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# A Framework for Reliability Based Design Optimization of Curvilinearly Stiffened Panels

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## Abstract

A method for decoupling reliability based design optimization problem into a set of optimization and reliability analysis is described. The inner reliability analysis and outer optimization are separated and performed in a sequential manner. In each iteration, the probability constraints are converted into equivalent deterministic constraints using the reliability analysis and then implemented in the deterministic optimization problem. Numerical results are given for an example of the developed framework to show that the sequential RBDO converges and obtains the objective function while satisfying the reliability constraints. The framework is tested for curvlinearly stiffened panels. The stiffeners are defined using a set of design variables that include the shape and the size of stiffeners. Using this framework, panel under four load cases, similar to the ones used in practical applications, are optimized while subjected to the probability constraints on buckling, stress, and crippling.

**Keywords:** Reliability, Optimization, Stiffened Panels

## 1. Introduction

The aerospace industry continuously seeks to reduce the weight of flight vehicle structures. There is an unending quest for both materials and fabrication techniques that can yield structures that are light weight yet are robust, durable, damage tolerant, and corrosion resistant. To that end, NASA Langley Research Center and others are investigating new approaches [1], such as Electronic Beam Free Form Fabrication, to fabricate aerospace structures using additive manufacturing. These approaches, which fall under the rubric of unitized structures, have made it possible to tailor a metallic structure, e.g. a curvilinearly stiffened panel, according to such operational requirements as high stresses and desired acoustic behavior without the panel undergoing buckling [2]. The flexibility that the curvature of stiffeners provides in addition to their location and orientation has made them appealing to aerospace industries. Since there was not any computational design environment available in the research/commercial domain for design optimization of curvilinearly stiffened plate, a framework called EBF3PanelOpt has been developed by Unitized structures group at Virginia Tech [3]. In addition to the development of optimization tool for curvilinearly stiffened panels, due to various uncertainties in loading, material properties, and geometry, an appropriate and efficient ways to model the uncertainties of stiffened panels is also required. There is an increasing realization, derived from the experience of the nuclear industry, that designing a structure by taking into account various uncertainties is a more rational approach than the current one, using safety factor [4]. Failure in aircraft structure can have catastrophic consequences, with resultant loss of life and of the aircraft. The use of reliability concepts in structural optimization is, thus, of vital importance in saving structural weight and, at the same time, maintaining an acceptable level of safety.

The conventional approach to formulate the Reliability Based Design Optimization (RBDO) is to utilize a double-loop optimization and uncertainty analysis which are nested one in another and interact in a synergistic manner to minimize the objective function while satisfying the probability constraints. The purpose of the optimization loop is to execute optimum search. The purpose of the uncertainty analysis loop is to evaluate the design and its uncertainty characteristics. At every iteration in the outer loop, the optimizer calls the uncertainty analysis which then requires to execute many simulations depending on the uncertainty analysis methods being used e.g. Monte Carlo Methods and First and Second Order Reliability Methods. The computational cost associated with the nested RBDO is very expensive due to the number of simulations for each uncertainty analysis at every optimization iteration [5]. Since the double loop procedure may be computationally impractical, researchers have studied several techniques to reduce the high computational expense of RBDO. To merge the double loop RBDO into one single level problem, Agarwal et al. [6] proposed to replace the lower-level inverse reliability analysis optimization problem by the corresponding first order necessary Karush-Kuhn-Tucker (KKT) optimality conditions at the upper level optimization. The proposed formulation is implemented in an augmented design space that consists of the original variables and the most probable point (MPP) of failure corresponding to each hard constraint. This formulation is mathematically equivalent to solving the original nested optimization if the constraint qualification conditions are satisfied. The issue with this proposed formulation is that the number of design variables is increased by the number of hard constraints. This increases the optimization computational cost. Chen, Hasselman, and Neill [7] also developed another method to convert the double loop RBDO into a single loop procedure by approximately finding MPP of each active constraint. The MPP is found by using the gradients of the constraints and the desired safety factor.

Another way of converting the double loop RBDO into a single loop procedure is to perform the optimization and uncertainty analysis sequentially. The double loop reliability constraints are formulated as deterministic constraints based on the uncertainty analysis. Then the equivalent constraints are used in the optimization to direct the optimal solution to the feasible region which satisfies the reliability requirement.

Agarwal and Renaud [8] developed decoupled method for RBDO. The deterministic optimization loop is separated from the reliability analysis loop. The MPPs are updated during the deterministic optimization by using a first-order Taylor series expansion about the design point from the preceding cycle. The sensitivities required to update the MPP are obtained using optimization at the MPP optimal solution. Elishakoff [9] studied the relationship between the safety factor and reliability levels and showed that in many cases the safety factors can be directly expressed by the required reliability levels. However, since the value of safety factor does not specify the reliability, he reported the probabilistic sufficiency factor [10] that is more related to the target reliability. A probabilistic sufficiency factor of 1.0 represents that the achieved probability of failure is equal to the target one. If a probabilistic sufficiency factor is less than one, the probability of failure exceeds the target and the design is not safe, and probabilistic sufficiency factor larger than one means the probability of failure is less than the target probability. Qu and Haftka [10] compared the probability of failure, safety index, and probabilistic sufficiency factor. They showed that the response surface approximation can have a better accuracy when it is fitted to the probabilistic sufficiency factor than to the probability of failure and the safety index. Furthermore, they showed that the probabilistic sufficiency factor provides more information in regions of low probability than the probability of failure or the safety index.

Wu et al. [11] also proposed a safety factor based RBDO by converting reliability constraints to the equivalent deterministic constraint with safety factor in the optimization cycle. Du and Chen [12] developed a sequential RBDO methodology. In their framework, the optimization is conducted by using the MPP of the previous design point and then reliability analysis is performed to update the MPP. The optimization and reliability analysis cycle is repeated until the objective convergence and the reliability requirement is achieved. Ba-abbad et al. [13] improved Du and Chen technique to distribute the reliability of the system over its components in an optimal way. In their technique at each iteration, first, the first-order reliability analysis is carried out to check if this design has acceptable reliability. Then, the performance measure analysis is performed to calculate the MPPs of the failure modes. Finally, the approximate deterministic optimization is conducted to find the optimum design and measure the maximum of the safety indices.

This research considers an efficient reliability based design optimization of curvilinearly stiffened panels. A sequential optimization and reliability analysis methodology is developed that utilizes the shape and size variables as design variables; the applied compression and shear in-plane loads, and the Young's modulus, all as random variables. The proposed approach, first, conducts the reliability analysis to find MPPs and the probability of satisfying the given constraints. Next, each probability constraint is converted to an equivalent deterministic constraint by using its MPP of previous iteration. By replacing the random variables with their MPPs, the current constraint is shifted to meet the desired reliability level. Then the deterministic shape and size optimization is performed to optimize the mass of structures while satisfying the equivalent constraints on buckling, stress, and crippling.

A method for decomposing the shape and size optimization problem is utilized to improve the efficiency and accuracy of developed framework. In the two-step optimization algorithm, the shape and size optimization process is divided into two parts, the first part involves calculation of the best stiffener curve that gives the maximum buckling load subjected to stress and crippling constraints, and the second step consists of a sizing optimization while keeping the stiffener curve unchanged to minimize the mass while satisfying the buckling, stress, and crippling constraints. It is necessary to employ an iterative approach between two steps in order to obtain an accurate optimal result. The updated design variables obtained by deterministic optimization are fed to the reliability analysis to find the probability of safety and update the MPPs. The optimization and reliability analysis cycle are repeated until the objective function is converged and the probabilistic constraints are satisfied. The sequential RBDO framework employs *EBF3PanelOpt*, a Computational Design Environment for panel with curvilinear stiffeners, to analyze the structures. *EBF3PanelOpt* is developed in a PYTHON programming environment. The finite element commercial software, Msc.PATRAN and Msc.NASTRAN are used to parametrically create and analyze a detailed finite element model of curvilinearly stiffened panels.

The developed sequential RBDO framework also utilizes DAKOTA, Design Analysis Kit for Optimization and Terascale Applications, for reliability analysis and design optimization. The present study includes a number of numerical examples to discuss the optimal design of curvilinearly stiffened panels subjected to probabilistic constraints. In these examples, various combinations of loading conditions such as uniform, linearly varying, and parabolically varying in-plane compression and shear loads are taken into account as random variables.

## 2. Reliability Based Design Optimization Framework

For deterministic design optimization, all the important parameters influencing the system are assumed to be well defined with known values. For stiffened panels, these parameters could include panel loading and material properties. Traditionally, uncertainties are accounted for by using safety factors in the design process. This approach often leads to overdesigning the system and the need to include uncertainty in designing the system becomes important and Reliability-based design optimization is being increasingly accepted by the industry. However, RBDO encounters computational issues when it is applied to a complex engineering design. Performing a reliability analysis for a given structures requires repeating the structural analysis for different sets of random variables, which can be computationally very expensive when using numerical methods such as finite element analysis. In order to reduce the computational time of RBDO, the framework can be reformulated by converting the probabilistic constraints into equivalent deterministic constraints, [11] and [12].

To perform the RBDO efficiently rather than utilizing the commonly used double-loop framework, the deterministic optimization and the reliability analysis are decoupled from one another. Various techniques have been developed to decouple the optimization and the reliability analysis. One of these techniques is the sequential optimization and reliability analysis (SORA) [12]. The main idea behind SORA is to perform optimization by applying the equivalent deterministic constraints, instead of using the reliability constraints. The constraints on the probability of satisfying constraints of a structure can be converted to the equivalent deterministic constraints by using the MPPs at the desired level of safety.

Most Probable failure Point (MPP) is the design point that has most significant contribution to the Probability of failure [14]. The MPP is defined in a standardized and independent coordinate system. In the transformation procedure, the design vector X is transformed into the vector of standardized, independent Gaussian variables, U. Generally MPP calculation can be formulated as an optimization problem:

$$\begin{cases} find\ U \\ \min\ \beta = (U^T U)^{1/2} \\ s.t\ g(U) = 0 \end{cases} \tag{1}$$

The shortest distance β from the origin to a point on the limit-state surface, g(U), is called reliability index [15]. Typical MPP-based reliability analysis methods include first and second order reliability methods (FORM/SORM). To calculate the probability of failure using FORM and SORM, first, MPP needs to be found. After finding the MPP and reliability index using Eq. (1), FORM and SORM approximate the probability of failure by using first or second order Taylor series expansion of limit state function at the MPP. For MPP based sequential RBDO, the random variables are replaced with their MPPs, therefore, the deterministic constraints are shifted to meet the desired reliability level. The calculated MPP is improved after each iteration to provide an accurate MPP for the deterministic optimization.

Based on the sequential optimization and reliability analysis technique, an RBDO framework is developed for curvilinearly stiffened panels, Figure 1. Since the deterministic optimization of curvilinearly stiffened panel is computationally expensive, first the reliability analysis of initial design ($d^0$) is conducted to find the MPP corresponding to the desired reliability (R). Then the constraints are evaluated at MPP, and used as equivalent deterministic constraints in optimization, $g_i(d, X_{MPP})$.The deterministic optimization is conducted using the equivalent constraints to find the optimum design. Once the optimum is found, the reliability analysis is performed at current optimum to find the updated MPPs and also calculate the probability of satisfying constraints, $P_s$. If objective function is not close to one obtained in the previous iteration or some constraints are violated, the iterative procedure will be continued until the objective function converges, the probability of satisfying constraints is larger than the desired system probability of safety, and the MPP also converges. In order to reduce the number of optimization iterations and find the optimal design faster, the optimum design obtained in the previous iteration is given as the initial design for the current optimization cycle.

## 3. Results

In this section, results for a test case, short column, two cases of curvilinearly stiffened panels subjected to uniform shear and compression in-plane loads, and two cases of curvilinearly stiffened panels subjected to linearly and parabolically varying shear and compression are presented. The test case considers RBDO of a rectangular short column with cross-sections design variables, and the applied loads and the yield stress as random variables. Finally, the application of developed RBDO framework for curvilinearly stiffened panels is demonstrated. The design variables include the shape and size variable of stiffened panels, and the in-plane loads and young modulus are defined as random variables. To find out the effect of in-plane load variations on the optimal mass of the panel, we carried four different sets of in-plane load distributions.

### 3.1 Short Column

This test problem involves the plastic analysis and design of a short column with rectangular cross section (width *b* and depth *h*) having uncertain material properties (yield stress *Y*) and subject to uncertain loads (bending moment *M* and axial force *P*), Cheng et al., 2006. The objective and limit state functions are defined as:

$$\begin{cases} f(d) = b.h \\ G(d,X) = 1 - \dfrac{4M}{bh^2Y} - \dfrac{F^2}{(bhY)^2} \end{cases} \tag{2}$$

The distributions for *P*, *M*, and *Y* are Normal (500, 100), Normal (2000, 400), and Lognormal (5, 0.5), respectively, with a correlation coefficient of 0.5 between *P* and *M*. An objective function of cross-sectional area and a target reliability index of 2.5 (cumulative failure probability $P_f \leq 0.00621$) are used in the design problem:

$$\begin{cases} min\, f(d) \\ s.t\ \beta \geq 2.5 \\ 5.0 \leq b \leq 15.0 \\ 15.0 \leq h \leq 25.0 \end{cases} \tag{3}$$

First, the reliability analysis of initial design is performed to find the MPP corresponding to the desired target reliability index of 2.5, see iteration zero in Figure 2. Next, the constraints, $\mathrm{G(d,X)}$ are evaluated at MPP, and used as equivalent deterministic constraints in optimization, $\mathrm{G(d,X_{MPP})}$. Once the optimum is found, the reliability analysis is performed at current optimum to find the updated MPPs and also to calculate the reliability index. It is noted from Figure 2 that the error of the reliability index is less than 0.5% after the first iteration, and the second and third iteration is conducted to guarantee the convergence of the objective function. The optimal design from sequential reliability design optimization and the comparison of performance are shown in Table 1.

### 3.2 Curvilinearly Stiffened Panels Subjected to Uniform Shear and Compression In-plane Loads

A simply supported rectangular plate of size 0.4064 × 0.5080m with material properties listed in Table 2 is studied under combined uniform shear and compression. The baseline panel configuration, loading, material properties, and design constraints are representative of typical aircraft structure for this design optimization study. All panel analyses, with or without stiffeners, are performed with NASTRAN using EBF3PanelOpt. The stiffened panel geometry and mesh are regenerated for each design point analysis during optimization. Reliability based design minimizes the mass of courvilinearly stiffened panel subjected to the constraints on buckling ($\lambda$), Kreisselmeier and Steinhauser (KS) and the crippling ($\sigma_{cc}$). The desired probability of safety (R) for all cases is 0.9998.
The first load case (L1) studied is a 0.508×0.406 *m* stiffened panel under combined shear and compression with dominant compression ($\mathrm{NY/NXY} = 4.36$). NY and NXY are normally with a mean of 308 kN/m and 71 kN/m, respectively and 15% coefficient of variation (COV). The distribution for E is lognormal with a mean of 73 GPa and 1% COV. The second load case (L2) has smaller ratio of shear and compressive load magnitudes ($NY/NXY = 1.13$). In this load case NY and NXY are normally distributed with a mean of 152kN/m and 134kN/m, respectively and 15% COV. The distribution, mean, and covariance for E are similar to previous case.

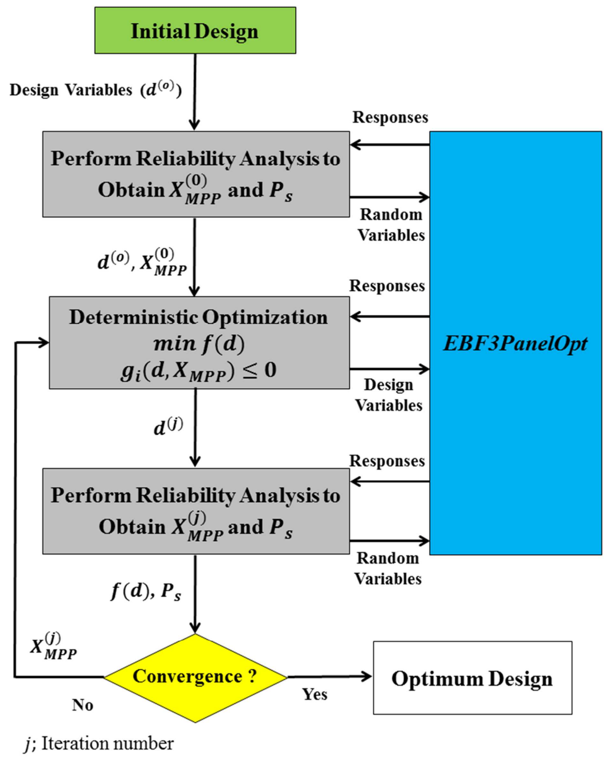



**Figure 1: The sequential framework for RBDO**

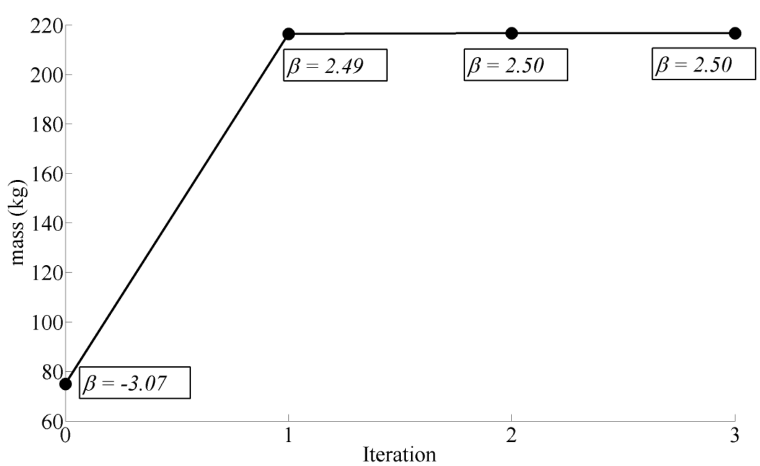



**Figure 2: Iteration history of the sequential optimization and reliability analysis of short column**

**Table 1: Reliability design optimization of short column**

| | mass | Reliability index | *NFE |
|---|---|---|---|
| Bi-level approach (Cheng et al., 2006) | 216.82 | 2.503 | 136 |
| Sequential RBDO | 216.7 | 2.500 | 72 |

*NFE is the number of function evaluation

**Table 2 Material properties of curvilinearly stiffened plate**

| | |
|---|---|
| Modulus of Elasticity | $73\times10^9$ *Pa* |
| Density | 2795 $kg/m^3$ |
| Poisson's Ratio | 0.33 |
| Yield stress | 427.4 *MPa* |

Following the sequential RBDO scheme presented in the previous section, the iteration histories for load case one are shown in Figure 3 (L1). It is shown that both the optimization and reliability analysis converges after two iterations. The panel weighs 2.031 kg, which is slightly lighter than the deterministic optimum of 2.032 kg. The applied loads for deterministic optimization are obtained after applying a factor of safety of 1.5 to the limit loads and the panels are designed for that loads. The deterministic optimum configuration is shown in Figure 4 (L1). The optimum objective, probability of safety, and shape and sizing design variable values for RBDO and deterministic optimization are shown in Table 3 (L1). A few observations are of interest here. First, since in the studied case the compression is the dominant load, using an appropriate safety factor (here it is 1.5) can give the desired probability of failure, as can be seen in Table 3. When there is only one important random variable, the safety factors can be directly expressed by the required reliability levels. However, in many cases, there does not exist a relationship between the safety factor and reliability levels. Furthermore, the buckling constraint is active for both configurations, which yield closely optimal results.

The optimum configuration and iteration histories for the second load case are shown in Figure 3 (L2) and the shape and sizing design variable values obtained from sequential RBDO and the related final optimum mass, and probability of safety for three constraints are shown in Table 3 (L2). The deterministic optimization result for the second load case using a factor of safety of 1.5 is presented in Figure 4 (L2). The optimum mass of the panel is 1.746kg, which is slightly lighter than the deterministic optimum of 1.794kg with the probability of safety 0.9999. For the second load case, the compression and shear are both important and one cannot be ignored. Therefore, using same safety factor for both loads may not result in the desired probability of failure and optimum mass obtained using RBDO. By comparing the sequential RBDO Figure 3 (L2) and deterministic optimization Figure 4 (L2), it further becomes evident that the deterministic optimization using safety factor did not yield the RBDO final configuration. It is important to note that changing the safety factor would change the shear and compression loads, but it does not change their ratio, and consequently it only changes the size variables while having no effect on the shape variables. The final configuration of curvilinear stiffeners is governed by the ratio of the shear and compression loads rather than their magnitudes.

### 3.3 Curvilinearly Stiffened Panels Subjected to Non-Uniform Shear and Compression In-plane Loads

It is seen from the results shown in the previous subsection that the influence of ratio of shear and compression loads on the final results is substantial. Furthermore, it is also important to understand the influence of the additional random variables, such as the linearly and parabolic varying loads, and study their effect on the optimal mass and probability of safety. In this subsection the curvilinearly stiffened panels under shear and compression loads with linearly and parabolically varying random distributions is studied. The rectangular panel has same dimensions and boundary conditions as discussed in previous subsection but is subjected to different loading condition. The linearly load distribution (L3) and parabolic load distribution (L4) are shown in Eq. (4).

$$Linear\ Load\ Case\!:\begin{cases} N_Y = N_{Y1} + N_{Y2}\left(1-\dfrac{2x}{a}\right) \\ N_{XY} = N_{XY1} + N_{XY2}\left(1-\dfrac{2x}{a}\right) \\ N_{XY} = N_{XY1} + N_{XY2}\left(1-\dfrac{2y}{b}\right) \end{cases}$$
$$Parabolic\ Load\ Case\!:\begin{cases} N_Y = N_{Y1} + N_{Y2}\left(1-\dfrac{2x}{a}\right) + 4N_{Y3}\dfrac{x}{a}(1-\dfrac{x}{a}) \\ N_{XY} = N_{XY1} + N_{XY2}\left(1-\dfrac{2x}{a}\right) + 4N_{XY3}\dfrac{x}{a}(1-\dfrac{x}{a}) \\ N_{XY} = N_{XY1} + N_{XY2}\left(1-\dfrac{2y}{a}\right) + 4N_{XY3}\dfrac{y}{a}(1-\dfrac{y}{a}) \end{cases} \quad (4)$$

As can be seen, for the linear load case, two random variables are defined for each of shear and compression loads. NY1 and NXY1 are normally distributed and uncorrelated with a mean of 152kN/m and 134kN/m, respectively and 15% COV, and NY2 and NXY2 are normally distributed with a mean of zero, and the standard deviation is 5% of means of NY1 and NXY1, respectively.

The distribution, mean, and covariance for E are similar to previous subsection. Starting from an initial design that satisfies all the constraints, the sequential RBDO is carried out for the third load case and it converged in three iterations with 33270 analyses. The history of the objective function and reliability constraints with respect to the iteration number are shown in Figure 5 (L3), and the optimal design variable values and mass along with probability of safety are presented in Table 4 (L3). By comparing Figure 4 (L1) and Figure 5 (L3), it is seen that the optimal design for linearly varying load case and those obtained for uniform load case are similar to each other, which is due to the small influence of load variation on structural response.

As for the parabolic load distribution, three random variables are defined for each of shear and compression loads. NY1 and NXY1 are normally distributed with a mean of 152kN/m and 134kN/m and 15% COV, NY2 and NXY2 are normally distributed and uncorrelated with a mean of zero, and the standard deviation is 5% of means of NY1 and NXY1, and NY3 and NXY3 are normally distributed and uncorrelated with a mean of zero, and the standard deviation is 2% of means of NY1 and NXY1.

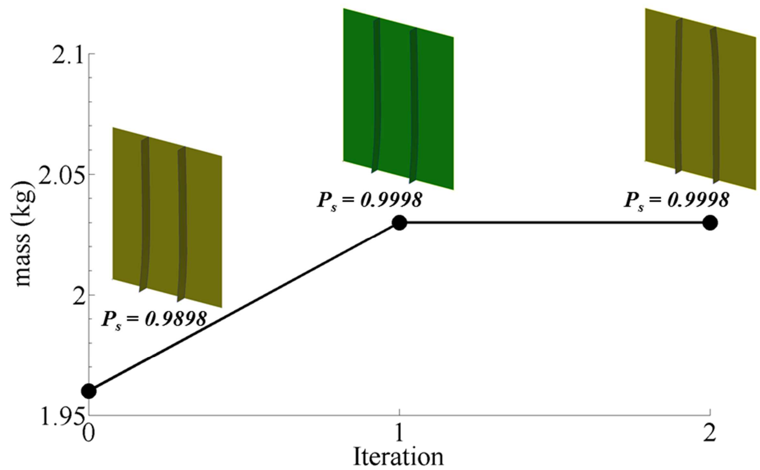
Table 4 compares the optimal design variables, objective function, probability constraints, and number of iterations of two load cases (L3 and L4) achieved by the sequential RBDO. Note that, as expected, the parabolically varying load case has objective function values larger than linearly varying load case which is due to the effect of third variable in the parabolic load distribution. However as shown in Figure 5, the stiffeners layouts obtained for two load cases are similar to each other. This can be explained by the fact that the first variable in the compression and shear loads (NY1 and NXY1) are dominant and they appear to be governing the final optimal layout and the other parameters (NY2, NXY2, NY3, and NXY3) change the size variables and consequently the weight of structure.

**Table 3: Optimum mass, constraint and design variable obtained for (L1) $NY/NXY = 4.36$ (L2) $NY/NXY = 1.13$**

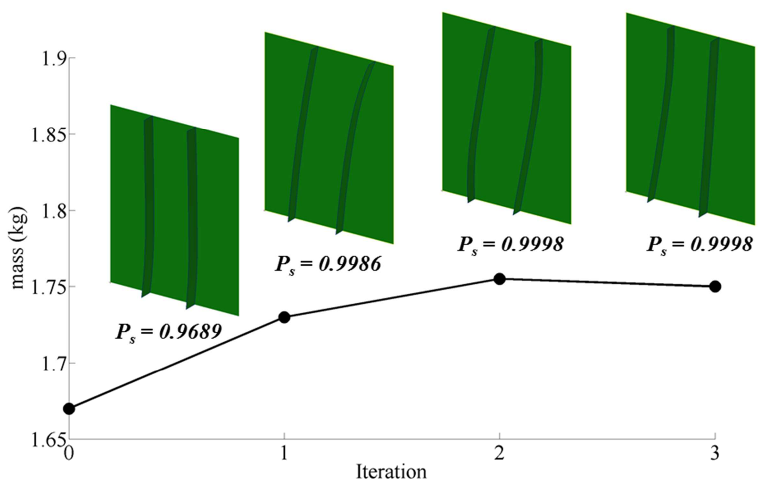
| Variable No. | Deterministic Optimization (L1) | Sequential RBDO (L1) | Deterministic Optimization (L2) | Sequential RBDO (L2) |
|---|---|---|---|---|
| *x1* | 0.0861 | 0.5806 | 0.6417 | 0.6393 |
| *x2* | 0.2052 | 9.8569 | 0.0361 | 0.7125 |
| *x3* | 0.2484 | 0.6736 | 0.2437 | 0.3899 |
| *x4* | 0.6403 | 0.1479 | 0.0651 | 0.0480 |
| *x5* | 0.1412 | 0.6481 | 0.1339 | 0.5619 |
| *x6* | 0.6613 | 03642 | 0.8111 | 0.3642 |
| *x7* | 0.7401 | 0.3123 | 0.7497 | 0.6698 |
| *x8* | 0.5812 | 0.0725 | 0.5701 | 0.1392 |
| *x9, m* | 0.0321 | 0.0318 | 0.0325 | 0.0318 |
| *x10, m* | 0.0428 | 0.0324 | 0.0317 | 0.0390 |
| *x11, m* | 0.0031 | 0.0031 | 0.0027 | 0.0026 |
| *x12, m* | 0.0024 | 0.0025 | 0.0021 | 0.0021 |
| *x13, m* | 0.0021 | 0.0021 | 0.0020 | 0.0020 |
| Mass, *kg* | 2.0324 | 2.0316 | 1.7941 | 1.7455 |
| $Prob[\lambda(d,X) \leq 1]$ | 0.9998 | 0.9998 | 0.9999 | 0.9998 |
| $Prob[KS(d,X) \leq 1]$ | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| $Prob[(\sigma_{cc}(d,X) \leq 1]$ | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| Number of evaluations | 10447 | 20849 | 11637 | 33270 |

## 4. Conclusion

An efficient reliability based design optimization framework is studied. A sequential optimization and reliability analysis methodology is developed. The sequential RBDO, first, conducts the reliability analysis to find MPPs and the probability of satisfying the given constraints. Next, each probability constraint is converted to an equivalent deterministic constraint by using its MPP of previous iteration. Since the changes in size and shape variables during the optimization process result in different kinds of changes to the structure's performance, a method for decomposing the shape and size optimization problem is utilized to improve the efficiency and accuracy of developed framework. In the two-step optimization algorithm, the shape and size optimization process is divided into two parts, the first part involves calculation of the best stiffener curve that gives the maximum buckling load subjected to stress and crippling constraints, and the second step consists of a sizing optimization while keeping the stiffener curve unchanged to minimize the mass while satisfying the buckling, stress, and crippling constraints. The present study includes a test case, short column, two cases of curvilinearly stiffened panels subjected to uniform shear and compression in-plane loads, and two cases of curvilinearly stiffened panels subjected to linearly and parabolically varying shear and compression.




(L1)




(L2)

**Figure 3: Iteration history of mass and probability of safety (L1) $NY/NXY = 4.36$ (L2) $NY/NXY = 1.13$**

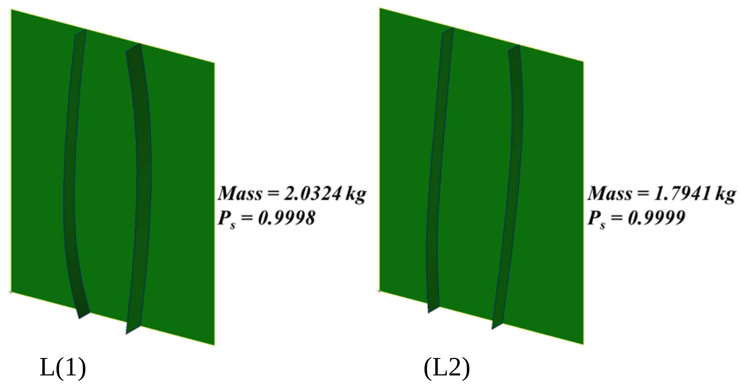



**Figure 4: Deterministic optimum configuration (L1) $NY/NXY = 4.36$ (L2) $NY/NXY = 1.13$**

**Table 4: Optimum mass, constraint and design variable obtained for linearly (L3) and parabolically (L4) varying load cases**

| Variable / Response | Sequential RBDO (L3) | Sequential RBDO (L4) | Variable / Response | Sequential RBDO (L3) | Sequential RBDO (L4) |
|---|---|---|---|---|---|
| *x1* | 0.6363 | 0.6337 | *x10, m* | 0.0324 | 0.0322 |
| *x2* | 0.6956 | 0.6258 | *x11, m* | 0.00266 | 0.0027 |
| *x3* | 0.3524 | 0.3727 | *x12, m* | 0.0022 | 0.0020 |
| *x4* | 0.0518 | 0.034 | *x13, m* | 0.0020 | 0.0021 |
| *x5* | 0.1339 | 0.1284 | *Mass, kg* | 1.7334 | 1.7404 |
| *x6* | 0.4556 | 0.5768 | *Prob[λ(d,X)≤1]* | 0.9998 | 0.9998 |
| *x7* | 0.6910 | 0.7586 | *Prob[KS(d,X)≤1]* | 0.9999 | 0.9999 |
| *x8* | 0.5530 | 0.5550 | *Prob[(σ_cc (d,X)≤1]* | 0.9999 | 0.9999 |
| *x9, m* | 0.0321 | 0.0325 | Number of evaluations | 33632 | 36557 |

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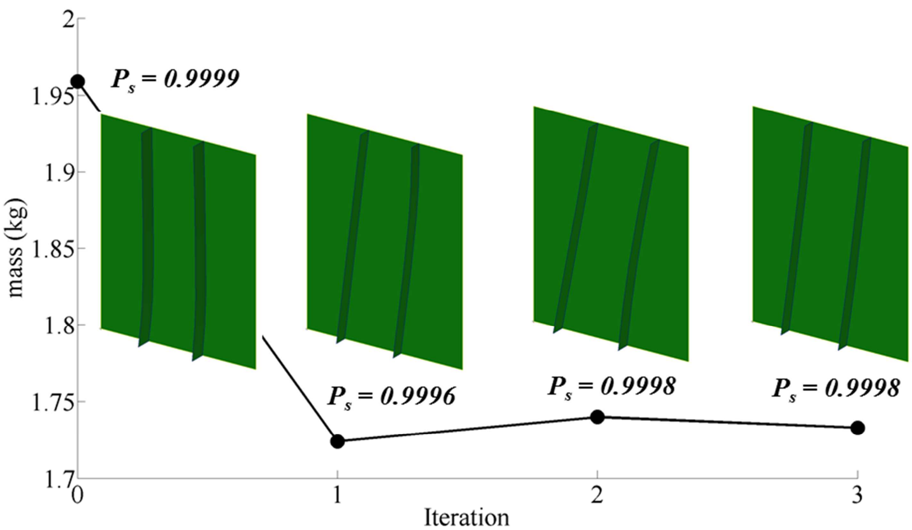
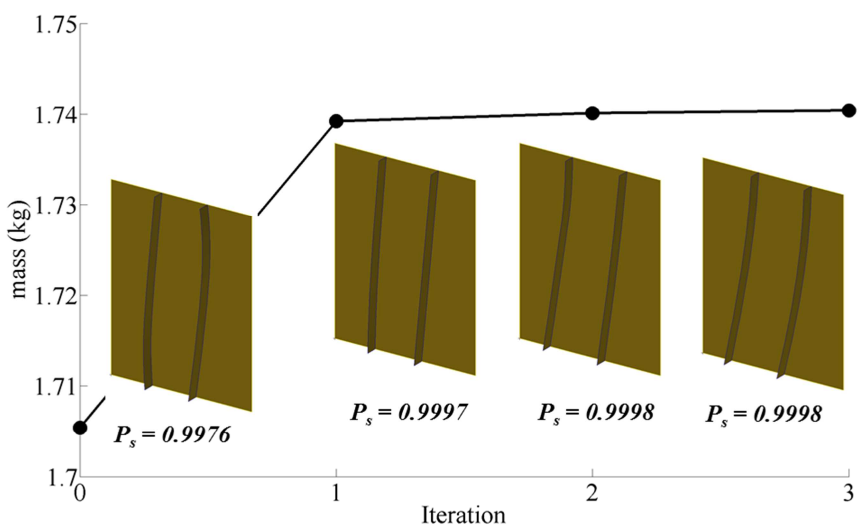



(L3)




(L4)

**Figure 5: Iteration history of mass and probability of safety for linearly (L3) and parabolically (L4) varying load cases**