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Investigation of Plate Structure Design under Stochastic Blast Loading

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1 Abstract

The protection of lightweight ground vehicle crews from rapid acceleration events in hostile operating environments is an active field of study in defense research. Vehicle up-armoring traditionally entails the upgrading of standard shells with additional sacrificial components to improve blast protection capabilities. Resulting weight increment diminishes effectiveness and speed, increases vulnerability, and wears out vehicle components more quickly. This research furthers efforts in shape optimization relevant to blast events by incorporating robust design methods and the use of LS-DYNA to apply dynamic loading conditions in FEA simulations. The objective of this investigation is to establish a reliability-based robust design (RBRDO) methodology in blast mitigation. This approach controls the reliability of the protective shell while reducing the variance of the deflection performance caused by the variation of the uncontrollable environmental factors, namely the magnitude and location of the blast with respect to the design space. The shape of the structure is parameterized using envelope constraints, which lead to non-intuitive designs. In order to address the numerical cost of the uncertainty quantification, this work adapts a Univariate Dimension Reduction (UDR) methodology. Where mass reduction is considered to be the key performance metric, results demonstrate an increase in performance in methods that employ complex, multi-variable basis functions, which are able to adapt to uncertain conditions.

2 **Keywords:** design under uncertainty, blast loading, uncertainty quantification

3 Introduction

This article investigates the automated design process of isotropic plates to mitigate the effects of blast loading. Framed against the problem of blast protection system design for vehicle applications, the performance of the designs is measured in terms of mass and deflection, as has been done in previous investigations of lightweight armor design [1]. There has been substantial work on analysis of the effects of blast loading on shell structures. Argod and Belegunda have shown significant improvement of structure design using velocity-field based optimization and have demonstrated the effects of different boundary conditions for plates of this kind [2]. Methods of blast energy absorption have been evaluated through extensive design investigations of composite materials [3, 4], as well as numerical [5] and experimental [6] simulations of sacrificial structures.

The fundamental optimization problem to be analyzed in the design of a structure for blast mitigation is that of minimizing the kinetic energy transfer from the blast wave to the solid body. The goal of such efforts is to develop a system that could absorb a significant amount of the energy released in a blast event such that the underlying structure may be preserved. This article will focus on various design techniques for isotropic plate structures utilizing contemporary uncertainty quantification (UQ) and reliability-based robust design optimization (RBRDO) methods to compare design methodologies in terms of optimized structures produced via formal optimization techniques such as sequential quadratic programing (SQP).

The objective of this investigation is to compare feasible design methodologies through the expansion of the problem dimension in order to reach the limits of performance. For the purposes of this study, three profiles are evaluated, each under the same loading and boundary conditions. The candidate profiles are defined by a basis shape and are referred to here as geometrically constrained designs (GCD). A GCD includes an analytical description of the structure's profile in the problem statement, and the small number of design variables describing the structure's profile in a GCD allows the use of traditional programming methods such as sequential quadratic programming.

Only recently have investigations into topography optimization employing stochastic methods emerged [7]. Of chief importance in the application of any design under uncertainty method is the quantification of uncertainties in performance as result of uncertain inputs. The quantification of how uncertainty propagates throughout the system is the general goal of such endeavors and is commonly referred to as uncertainty quantification (UQ) [8]. The recent work of Chen and Lee has set out to examine the various methods of UQ and investigate the relative merits of each with respect to engineering test problems [8]. This investigation is focused on the application of a method that falls into the numerical integration based category of UQ methods. The univariate dimensional reduction method (UDR) is based in the decomposition of a multi-dimensional function into sum of several one-dimensional functions [9, 10].

This method is well-suited for the problems at hand as it is fairly computationally efficient for problems involving few random inputs [11].

This work is divided into three primary phases. Phase one deals with the generation of optimized plate structures under deterministic loading cases to create a baseline for comparison. Phase two involves the introduction of uncertainty into the design and the investigation of the UDR uncertainty quantification technique in application to FEA-based structural optimization. Phase three applies the RBRDO method to investigate the generation of reliable and robust plate structures under stochastic loading conditions.

4 Deterministic Loading Case

In blast protection system design for vehicle applications, there are two primary performance measures of critical relevance: weight and cabin penetration. Weight is privileged as a key performance measure stemming from the need for lightweight, compact structures that can be fitted to existing vehicle designs without extensive re-design of the frame or other vehicle systems. The goal is to design light structures that do not adversely affect vehicle performance. Large deflections of the plate structure due to a blast event can cause penetration into the passenger cabin of the vehicle and result in occupant injury. In this way, cabin penetration can be seen as a means of quantifying the degree to which the blast energy has been mitigated; if the deflection of the plate structure post-blast exceeds a certain amount, the design is unsuccessful. In consideration of these performance measures, the objective function is formulated as the mass of the plate structure and is minimized subject to displacement constraints (cabin penetration) and design space limitations. The general optimization problem for deterministic cases addressed in this paper is

$$\begin{aligned} &\text{find} && \boldsymbol{d} \\ &\text{minimize} && M(\boldsymbol{d}) \\ &\text{subject to} && P_c(\boldsymbol{d}) - P_{c\,max} \leq 0 \\ & && S(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{d}) = 0 \\ & && \boldsymbol{d}^L \leq \boldsymbol{d} \leq \boldsymbol{d}^U \end{aligned} \tag{1}$$

where $\boldsymbol{d} \in \mathbb{R}^{\boldsymbol{n}}$ is the set of all design variables characterizing the shape and thickness of the plate, $M(\boldsymbol{d})$ is the plate's mass, $P_c(\boldsymbol{d})$ is the penetration after the blast event with respect to datum plane, $P_{c\,max}$ is the maximum allowable value for penetration. The envelope constraint $S(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{d})$ is a function of the nodal coordinates $\mathbf{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$. The box constraint $\boldsymbol{d}^L$ and $\boldsymbol{d}^U$ are the lower and upper bounds for the design variables, respectively. The envelope constraint S is progressively relaxed so the design space is expanded to contain more design variables. This allows increasing the performance design problem at expenses of more complex topographies.

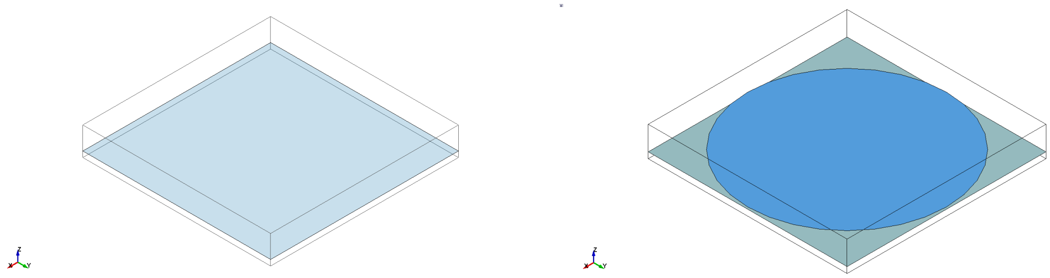
4.1 Geometrically Constrained Designs

Three envelope constraints are considered in this work. These envelope constraints define geometrically constrained designs whose shape is can be characterized through only a few design variables. A description of each geometrically constrained design along with a brief motivation for each is given here:

- **Flat Plate Design.** This topography for flat plane is defined by the condition in which the all the $z-$coordinate values for every node is the same. This design is regarded as the baseline for comparison with each progressive candidate design.
- **Pyramid Profile Design.** Methods of blast mitigating structure design have been evaluated extensively through experimental efforts in the development of lightweight V-shaped hulls, and have demonstrated that V-shaped designs can mitigate the effects of blast events [12]. The pyramid profile design is an improvement of the V-shape design used in concept designs, due to the fact that it is constrained on four sides.
- **Polynomial Function Design.** An additional design methodology is examined which relaxes the problem further. In the polynomial function case, a function of several variables is used to generate a complex curve: $f(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$. The plate design is achieved as a surface of revolution by rotating the curve about the $z-$axis.

4.2 Design Domain

The design domain is chosen in such a way as to simulate under-vehicle conditions. The datum plane is an arbitrary distance from the lower plane of the design space to account for the distance between an under-vehicle plate structure and the passenger cabin. This distance allows for some penetration below the datum plane without penetrating the cabin area. In order to privilege the display of the optimized plate designs, the full design domain has been modeled from beneath or "upside down" orientation, resulting in a 1 m by 1 m by 0.15 m three-dimensional space with the datum plane distance δ from the bottom plane as shown in Figure 1 (a). For this orientation, a node is considered to penetrate the cabin if its nodal $z-$coordinate is less than -0.03 m. While the flat plate and pyramid design methods examined here make use of the full design domain, the polynomial function design utilizes only the center portion of the full domain. This domain has its origins at the center of the plate and extends radially as shown in Figure 1 (b). This distinction is noted in the comparison of results.



(a) (b)

Figure 1: Full design domain representation (a) with datum plane (z = 0) shown, and radial design domain representation (b) with datum plane (z = 0) shown

4.3 Optimization Results: Deterministic Case

The optimization problem as described by Eq. (1) is solved using gradient-based sequential quadratic programming techniques; all candidate optimization problems are solved using an active set algorithm. The non-linear programming algorithm is described by Powell [13] and incorporated in Matlab. The numerical results for the convergent designs under the deterministic loading case are given by Table 1. The flat plate design represents the least complex topography for a plate structure and is provided here as a baseline for comparison.

Table 1 Numerical results, deterministic loading case

Method	**No. of design variables**	**No. of iterations**	**No. of function calls**	**Mass of converged design (kg)**	**Mass reduction from baseline (%)**
Flat Plate Design	1	18	71	226.4	0.0
Pyramid Profile Design	2	17	121	99.1	-56.2
Polynomial Function Design	5	173	1398	113.1	-50.0

It is demonstrated that the pyramid profile design is far superior to the baseline flat plate topography. Convergent results demonstrate an increase in height to the boundary of the design domain while exhibiting a thickness less than half of that of the baseline design, resulting in a structure of significantly smaller mass, as can be seen Figure 2.

The optimized result for the polynomial function design method also demonstrates an improvement over the baseline design in terms of mass reduction, but demonstrates an increase in mass when compared to the pyramid profile due to an increase in thickness in the converged results. It is thought that this increase in thickness is in compensation for a feature of this design not present in the previous two design methods mentioned here. Due to the fact that the design is generated as a surface of revolution, which is inherently circular, while the shape of the plate as described by the domain is square; the whole design domain is not used on the generation of the plate profile. In turn, this creates an inherent weakness in the plate structure in the portion of the plate beyond the area described by the surface of revolution – i.e. the flat portions of the plate outside of the bulging center portion.

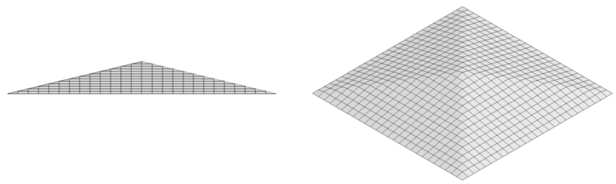


Figure 2: Optimized pyramid profile design in side view and isometric view). Convergent numerical results: thickness = 12.4 mm, height = 120 mm, and mass = 99.1 kg.

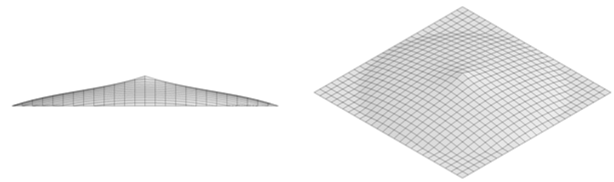


Figure 3 Optimized polynomial function design in side view and isometric view. Thickness = 14.1 mm, C_0 = 0.112, C_1 = -0.332, C_2 = 0.642, C_3 = -0.859, mass = 113.1 kg.

In general, it can be concluded that the less complex topography, i.e. the pyramid profile design method, out-performs the more complex polynomial function design in terms of weight reduction and computational cost. The pyramid profile design has a lesser value for mass and required over 1200 fewer function evaluations to reach a convergent solution.

5 Uncertainty Quantification

The objective of this phase of the investigation is the evaluation of recently proposed methods for uncertainty quantification (UQ) for application in reliability-based robust design optimization for plates under stochastic loading. Due to the highly non-linear problems associated with the response to blast loading, the two methods for uncertainty quantification used here fall under the numerical integration based category. Numerical integration based methods first approximate the statistical moments of the response and then the probability density function can be approximated using empirical distribution systems [14]. The univariate dimensional reduction method (UDR) as proposed by Rahman and Xu [9, 10] decomposes the performance function into the sum of several univariate functions, which can become costly when the number of random variables is large.

The end result of this UQ method is the approximation of the first two ordinary statistical moments of the system response, mean and variance, which can be used to characterize the probability distribution of system performance. This probabilistic information can be used to develop new designs that incorporate uncertainty and in turn increase confidence in product performance.

5.1 Univariate Dimensional Reduction (UDR)

The univariate dimensional reduction method is based in the decomposition of a multi-dimensional function into sum of several one-dimensional functions. Consider the decomposition of the performance function $g(\boldsymbol{X})$ as shown:

$$g(\boldsymbol{X}) \cong \hat{g}(\boldsymbol{X}) = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} g(\mu_1, \dots, X_i, \dots \mu_N) - (N-1) * g(\mu_1, \dots \mu_N) \tag{2}$$

where μ_i is the mean value of the random variable X_i, $g(\mu_1, \dots, X_i, \dots \mu_N)$ is a random response that depends on the ith random variable, and N is the total number of random variables. It can be demonstrated that a Taylor series expansion of the approximation function $\hat{g}(\boldsymbol{X})$ contains all of the single variable terms of the Taylor series of $g(\boldsymbol{X})$, which means that the approximation error is due only to the terms with two or more variables. A demonstration of the Taylor series expansion of the univariate approximation, complete with determination of the residual error can be found in Xu and Rahman [9].

This univariate decomposition can be applied to the multi-dimensional integral for moment calculation given as

$$E\left[\left(g(\boldsymbol{X})\right)^k\right] = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \left(g(\boldsymbol{X})\right)^k f_x(\boldsymbol{X}) d\boldsymbol{X} \tag{3}$$

where $f_x(\boldsymbol{X})$ is the joint probability density function of $\boldsymbol{X}$, and $E$is the expectation operator. According to the moment based integration rule [9], which is similar to Gaussian quadrature techniques for numerical integration, the statistical moments of a function can be obtained through the numerical approximation of Eq. (3) expressed by the following:

$$E\left[\left(g(\boldsymbol{X})\right)^k\right] = \sum_{i=1}^{n} w_i g^k(x_i) \tag{4}$$

where w_i are the weights associated with the quadrature and x_i are the quadrature points. If the probability density function (PDF) of the random input variables are known, for example standard normal distribution, then the weights and quadrature points can be found via the moment based quadrature rule (MBIR) [15]. Similar to Gaussian quadrature, the degree of precision is $2n - 1$. Therefor for highly nonlinear functions, more quadrature points will result in a higher accuracy of approximation.
Through the combination of Eq. (2), (3) , and (4) the mean and variance of the performance function can be expressed as

$$\mu_G = E[g(\boldsymbol{X})] \cong \sum_{j=1}^{n}\sum_{i=1}^{N} w_{ji}\left[g(\mu_1, \dots, \mu_{i-1}, X_{ji}, \mu_{i+1}, \dots, \mu_N) - (N-1)g(\mu_1, \dots, \mu_N)\right] \tag{5}$$

and

$$\sigma_G^2 = E[g^2(\boldsymbol{X})] - (E[g(\boldsymbol{X})])^2 \cong \sum_{j=1}^{n}\sum_{i=1}^{N} w_{ji}\left[g^2(\mu_1, \dots, \mu_{i-1}, X_{ji}, \mu_{i+1}, \dots, \mu_N) - (N-1)g^2(\mu_1, \dots, \mu_N)\right] - \mu_G^2 \tag{6}$$

where *n* is the number of quadrature points and *N* is the number of random variables. It can be understood from the above equations that the UDR method involves a double approximation to calculate the statistical moments: the first approximation which decomposes the function into the sum of several univariate functions, and a second to approximate the multi-dimensional integral for moment calculation. Due to highly nonlinear performance functions observed in the investigation of blast loading of plates, five quadrature points are used for all approximations as shown in Table 2.

Table 2 MBIR Quadrature Points and Weights

MBIR Quadrature Points, x_i	Weights, w_i
± 2.856970	0.011257
± 1.355626	0.222076
0.0	0.533333

The computational efficiency of this method is dependent on the number of random variables and the number of quadrature points used in the approximation. Generally speaking, assuming symmetric distribution of the random variables, the number of function evaluations of $g(\boldsymbol{X})$ necessary for the approximation of the first two statistical moments is

$$\text{No. of function evaluations} = nN + 1 \tag{7}$$

where *n* is the number of quadrature points and *N* is the number of random variables.

5.2 Plate Design Application

In order to apply the methods of uncertainty quantification to the area of plate design, numerical function evaluations are replaced by finite element simulations to calculate the maximum deflection of the plate under uncertain loading conditions. In this way, the statistical moments of the protection response can be approximated for a given design. The amount of deflection below the datum plane, $P_c(\boldsymbol{d})$ is now a function of both the design variables $\boldsymbol{d}$ and the vector of random variables $\boldsymbol{X}$, and can be stated as $P_c(\boldsymbol{d}, \boldsymbol{X})$. This creates a need for the introduction of uncertain quantities into the design.
Staying within the framework of vehicle protection, there are two chief candidates for sources of epistemological uncertainty: blast magnitude and blast location. The deterministic loading cases as previously investigated assumed a single loading case, that being a charge of 5 kg TNT located at a position of 40 cm above the plate dead center. In order to investigate the uncertainty propagation throughout the system, those deterministic conditions are replaced by random variables, each considered to have standard normal distribution using the deterministic cases as the mean values. The magnitude of the blast X_M is therefore defined as a normally distributed random variable with a mean of 5 kg TNT a variance of 1 kg TNT, or $X_M \sim N[5,1]$. Considering all the cases which fall under plus/minus three standard deviations, this allows the blast magnitude to vary between a relatively small blast magnitude of 2 kg TNT to a relatively large magnitude of 8 kg TNT. Similarly, the x-coordinate of the blast location, X_L, is defined as a standard normal random variable having with a mean value of 0 m and a variance of 0.0625 m, or $X_L \sim N[0,0.0625]$. This allows for the location of the blast to be modeled as occurring to the left or right of the vehicle, while the majority of the points will fall within $\pm$ 0.5 m of dead center.

To evaluate the methods for uncertainty quantification, the uncertain loading conditions are applied to the convergent structures generated under deterministic loading cases and the statistical moments of the protection performance value μ_P and σ_P are calculated. These deterministic designs represent cases of few design variables (base and pyramid) and a case of an increased number of design variables (polynomial), in order to investigate the application and accuracy of the uncertainty quantification methods across the range of design domains.

5.3 Uncertainty Quantification Results

As shown in Table 3, the values found via UDR approximation compare favorably with those generated by the Monte Carlo simulation of 10,000 samples. Since both random variables are normally distributed, the computation cost of the UDR method is reduced to

$$\text{No. of function evaluations} = n(N-1)+1 \tag{8}$$

for a total of 9 FEA simulation to calculate the low order moments of performance.

For all deterministic design cases, the relative error of the mean calculation with respect to the Monte Carlo results was within 10%, while the mean values for the base and pyramid designs was with 2%. Similar results were found for the standard deviation, with the largest error being observed in the pyramid design, at approximately 8%.

Table 3 Numerical results for UDR method in plate design application.

	Flat Plate		**Pyramid**		**Polynomial**	
Method	**Mean (μ)**	**St. Dev. (σ)**	**Mean (μ)**	**St. Dev. (σ)**	**Mean (μ)**	**St. Dev. (σ)**
UDR	-0.0252	0.0096	-0.0522	0.0321	-0.0434	0.0254
Monte Carlo	-0.0256	0.0092	-0.0512	0.0297	-0.0459	0.0247
% Error	-1.68	4.47	1.95	8.01	-5.53	3.00

6 Reliability-based Robust Design Optimization (RBRDO)

Reliability-based robust design optimization (RBRDO) is a hybridization which combines robust design optimization (RDO) and reliability-based robust design optimization (RBDO) to achieve multiple goals simultaneously [11, 16]. This class of optimization combines the variance reduction goal of RDO with the probability constraint based optimization of RBDO into a new optimization problem expressed as

$$\begin{aligned} &\text{find} && \boldsymbol{d} \\ &\text{minimize} && F_C(\boldsymbol{d}, \mu_P, \sigma_P^2) \\ &\text{subject to} && P[P_c(\boldsymbol{d}, \boldsymbol{X}) > P_{c\,max}] \le P_f^T \end{aligned} \tag{9}$$

where $\boldsymbol{d} \in \mathbb{R}^{\boldsymbol{n}}$ is the set of all design variables, $F_C(\boldsymbol{d}, \mu_P, \sigma_P^2)$ is the cost function, and $P[P_c(\boldsymbol{d}, \boldsymbol{X}) > P_{c\,max}]$ is the probability that the deflection of the plate will exceed the maximum allowable value $P_{c\,max}$ constrained to be less than the target probability P_f^T. This hybridization allows the designer to minimize the variance of the design with respect to random inputs as in robust design optimization, while including the failure constraints that are the hallmark of reliability-based design.

Characterization of the cost function $F_C(\boldsymbol{d}, \mu_P, \sigma_P^2)$ has a substantial impact on the outcome of the optimization, and there various ways that the cost function may be formulated [16]. In the context of design of lightweight structures for blast mitigation, two different cost function formulations are investigated. The first formulation is based on the "nominal-the-best" type as proposed by Chandra and found in the works of Choi and Lee [17, 18]:

$$F_C(\boldsymbol{d}, \mu_P, \sigma_P^2) = w_1 \left(\frac{\mu_P - \mu_T}{\mu_{P_0} - \mu_T} \right)^2 + w_2 \left(\frac{\sigma_P}{\sigma_{P_0}} \right)^2 \tag{10}$$

where μ_T is the target nominal value for the mean of the performance function, and w_1 and w_2 are the weights to be chosen by the designer. To reduce the dimensionality problem of the two terms, each is divided by an initial value μ_{P_0} and σ_{P_0}. The value of the cost function is dependent on terms associated with both the mean μ_P and variance σ_P of the performance function, thus it is a bi-objective problem. The optimum value of this bi-objective problem is heavily dependent on the weights of each term; Variation of the weight values is investigated in the numerical applications section of this thesis. The second cost function investigated is based on a formulation used for optimum design for crashworthiness by Lee *et al.* [11] given as

$$F_C(\boldsymbol{d}, \mu_P, \sigma_P^2) = w_1\left(\frac{M(\boldsymbol{d})}{M_0(\boldsymbol{d}_0)}\right) + w_2\left(\frac{\sigma_P}{\sigma_{P_0}}\right)^2 \quad (11)$$

where $M(\boldsymbol{d})$ is the mass of the structure and $M_0(\boldsymbol{d}_0)$ is the mass of the structure calculated as a function of the initial design variable values.

6.1 Application to Plate Design

As a demonstrative example, the general implementation of RBRDO in the polynomial function plate design is

$$\begin{aligned}
&\text{find} && c, C_0, C_1, C_2, C_3 \\
&\text{minimize} && F_C(c, C_0, C_1, C_2, C_3, \mu_P, \sigma_P^2) \\
&\text{subject to} && P[P_c(\boldsymbol{d}, \boldsymbol{X}) > P_{c\,max}] \le 0.05 \\
& && x_i^2 + y_i^2 = r_i^2 \\
& && \boldsymbol{z} - C_0 + C_1 r + C_2 r^2 + C_3 r^3 = 0 \\
& && h = 120\ mm \\
& && 7.5\text{ mm} \le c \le 30\text{ mm} \\
& && C_0 - \text{h} \le 0 \\
& && C_0 + RC_1 + R^2 C_2 + R^3 C_3 = 0
\end{aligned} \quad (12)$$

where c, C_0, C_1, C_2, C_3 is the set of all design variables characterizing the shape and thickness of the plate, consisting of the thickness of the plate and the four polynomial constants. $\boldsymbol{X} \sim N[\mu, \sigma^2]$ represents the normally distributed random variables for blast magnitude and location. The statistical moments of the protection performance are μ_P and σ_P^2. The failure criterion, $P_{c\,max}$, and the probability constraint, $P[P_c(\boldsymbol{d}, \boldsymbol{X}) > P_{c\,max}] \le 0.05$, are implemented such that there is a 5% probability that the maximum deflection of the structure beneath the datum plane will exceed 3 cm after the blast. The objective function, $F_C(c, C_0, C_1, C_2, C_3, \mu_P, \sigma_P^2)$, is a function of all of the design variables as well as the statistical moments of performance. Characterization of the cost function can have a substantial impact on the outcome of the optimization. In order to investigate the effect the cost function on the optimization results, two different formulations are used in the optimization problem: a mean-based cost function and a mass-based cost function.

6.2 Comparison of RBRDO Results

In the interest of comparing numerical results, three different cost function characterizations are applied to the design methods: the flat plate design, the pyramid design, and the polynomial function design. Table 4 gives the results of all optimization methods for the flat plate design. The deterministic design has an approximately 30% probability of failure when uncertain loading conditions are considered. Interestingly, the remaining optimization methodologies each produce nearly identical results. The mass of the optimized structure is approximately 262 kg with a 5% probability of failure in all cases These results are indicative of a lack of robustness in the design methodology itself. In order to satisfy the failure constraint present RBRDO, the thickness of the plate must be increased. Since there are no other design variables, as soon as the thickness is such that is satisfies the constraint the design is considered converged. Put simply, the only way to increase the reliability of the structure under uncertain conditions is to increase the plate thickness.

Table 4 Numerical results for optimization of flat plate design.

Flat Plate Optimization	Mass	No. of iterations	No. of F.E.	Mean (μ)	St. Dev. (σ)	Prob. Of Failure
Deterministic Design	226.32	18	71	-0.0252	0.0096	0.3085
RBRDO (Mean-based cost function)	262.30	10	621	-0.0164	0.0083	0.0506
RBRDO (Mass-based cost function), Weights Even	262.40	10	207	-0.0164	0.0083	0.0506
RBRDO (Mass-based cost function), Weights Skewed	262.33	10	306	-0.0164	0.0083	0.0506

The numerical results for the pyramid design, given by Table 5, demonstrate a similar lack of robustness. The structures produced by the various optimization methods again produced nearly identical in terms of mass and reliability. The exception to this trend is the even-weighted mass-based RBRDO method, which privileges minimization of the variance above minimization of the mass and thus produces a heavier structure with a higher degree of reliability. As was the case in optimization of the flat plate, the convergent structures are nearly identical with only the thickness varying from design to design.

Table 5 Numerical results for optimization of pyramid design.

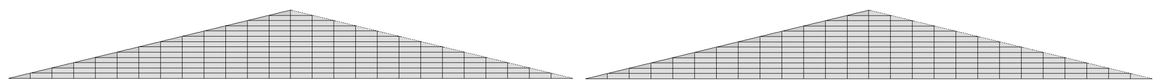
Pyramid Optimization	Mass	No. of iterations	No. of F.E.	Mean (μ)	St. Dev. (σ)	Prob. Of Failure
Deterministic Design	99.31	17	121	-0.0522	0.0321	0.7554
RBRDO (Mean-based cost function)	135.39	10	306	-0.0106	0.0118	0.0500
RBRDO (Mass-based cost function), Weights Even	144.39	5	135	-0.0080	0.0088	0.0062
RBRDO (Mass-based cost function), Weights Skewed	135.37	10	450	-0.0106	0.0118	0.0501

In the case of the polynomial function design method, however, some real variation in the convergent structures produced is demonstrated. As given in Table 6, the results for the optimization procedures show distinct structures generated by the design under uncertainty procedures. The RBRDO optimization results clearly demonstrate structures that vary as function of the optimization method. The mean-based RBRDO procedure, which defines a target value for the mean, produces the structure with the largest mass value at ~175 kg. The mass-based RBRDO procedure, which in turn privileges reduction of mass, produces lighter structures. Of particular interest is the result for the skewed-weights mass-based RBRDO procedure, which yielded a structure of ~131 kg and probability of failure of only ~5%. This result exceeds the performance of the pyramid design produced by the same optimization procedure, in contrast to the deterministic results in which the pyramid out performs the polynomial function.

Table 6 Numerical results for optimization of polynomial design.

Polynomial Optimization	Mass	No. of iterations	No. of F.E.	Mean (μ)	St. Dev. (σ)	Prob. Of Failure
Deterministic Design	112.38	173	1398	-0.0434	0.0254	0.7011
RBRDO (Mean-based cost function)	175.62	7	936	-0.0158	0.0086	0.0494
RBRDO (Mass-based cost function), Weights Even	146.43	10	837	-0.0058	0.0039	0.0001
RBRDO (Mass-based cost function), Weights Skewed	131.29	12	1386	-0.0135	0.0100	0.0495

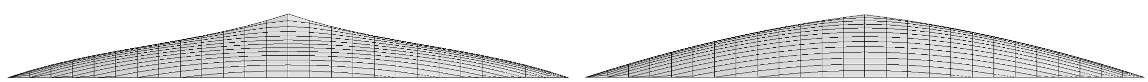
The reason for this variation of results stems from the additional design variables present in the polynomial design which allow for the generation of more complicated topography. Consider Figure 4, which shows a side by side comparison of the pyramid structure produced via deterministic optimization (left) and the structure produced via RBRDO optimization (right). The structures have identical topographies with a height of 120 mm, but have different thickness and thus different mass.



(a) (b)

Figure 4 Finite element models of the pyramid structures produced by deterministic optimization (a) and RBRDO optimization (b).

In contrast, consider the polynomial design structures produced via deterministic optimization (left) and the structure produced via RBRDO optimization (right), as shown by Figure 5. In this case, we see two distinct topographies, as the shape was able to conform to the optimization constraints in terms of shape and thickness.



(a) (b)

Figure 5 Finite element models of the polynomial function structures produced by deterministic optimization (a) and RBRDO optimization (b).

7 Conclusion

This work deals with the robust and reliable design of blast mitigating shell structures. The shape of the structures is parameterized using three envelope constraints, namely flat plate, pyramid profile, and a polynomial function. The relaxation of the design allows to obtaining structures with increased complexity. The structures are first optimized via gradient-based, SQP techniques under deterministic loading conditions, where the measures of performance are mass and dynamic deflection. Uncertainty is introduced into the loading conditions, and the propagation of the uncertainty into plate performance is quantified through an adaptation of the univariate dimensional reduction method. Through the application of this method of uncertainty quantification, the development and evaluation of design under uncertainty methodology for blast-resistant component design has been accomplished.

Topography optimization of plate structures subject to stochastic blast loads is performed. Uncertainty in introduced in terms of the location and magnitude of the blast. As a method of uncertainty quantification, the univariate dimensional reduction method is found to compare favorably with a quasi-Monte Carlo simulation in this application at a drastic reduction of computational cost.

Utilizing various cost-function formulations, reliability-based robust design optimization has been performed to obtain designs that optimize a merit function while ensuring a target reliability level. There are two primary conclusions to be made from the comparative results. The first: for the design of lightweight structures where mass is considered to be the key performance metric, unless there are enough design variables such that the structure can adapt to uncertain conditions under various constraint criteria, all methods for design under uncertainty will produce similar or identical results. The second: the more complex design method, that is the polynomial function design with more design variables, is able to produce structures that out-perform less complex designs.

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