane shear moduli, G_{LT}, G_{LW} and G_{TW}.

$$Q_{11} = \frac{E_{LT}}{(1-\nu_{LT}\nu_{TL})} \qquad Q_{22} = \frac{E_{TL}}{(1-\nu_{LT}\nu_{TL})} \qquad Q_{12} = \frac{\upsilon_{TL}E_{LT}}{(1-\nu_{LT}\nu_{TL})} \qquad Q_{66} = G_{LT} \qquad Q_{44} = G_{LW} \qquad Q_{55} = G_{TW} \tag{7}$$

The sensitivity of the compliance to the fiber orientation requires Eq. (3) to be differentiated with respect to θ. Performing the matrix-vector calculation in Eq. (3) results in the equation for the compliance below.

$$\begin{aligned} \varepsilon^T C\varepsilon = {} & A_1 cos^4\theta + A_2 sin^4\theta + A_3 cos^3\theta sin\theta + A_4 cos\theta sin^3\theta + A_5 cos^2\theta sin^2\theta \\ & + A_6 cos^2\theta + A_7 sin^2\theta + A_8 cos\theta sin\theta \end{aligned} \tag{8}$$

where A are constant coefficient values shown below:

$$A_1 = \varepsilon_x\left[\left(\varepsilon_x Q_{11} + \varepsilon_y Q_{12} + \varepsilon_z Q_{16}\right)\right] + \varepsilon_y\left[\left(\varepsilon_x Q_{12} + \varepsilon_y Q_{22} + \varepsilon_z Q_{26}\right)\right] + \varepsilon_z\left[\left(\varepsilon_x Q_{16} + \varepsilon_y Q_{26} + \varepsilon_z Q_{66}\right)\right]$$
$$A_2 = \varepsilon_x\left[\left(\varepsilon_x Q_{22} + \varepsilon_y Q_{12} - \varepsilon_z Q_{26}\right)\right] + \varepsilon_y\left[\left(\varepsilon_x Q_{12} + \varepsilon_y Q_{11} - \varepsilon_z Q_{16}\right)\right] + \varepsilon_z\left[\left(-\varepsilon_x Q_{26} - \varepsilon_y Q_{16} + \varepsilon_z Q_{66}\right)\right]$$
$$\begin{aligned} A_3 = {} & \varepsilon_x\left[\left(-4\varepsilon_x Q_{16} + 2\varepsilon_y(Q_{16} - Q_{26}) - \varepsilon_z(Q_{11} - Q_{12} - 2Q_{66})\right)\right] + \varepsilon_y\left[\left(2\varepsilon_x(Q_{16} - Q_{26}) + 4\varepsilon_y Q_{26} + \right.\right. \\ & \left.\left.\varepsilon_z(Q_{12} - Q_{22} + 2Q_{66})\right)\right] + \varepsilon_z\left[\left(\varepsilon_x(Q_{11} - Q_{12} - 2Q_{66}) + \varepsilon_y(Q_{12} - Q_{22} + 2Q_{66}) + 2\varepsilon_z(Q_{16} - Q_{26})\right)\right] \end{aligned}$$
$$A_4 = \varepsilon_x\left[\left(-4\varepsilon_x Q_{26} + 2\varepsilon_y(Q_{26} - Q_{16}) - \varepsilon_z(Q_{12} - Q_{22} - 2Q_{66})\right)\right] + \varepsilon_y\left[\left(2\varepsilon_x(Q_{16} + Q_{26}) + 4\varepsilon_y Q_{16} + \right.\right.$$

$$\varepsilon_z(Q_{11} - Q_{12} + 2Q_{66})\Big)\Big] + \varepsilon_z\Big[\Big(\varepsilon_x(Q_{12} - Q_{22} + 2Q_{66}) + \varepsilon_y(Q_{11} - Q_{12} - 2Q_{66}) + 2\varepsilon_z(-Q_{16} + Q_{26})\Big)\Big]$$

$$A_5 = \varepsilon_x\Big[\Big(\varepsilon_x(2Q_{12} + 4Q_{66}) + \varepsilon_y(Q_{11} + Q_{22} - 4Q_{66}) + \varepsilon_z(-3Q_{16} + 3Q_{26})\Big)\Big] + \varepsilon_y\Big[\Big(\varepsilon_x(Q_{11} + Q_{22} - 4Q_{66}) + \varepsilon_y(2Q_{12} + 4Q_{66}) + \varepsilon_z(3Q_{16} - 3Q_{26})\Big)\Big] + \varepsilon_z\Big[\Big(\varepsilon_x(-3Q_{16} + 3Q_{26}) + \varepsilon_y(3Q_{16} - 3Q_{26}) + \varepsilon_z(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\Big)\Big]$$

$$A_6 = R_x(R_xQ_{44} + R_yQ_{45}) + R_y(R_xQ_{45} + R_yQ_{55})$$

$$A_7 = R_x(R_xQ_{55} - R_yQ_{45}) + R_y(-R_xQ_{45} + R_yQ_{44})$$

$$A_8 = R_x\Big(-2R_xQ_{45} + R_y(Q_{44} - Q_{55})\Big) + R_y(R_x(Q_{44} - Q_{55}) + 2R_yQ_{45})$$

The resulting formula for $\varepsilon^T C \varepsilon$ can then be differentiated in terms of θ:

$$\varepsilon^T \frac{\partial C}{\partial \theta}\varepsilon = -4A_1cos^3\theta sin\theta + 4A_2cos\theta sin^3\theta + A_3(cos^4\theta - 3cos^2\theta sin^2\theta) + A_4(3cos^2\theta sin^2\theta - sin^4\theta) + 2A_5(cos^3\theta sin\theta - cos\theta sin^3\theta) - 2A_6cos\theta sin\theta + 2A_7cos\theta sin\theta + A_8(cos^2\theta - sin^2\theta) \tag{9}$$

The sensitivity of the overall structural compliance to the elemental fiber angle is evaluated using the energy based method from Luo and Gea [12]. This energy based method considers the effect of the fiber orientation on both the strain and stress field. It has been shown to produce numerically more optimal orthotropic material structures then the strain or stress based methods alone.

To combine the stress and strain fields as a function of fiber angle an inclusion model is used to estimate the change in the strain and stress field caused by a change in the element fiber orientation ($\Delta\theta_e$) in terms of the current strain and stress, ε_0 and σ_0, and the energy factor α; that is defined as the ratio of the change in elemental strain energy to the change in body strain energy caused by $\Delta\theta_e$.

$$\varepsilon_e = (1-\alpha)\varepsilon_0 + \alpha S_e\sigma_0 \tag{10}$$

Substituting this into the objective function Eq. (2) gives the formulation.

$$\frac{\partial E}{\partial \theta_e} = (-1 + 2\alpha - \alpha^2)\varepsilon_0^T \frac{\partial C}{\partial \theta_e}\varepsilon_0 + \alpha^2\sigma_0^T\frac{\partial S}{\partial \theta_e}\sigma_0 + (2\alpha^2 - 2\alpha)\varepsilon_0^T\frac{\partial C}{\partial \theta_e}\sigma_0 \tag{11}$$

where S is the compliance matrix, calculated from Q^{-1}. Eq. (17) can be solved using the formulation for $\varepsilon^T(\partial C/\partial \Theta)\varepsilon$ in Eq. (8) and the equivalent formulations for $\varepsilon^T(\partial S/\partial\Theta)\varepsilon$ and $\varepsilon^T(\partial C/\partial\Theta)\sigma$. The complete derivations can be found in Luo and Gea [12].

Following in the sensitivity computation, Eq. (13), a new fiber angle is determined by Eq. (12).

$$\begin{aligned} &while\ iteration\ \ k < 10 \qquad \Delta\theta_e = \Delta\theta_{max}\left(\frac{S_e}{|S_{max}|}\right) \\ &Then{:}\quad \Delta\theta_e = \Delta\theta_{max}\left(\frac{S_e}{|S_{max}|}\right)\left(\frac{\left|C_{max}^{k-n} - C_{min}^{k-n}\right|}{C_{max}^{k-n}}\right) \qquad Where{:}\ \ 1 \le n \le 10 \end{aligned} \tag{12}$$

where $\Delta\theta_{max}$ is the maximum allowable change in one iteration given by the user. If $\Delta\theta_{max}$ is too small, it can delay convergence. If too large, it can induce instability and oscillation during optimization.

The change in the elemental fiber angle is interpreted in terms of the level set function in order to update the structure. The sensitivity of the elemental fiber angle to the nodal level set function values is given by differentiating (1) with respect to the level set function.

The nodal values can be differentiated over an element from the shape functions, N_i at node i.

$$\frac{\partial\varphi}{\partial x} = \sum_{i=1}^{4}\frac{\partial N_i}{\partial x}\varphi_i \qquad\qquad \frac{\partial\varphi}{\partial y} = \sum_{i=1}^{4}\frac{\partial N_i}{\partial y}\varphi_i \tag{13}$$

Using the standard results for differentiating the arctangent function, the sensitivity becomes,

$$\frac{\partial \theta_e}{\partial \varphi_i} = \frac{\frac{\partial N_i}{\partial y}\left(\frac{\partial N_i}{\partial x}\varphi_i + P_i\right) - \frac{\partial N_i}{\partial x}\left(\frac{\partial N_i}{\partial y}\varphi_i + Q_i\right)}{\left(\frac{\partial N_i}{\partial x}\varphi_i + P_i\right)^2 \left[\left(\frac{\frac{\partial N_i}{\partial y}\varphi_i + Q_i}{\frac{\partial N_i}{\partial x}\varphi_i + P_i}\right)^2 + 1\right]} \tag{14}$$

where $P_i = \sum_{\substack{p=4 \\ p \neq i}}^{4} \frac{\partial N_p}{\partial x} \varphi_p$ and $Q_i = \sum_{\substack{q=4 \\ q \neq i}}^{4} \frac{\partial N_q}{\partial y} \varphi_q$

Multiplying the result, $\partial\theta_e/\partial\varphi_i$, by the result of the energy equation $\partial\emptyset/\partial\theta_e$ gives the change in level set function in node *i* for element *e*. Since each node is attached to four elements, the level set update is calculated from the sensitivity of all four elements by (15).

$$\frac{\partial E}{\partial \varphi_i} = \sum_j \frac{\partial E}{\partial \theta_j} \frac{\partial \theta_j}{\partial \varphi_i} \tag{15}$$

where *j* denotes the elements surrounding node *i*.

4.4 Initialization of the Level Set Function

Optimizing the path of the lsf = 0 line ensures the continuity of fiber paths although it limits the search space and can easily find a local optimum. Since the solution can be highly dependent on the initial solution, a strategy for selecting the initial solution is investigated. We found that a consistently good solution is obtained by using the solution from an isotropic topology optimization as the starting solution. A topologically optimum solution has near uniform strain energy along the structural boundary where lsf = 0, unless the boundary is limited by the edge of the design domain. The structural boundary of the topological optimum is treated as the starting solution for the fiber angle optimization problem of a continuous panel. In this paper the structure of all the test problems are initialized using the 2D level set topology optimization method from Dunning and Kim [20].

5. Results and Discussion

To test the performance of the level set fiber orientation optimization method it is used to optimize the orientation of orthotropic material in three simple in plane loaded test models, a cantilever beam, a centrally loaded beam constrained at both ends, and a bridge structure loaded along the top edge, as shown in figure 3. All three models have a 40x20 bilinear shell element mesh with the material properties E_L = 137.9 GPa, E_T = 10.34 GPa, ν_{LT} = 0.29, ν_{TL} = 0.021, G_{LT} = 6.89 GPa, G_{LW} = 3.7 GPa and G_{TW} = 6.89 GPa, and are optimized with $\Delta\theta_{max}$ = 5°.

For reference, these problems were first run by the element based approach with $\Delta\theta_{max}$ = 45° [12]. The optimum solutions do not restrict the fibre angle continuity therefore the searches a larger design space at the risk of finding an unmanufacturable solution. The best optimal results were obtained from an initial uniform fiber orientation of 0° for the cantilever and centrally loaded models and 90° for the bridge model, with the solutions shown in figure 4. The problems were also run by the level set approached discussed in Section 4. The initial solutions are obtained from isotropic topology optimisation and are shown in figures 5a, 7a and 8a. The current implementation of the method requires the fiber angle to be initialized from a single boundary line. To create simple single boundary the level set topology optimization method is run without the hole insertion method [20], as demonstrated by the cantilever beam, figure 5a. However multiple boundary results were still produced for the center loaded and bridge beams, in these cases the longest single boundary, highlighted in red in figures 6a and 7a, were selected to initialize the lsf = 0 line for the fiber orientation stage of the optimization procedure. In the initial and final level set function line plots in plots figure 5, 6, 7 b and c the red line shows the current path of the lsf = 0 line.

In addition to the objective function, we introduce another metric for the comparative study, a *fiber continuity score* (FCS). In each of the solutions the optimal orientation of the fiber with in each element is compared to the eight neighboring elements. The elemental FCS is calculated from the percentage of neighboring elements whose fiber angles differ by less than 10°, as shown in Eq.18 where n_e is the number of elements and n_{ne}^i is the number of elements neighboring element i. This provides a numerical value indicating the continuinty of fibre angles between elements.

$$FCS = \frac{\sum_{i=1}^{n_e}\left[\sum_{j=1}^{n_{ne}^i}\begin{cases} 1, & -10° \leq (\theta_i - \theta_j) \leq 10° \\ 0, & otherwise \end{cases}\right]}{\sum_{i=1}^{n_e}(n_{ne}^i)} \tag{16}$$

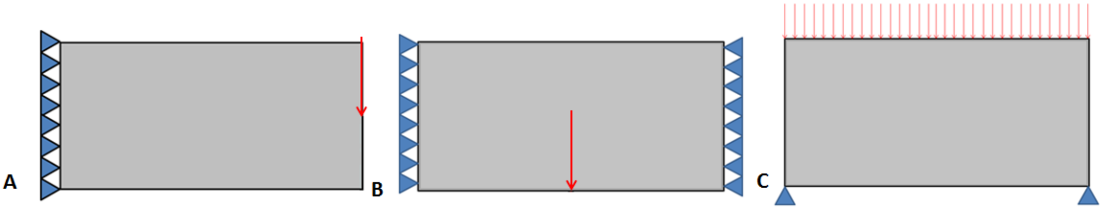

Figure 3: Test models. A: Cantilever Beam. B: Centrally Loaded Beam. C: Bridge

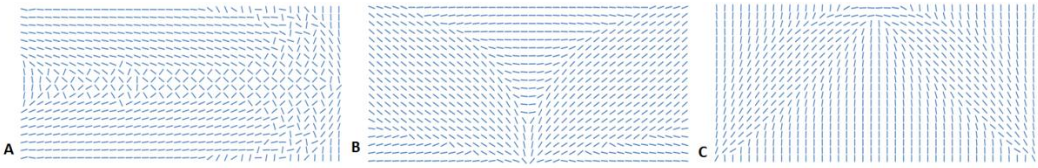

Figure 4: Optimal elemental fiber angle plots calculated by the elemental method using energy base sensitivity analysis. A: Cantilever Beam. B: Centrally Loaded Beam. C: Bridge

5.1 Cantilever Beam

The cantilever beam, results shown in figure 5, was initialized from the 2D level set result optimized with a volume constraint of 50% (figure 5A), creating initial fiber paths of a slight angle that steadily converge on the symmetry line (figure 5B). Fiber optimization increases the curvature towards the loaded end of the plate, figure 5C. The resulting optimal fiber paths are close to horizontal near the constrained edge of the plate, to resist the bending induced extension and compression of the plate, with more vertical fiber orientations near the opposite end to support the vertical load at the application point. The optimization reduces the compliance of the model by 40% from the initial structure to 19.67, while still maintaining a continuous fiber path, with a FCS value of 91.6%. The minimum radius manufacturing restriction, which has not been included in the optimization problem, is violated along the symmetry line of the model. The optimisation history is compared with that of the elemental optimisation in figure 6, both showing monotonic convergence. The elemental method solution of figure 4A has a compliance of 16.22, 17.6% lower than the level set method, with a much less continuous structure, FCS of 66.0%.

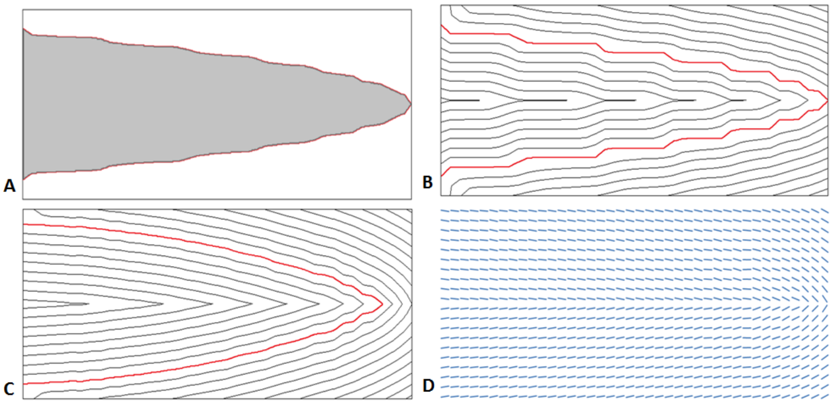

Figure 5: Optimal orientation of fiber tow paths in the cantilever beam using the level set method. A: level set topology optimization result. B: Initial tow paths defined by level set lines with integer values. C: Optimal tow paths defined by level set functions. D: Optimal elemental fiber angle plots. Optimized boundary line (lsf = 0) in red in B and C.

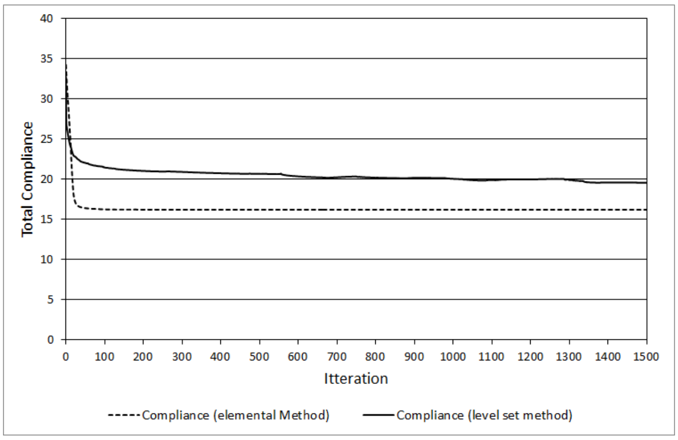

Figure 6: Graph showing the convergence of the elemental and level set methods of fiber orientation when applied to the cantilever beam. Elemental plot extended beyond convergence for compassion purposes.

5.2 Centrally Loaded Beam

The centrally loaded beam, results shown in figure 7, is initialized using the topology optimization result optimized with a volume constraint of 25% (figure 7A), producing an initial structure of roughly 45° angled fiber paths pointing towards the center although the structure flattens at the edges and steepens towards the load point (figure 7B). The optimization procedure makes only small modifications of the structure, smoothing the fiber paths so that all the fiber angles are around 45°connecting the load point to the constrained edge (figure 7C). These modifications reduce the overall compliance by 10% to 2.54, although again the fiber paths are entirely continuous, only violated at the symmetry point, with an FCS value of 96.0%. In comparison the elemental method produces an optimal structure with four different regions of continuous fibers, a v-shaped path of fibers around 45° supporting the load point with three regions of horizontal fibers in the gap resisting the local shear deformation of the structure. This arrangement has a structural advantage with compliance of 1.98, 28.3% lower than the level set result. The fiber path is relatively continuous, FCS of 84.9%, only violated where the different fiber orientation regions meet.

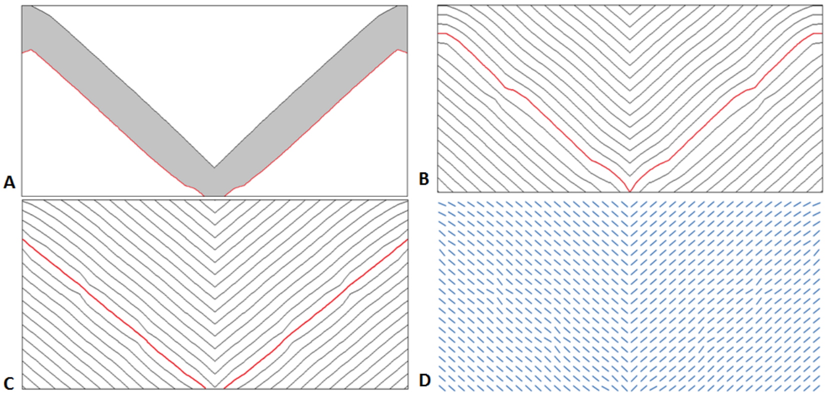

Figure 7: Optimal orientation of fiber tow paths in the centrally loaded model calculate using the level set method. A: level set topology optimization result. B: Initial tow paths defined by level set lines with integer values. C: Optimal tow paths defined by level set functions. D: Optimal elemental fiber angle plots. Optimized boundary line (lsf = 0) in red in B and C.

5.3 Bridge

The bridge, results shown in figure 8, is initialized using the level set topology optimization with a volume constraint of 35% producing initial fiber paths arching across the structure, figure 8A. The optimization procedure narrows this arch, moving the orientation of the fiber angle closer to the vertical to better support the loading on the structure. This further reduces the compliance by 9% to 2.04. The fiber path is entirely continuous apart from along the symmetry line; the FCS value is 94.04%. The elemental method produces structure similar to the centrally loaded method, but inverted, with a v-shaped region of 45° fiber paths supporting the center of loaded top edge, and three regions of vertical fiber angles in the gaps. The elemental method produces an optimal compliance of 1.57; 29.9% lower than the level set method with FCS of 84.1%

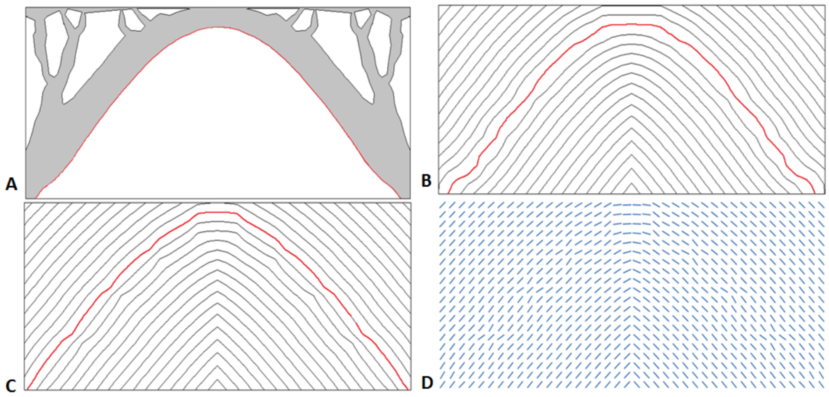

Figure 8: Optimal orientation of fiber tow paths in the bridge model calculate using the level set method. A: level set topology optimization result. B: Initial tow paths defined by level set lines with integer values. C: Optimal tow paths defined by level set functions. D: Optimal elemental fiber angle plots. Optimized boundary line (lsf = 0) in red in B and C.

5.4 Discussion of results

The results demonstrate that the level set method of fiber optimization is able to optimize the orientation of continuous tow paths. The compliance of all the level set results is higher than that obtained using the elemental method; this is to be expected as the level set method is constrained to continuous tow paths. Comparing the optimum solutions between the element and level set based solutions, it is interesting to note that of the three case studies presented in this paper, the largest difference in FSC shows the smallest difference in compliance (the cantilever beam), whilst the bridge showed the smallest difference in FSC and the largest difference in compliance. In the case of the cantilever beam, the most significant difference is the central section where the element based approach finds an alternate diagonal fiber configuration, giving a low FSC value, suggesting that the use orthotropic material in this region may be sub-optimal. Therefore the fiber continuity enforced by the level set method in this section thus does not significantly compromise the overall compliance. In the cases of the centrally loaded and bridge beams, the uniform vertical or horizontal fiber regions of the element based solutions suggests that this local orientation of the orthotropic material is optimal, thus the level set method of enforcing fiber continuity more significantly compromises the structural performance. In practice, the horizontal/vertical fiber regions and the diagonal regions, in the element based results for the bridge and central loaded beams, could be manufactured in a single ply using tow steering by cutting tows at the end of each uniform region, a technique that the current level set method does not account for. However since tow cutting can reduce stiffness and strength of the final structure [5] there is a benefit in considering the alternative continuous solutions and manufacture them by multiple plies with continuous fiber paths.

While the results demonstrate the potential of the level set fiber orientation optimization method the current implementation has a few draw backs that should be addressed. The requirement for a single boundary from the level set topology results requires that the lsf=0 line be selected by the user if the topology optimization geometry is complicated. In these simple test models the selection of the longest continuous boundary as the initial lsf = 0 line was relatively simple. However under more complicated loading conditions the choice of initial lsf =0 line could be less intuitive. Similarly using the local sensitivities around a single line to optimize the fiber orientation in the whole structure is a reasonable estimate in the simple test models shown above, under more complex loading conditions more global fiber sensitivities will need to be considered.

6. Conclusion

This paper proposes a new method for optimizing the orientation of fiber composites. The proposed method takes a two step-approach where the first step applies an isotropic level set topology optimisation to obtain the initial solution. The structural boundary where lsf = 0 is taken as the primary tow path which is then optimized at the second step. By using a continuous level set function to define and optimize the fiber orientation, it is demonstrated that the continuous and manufacturable tow paths could be obtained. The level set method successfully improves the optimality of fiber orientation in three test models from their initial designs while maintaining continuous fiber paths throughout the structure. Continuing work is underway to refine the selection of the initial solution and consider the global fiber orientation sensitivities during the update procedure. This paper demonstrates the feasibility of the level set function approach to optimization of continuous tow paths for fiber composites.

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