

## DESIGN OPTIMIZATION OF STRUCTURAL ACOUSTIC PROBLEMS USING FEM-BEM

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### ABSTRACT

*A design optimization procedure of a noise-vibration-harshness (NVH) problem of a complicated vehicle structure is presented by assuming the acoustic pressure does not affect the structural vibration. The steady-state dynamic behavior of the vehicle is calculated from the frequency response finite element analysis, while the sound pressure level within the acoustic cavity is calculated from the boundary element analysis. A reverse solution process is employed for the design sensitivity calculation using the adjoint variable method. The adjoint load is obtained from the acoustic boundary element re-analysis, while the adjoint solution is calculated from the structural dynamic re-analysis. The evaluation of pressure sensitivity only involves a numerical integration process over the structural part where the design variable is defined. A design optimization problem is formulated and solved, where the structural weight is reduced while the noise level in the passenger compartment is lowered.*

### KEYWORDS

Structural Acoustics, Design Optimization, Finite Element Analysis, Boundary Element Analysis, Design Sensitivity Analysis, Adjoint Variable Method

### 1. INTRODUCTION

The design of a comfortable vehicle increasingly draws attention to engineers according to the customer's preference. Especially, the structural-acoustic performance of a commercial vehicle becomes an important issue in the design process. The purpose of this paper is to show the feasibility of design optimization in order to minimize the vehicle's weight subjected to the structural-acoustic constraints. To arrive at this goal, the following tools are required: (1) an accurate and

efficient numerical method to evaluate the structural-acoustic performance, (2) an accurate and efficient design sensitivity analysis method to calculate the gradient information, (3) constrained, nonlinear design optimization algorithm, and (4) integrated design environment in which a design engineer can efficiently work throughout multidisciplinary environment. In this paper, an example of these four important requirements is presented.

Many numerical methods have been developed to simulate the structural-acoustic performance of a passenger vehicle. The finite element method [1], the boundary element method [2], the statistical energy analysis [3,4], and the energy flow analysis [5,6,7] are a short list of methods that can be used for the purpose. Different methods must be used based on the design interest. For example, the finite element analysis (FEA) and boundary element analysis (BEA) can be used for simulation in the low-frequency range, while the statistical energy analysis and energy flow analysis can be used for the high-frequency range. In this paper, the former methods are employed to simulate the vehicle's structural-acoustic performance in the 1–100 Hz frequency range. A commercial finite element code MSC/NASTRAN [8] is used to simulate the frequency response of the vehicle structure, while a boundary element code COMET/ACOUSTICS [9] is used to calculate the sound pressure level in the cabin compartment based on the velocity information obtained from the finite element code. That is, the simulation procedure is sequential and uncoupled based on the assumption that the vibration of the air does not contribute to the structural vibration.

Design sensitivity analysis (DSA) calculates the gradient information of the structural-acoustic performance with respect to the design variables, which is the panel thickness of the vehicle. Many research results [10–18] have been published in design sensitivity analysis (DSA) of structural-acoustic problems using FEA and BEA. While the direct differentiation method in DSA follows the same solution process as the response analysis, the adjoint variable method follows a reverse process. One of the challenges of the adjoint variable method in sequential acoustic analysis is how to formulate this reverse process. The sequential adjoint variable method with a reverse solution process developed by Kim et al. [19] is used, in which the adjoint load is obtained from boundary element re-analysis, and the ad-

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joint variable is calculated from structural dynamic re-analysis.

The importance of the integrated design environment increases as many disciplines are link together during design procedure. Finite element analysis, boundary element analysis, design parameterization, design sensitivity analysis, design optimization algorithms are needed to be integrated in the design optimization of the structural-acoustic problems. The design sensitivity analysis and optimization tool (DSO) [20] is used as an integrated design environment in this paper. Graphic user interface is provided for design engineer can perform design parameterization, structural-acoustic analysis, design sensitivity analysis, and design optimization.

The proposed sequential structural-acoustic analysis and DSA using the adjoint variable method are applied to the optimization of a next generation concept vehicle model, by which the vehicle weight is minimized while the sound pressure level is constrained. A design optimization problem is formulated and solved, where the structural weight is reduced while the noise level in the passenger compartment is lowered.

## 2. STRUCTURAL ACOUSTIC ANALYSIS

In the case that the acoustic cavity is composed of air, the coupling effect of acoustic medium to the structural behavior can be ignorable. Thus, the analysis procedure becomes sequential, as explained below.

First, using the continuum energy forms, the variational equation for frequency response of a structural system can be written as:

$$j\omega d_u(\mathbf{v}, \bar{\mathbf{z}}) + \kappa a_u(\mathbf{v}, \bar{\mathbf{z}}) = \ell_u(\bar{\mathbf{z}}), \quad \forall \bar{\mathbf{z}} \in Z \quad (1)$$

where  $\omega$  denotes excitation frequency,  $\phi$  is the structural damping coefficient,  $j = \sqrt{-1}$ ,  $\kappa = (1 + j\phi) / j\omega$ ,  $d_u(\bullet, \bullet)$  is the kinetic sesqui-linear form,  $a_u(\bullet, \bullet)$  is the structural sesqui-linear form, and  $\ell_u(\bullet)$  is the load semi-linear form. The definitions of the sesqui-linear and semi-linear forms can be found in Kim et al. [19].

After finite element approximation, the following form of the matrix equation can be obtained from (1):

$$[j\omega \mathbf{M} + \kappa \mathbf{K}]\{\mathbf{v}\} = \{\mathbf{F}\} \quad (2)$$

where  $[\mathbf{M}]$  and  $[\mathbf{K}]$  are the mass and stiffness matrices of the structure, respectively, and  $\{\mathbf{v}\}$  is nodal velocity vector. Force a given excitation frequency  $\omega$ , (2) can be solved using the standard frequency response analysis, which is available in most commercial finite element codes. Note that all variables that appear in this section are complex variables.

Then using a boundary element method [2], the sound pressure at any point inside the acoustic cavity due to structural vibration,  $\mathbf{v}$ , can be calculated as

$$b(\mathbf{x}_0; \mathbf{v}) + e(\mathbf{x}_0; p_s) = \alpha p(\mathbf{x}_0) \quad (3)$$

where  $p$  is the sound pressure at any point  $\mathbf{x}_0$  inside the acoustic cavity,  $p_s$  is the sound pressure vector at the surface of the structural domain,  $b(\mathbf{x}_0; \bullet)$  and  $e(\mathbf{x}_0; \bullet)$  are linear integral forms that correspond to the BEM governing equation [19], constant  $\alpha$  is equal to 1 for  $\mathbf{x}_0$  inside the acoustic volume, 0.5 for  $\mathbf{x}_0$  on a smooth boundary surface, and 0 for  $\mathbf{x}_0$  outside the acoustic volume.

At a discrete point  $\mathbf{x}_0$ , inside (acoustic cavity) or boundary (surface), (3) can be approximated by

$$\{\mathbf{b}(\mathbf{x}_0)\}^T \{\mathbf{v}\} + \{\mathbf{e}(\mathbf{x}_0)\}^T \{\mathbf{p}_s\} = \alpha p(\mathbf{x}_0) \quad (4)$$

However, (4) cannot be solved unless the surface pressure  $p_s$  in (3) or its discrete version  $\{\mathbf{p}_s\}$  in (4) is available. Thus, the BEM procedure is divided by two: (1) calculate the surface pressure at each boundary node by assigning  $\mathbf{x}_0$  to be boundary node location from (4) and (2) calculate the pressure in the acoustic cavity by assigning  $\mathbf{x}_0$  to be internal point.

Conceptually, the first procedure can be denoted as a mapping between structural velocity,  $\mathbf{v}$ , and surface pressure,  $p_s$ , as

$$A(p_s) = B(\mathbf{v}) \quad (5)$$

where,  $A(\bullet)$  and  $B(\bullet)$  are the corresponding integral forms in (3) evaluated at structural surface. Practically, equation (4) is solved for each boundary node, which can be represented using a matrix notation, as

$$[\mathbf{A}]\{\mathbf{p}_s\} = [\mathbf{B}]\{\mathbf{v}\} \quad (6)$$

After calculating nodal pressure  $\{\mathbf{p}_s\}$ , the pressure in the acoustic cavity can be calculated from (3) or (4), as

$$\begin{aligned} p &= \{\mathbf{b}\} + [\mathbf{B}]^T [\mathbf{A}]^{-T} \{\mathbf{e}\}^T \{\mathbf{v}\} \\ &\equiv \{\mathbf{b}\} + [\mathbf{B}]^T \{\boldsymbol{\eta}\}^T \{\mathbf{v}\} \\ &\equiv \{\mathbf{L}\}^T \{\mathbf{v}\} \end{aligned} \quad (7)$$

where  $[\mathbf{A}]$ ,  $[\mathbf{B}]$ ,  $\{\mathbf{b}\}$ ,  $\{\mathbf{e}\}$  are the matrices or vectors corresponding to the linear integral forms, and  $\{\boldsymbol{\eta}\}$  is the so-called *first adjoint variable vector*. The first adjoint variable vector is used in calculating the acoustic response and then the *adjoint load*, and it is the solution of the following linear algebraic equation

$$[\mathbf{A}]^T \{\boldsymbol{\eta}\} = \{\mathbf{e}\} \quad (8)$$

Here  $\{\boldsymbol{\eta}\}$  is only a function of the geometry of the acoustic cavity and properties of the sound field, and it is independent of the excitations in the system. The term  $\{\mathbf{L}\}$  in (7) is the so-called *adjoint load*, since it will be used in (12) as a load vector for obtaining the *second adjoint variable vector*. In addition,  $\{\mathbf{L}\}$  has an important physical meaning: each element in  $\{\mathbf{L}\}$  represents the contribution to the sound pressure of a unit velocity excitation at a corresponding structural node.

### 3. SEQUENTIAL SENSITIVITY ANALYSIS

Design sensitivity analysis (DSA) is an essential process in the gradient-based structural optimization process, for example, in the optimization process mentioned in the previous subsection. Here, a sequential structural-acoustic design sensitivity analysis method is presented in which structural and the acoustic behaviors are de-coupled. The method based on the continuum forms is briefly described. The original development work and a more detailed formulation can be found in [19].

Let there exist  $n$  design variables such that the design vector  $\mathbf{u} = \{u_1, u_2, \dots, u_n\}^T$ . In the following derivations, only one design variable  $u$  will be considered for notational convenience. Differentiating (3) and (1) with respect to a design variable  $u$  yields

$$\frac{\partial p}{\partial u} = b(\mathbf{x}_0; \frac{\partial \mathbf{v}}{\partial u}) + e(\mathbf{x}_0; A^{-1} \circ B(\frac{\partial \mathbf{v}}{\partial u})) \quad (9)$$

and

$$\begin{aligned} j\omega d_u(\frac{\partial \mathbf{v}}{\partial u}, \bar{\mathbf{z}}) + \kappa a_u(\frac{\partial \mathbf{v}}{\partial u}, \bar{\mathbf{z}}) \\ = \ell'_{\delta u}(\bar{\mathbf{z}}) - j\omega d'_{\delta u}(\mathbf{v}, \bar{\mathbf{z}}) - \kappa a'_{\delta u}(\mathbf{v}, \bar{\mathbf{z}}), \quad \forall \bar{\mathbf{z}} \in Z \end{aligned} \quad (10)$$

where  $\partial p/\partial u$  denotes the sensitivity of sound pressure  $p$  with respect to the design variable  $u$ ;  $\partial \mathbf{v}/\partial u$  denotes sensitivity of structural velocity  $\mathbf{v}$  with respect to  $u$ ; and  $d'_{\delta u}$ ,  $a'_{\delta u}$ , and  $\ell'_{\delta u}$  are variations of the kinetic sesqui-linear form, structural sesqui-linear form, and load semi-linear form, respectively [19]. A standard sensitivity calculation procedure (direct differentiation method) is then obtained as follows. First, (10) is solved for the velocity sensitivity  $\partial \mathbf{v}/\partial u$ , and then the result is substituted into (9) to obtain  $\partial p/\partial u$ . It is seen that this procedure needs to be repeated for each design variable used in the design problem. If the number of design variables is large, then the calculations will become extensive.

However, if the *second adjoint variable vector*  $\boldsymbol{\lambda}$  is defined as a solution of the following equation

$$\begin{aligned} j\omega d_u(\bar{\boldsymbol{\lambda}}, \boldsymbol{\lambda}) + \kappa a_u(\bar{\boldsymbol{\lambda}}, \boldsymbol{\lambda}) \\ = b(\mathbf{x}_0; \bar{\boldsymbol{\lambda}}) + e(\mathbf{x}_0; A^{-1} \circ B(\bar{\boldsymbol{\lambda}})), \quad \forall \bar{\boldsymbol{\lambda}} \in Z \end{aligned} \quad (11)$$

or equivalently in the discrete form as

$$[j\omega \mathbf{M} + \kappa \mathbf{K}]\{\boldsymbol{\lambda}^*\} = \{\mathbf{L}\} \quad (12)$$

then (9) can be simplified as

$$\frac{\partial p}{\partial u} = \ell'_{\delta u}(\boldsymbol{\lambda}) - j\omega d'_{\delta u}(\mathbf{v}, \boldsymbol{\lambda}) - \kappa a'_{\delta u}(\mathbf{v}, \boldsymbol{\lambda}) \quad (13)$$

Note that both adjoint variable vectors,  $\{\boldsymbol{\eta}\}$  defined in (8) and  $\{\boldsymbol{\lambda}^*\}$  defined in (12), are independent of the design variable. And  $\{\boldsymbol{\lambda}^*\}$  in (12) is the complex conjugate of the adjoint variable  $\{\boldsymbol{\lambda}\}$  due to the sesqui-linear form of  $d_u(\boldsymbol{\cdot}, \boldsymbol{\cdot})$  and  $a_u(\boldsymbol{\cdot}, \boldsymbol{\cdot})$  [19]. Therefore, (8) and (12) only need to be solved once regardless of how many design variables are used in the design problem. Solving (8) and (12) are the major calculations in the sensitivity analysis process. When a large number of design variables are used, which is the usual case in a structural optimization problem, significant savings can be obtained in terms of the computational cost. This is the major benefit from the use of the adjoint variables. More detailed discussions regarding this can be found in [19].

Also note that if the selected design variable only affects the properties of a substructure, a component in the structural system, then from (13), we can have

$$\frac{\partial p}{\partial u} = \ell'_{\delta u}(\boldsymbol{\lambda}_e) - j\omega d'_{\delta u}(\mathbf{v}_e, \boldsymbol{\lambda}_e) - \kappa a'_{\delta u}(\mathbf{v}_e, \boldsymbol{\lambda}_e) \quad (14)$$

where  $\mathbf{v}_e$  and  $\boldsymbol{\lambda}_e$  are components of the nodal velocity response and adjoint variable vectors with respect to the substructure, or component; and  $d'_{\delta u}$ ,  $a'_{\delta u}$ , and  $\ell'_{\delta u}$  are variations of the energy forms with respect to the design change in the corresponding substructure or component. Furthermore, for a general structural or performance measure which can be expressed by an integral form  $\psi = h(p(\mathbf{x}_0), u)$ , its sensitivity can be evaluated by

$$\frac{\partial \psi}{\partial u} = \frac{\partial h}{\partial u} + \ell'_{\delta u}(\boldsymbol{\lambda}) - j\omega d'_{\delta u}(\mathbf{v}, \boldsymbol{\lambda}) - \kappa a'_{\delta u}(\mathbf{v}, \boldsymbol{\lambda}) \quad (15)$$

More detailed descriptions of the sensitivity of different performance measures and its calculation method can be found in [19].

#### 4. DESIGN OPTIMIZATION PROCEDURE

For structural-acoustic problem, the sequential FEA-BEA analysis calculates the performance measure (noise and vibration), and the adjoint variable method for DSA calculates the sensitivity of the performance measure. This information is utilized by the optimization program to search for the optimum design.

The gradient-based optimization algorithms are commonly used in engineering design and optimization. The performance measure and its sensitivity are required for the gradient-based optimization process. Figure 1 shows the computational procedure for the optimization of the sequential structural-acoustic problem using a gradient-based optimization algorithm. Once the design variable, cost function, and design constraints are defined, the proposed sequential FEA-BEA and reverse adjoint variable DSA method are employed to compute the performance measure and their sensitivity, which will be input to the optimization program to search for the optimum design. The process will loop until an optimum design is achieved.

#### 5. NUMERICAL EXAMPLE – NVH OPTIMIZATION OF A VEHICLE

One of important applications of the proposed

method is structure-borne noise reduction of a vehicle. Figure 2 shows finite element and boundary element models of a next generation hydraulic hybrid vehicle [19]. In addition to the powertrain vibration and wheel/terrain interaction, the hydraulic pump is a source of vibration, considered as a harmonic excitation. Because of this additional source of excitation, vibration and noise is more significant than with a conventional powertrain. The object of the design optimization is to minimize the vehicle weight as well as maintaining noise and vibration at the driver's ear position to the desirable levels.

From the powertrain analysis and rigid body dynamic analysis, the harmonic excitations at twelve locations are obtained. Frequency response analysis is carried out on the structural FE model using MSC/NASTRAN to obtain the velocity response, corresponding to the frequency range up to 100 Hz. COMET/Acoustic [9] is employed to obtain the acoustic pressure performance measure in the acoustic domain as shown in Figure 2. Once the acoustic performance measure and sensitivity information are obtained according to the procedure illustrated in Figure 1, the sequential quadratic programming algorithm in DOT (Design Optimization Tool) [21] is used to search for the optimum design.

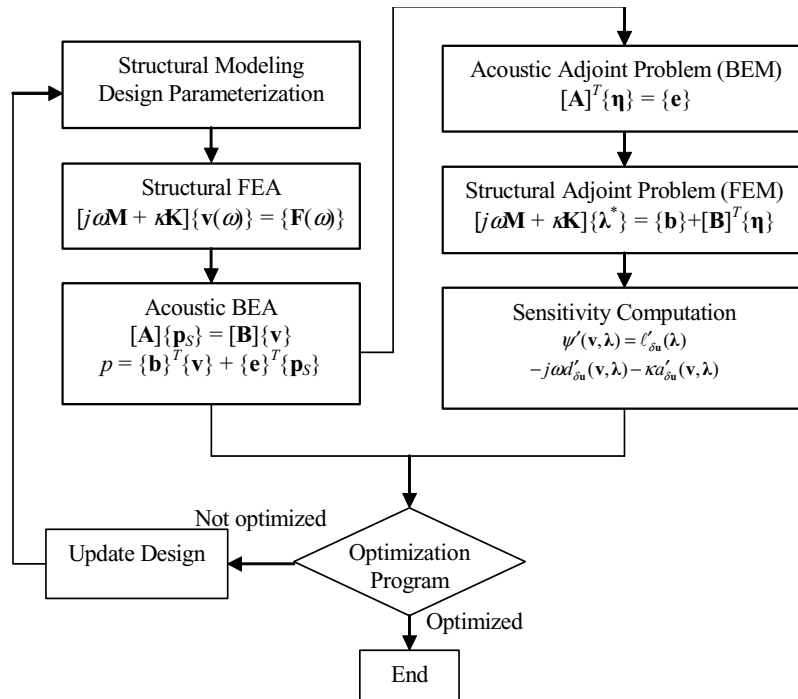


Figure 1. Design Optimization Procedure

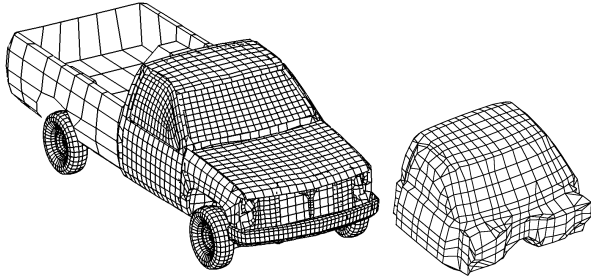


Figure 2. Vehicle FE model and Acoustic BE model of the Cabin Compartment

### 5.1 FEA-BEA NVH Analysis and Adjoint Sensitivity Analysis

The sequential FEA-BEA analysis is performed on the vehicle model. In this example, the noise level at the passenger compartment is chosen as the performance measure, and vehicle panel thicknesses are chosen as design variables. The sound pressure level frequency response up to 100 Hz at the driver's ear position is obtained and illustrated in Figure 3 and the numerical results at selected frequencies are shown in Table 1.

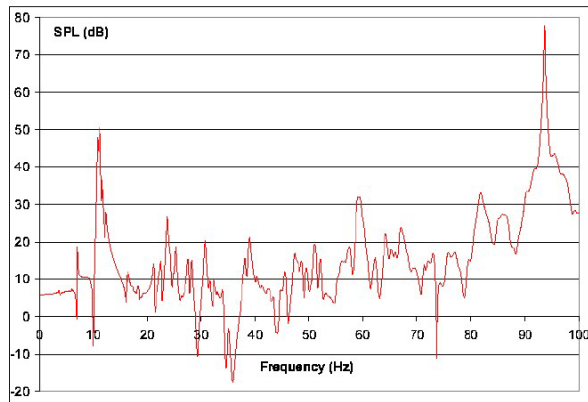


Figure 3. Frequency Response of Sound Press at Driver's Position (Initial Design)

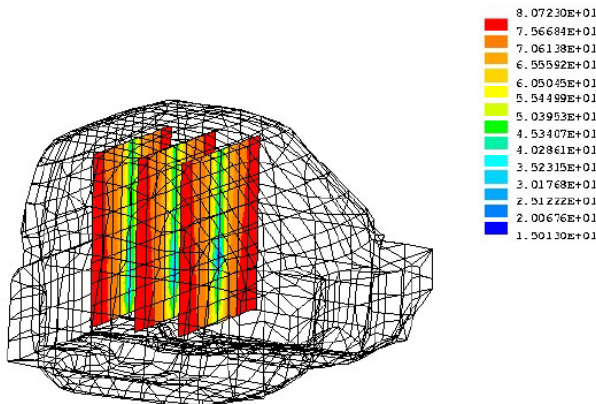


Figure 4. Sound Pressure Plot at 93.6 Hz (Max. 77.78 Db)

Table 1. Sound Pressure Levels at Driver's Ear Position (Selected Frequencies)

Frequency (Hz)	Pressure (kg/mm-sec <sup>2</sup> )	Phase Angle (Degree)
7.1	0.78747E-04	247.53
10.8	0.22699E-02	211.33
11.2	0.31111E-02	129.66
23.7	0.20014E-03	282.12
30.7	0.95681E-04	271.96
39.0	0.10764E-03	253.43
47.3	0.64318E-04	66.916
57.4	0.79145E-04	101.45
59.2	0.36350E-03	16.567
67.0	0.14299E-03	226.53
81.8	0.41087E-03	264.22
93.6	0.71486E-01	64.267

The highest sound pressure level occurs at 93.6 Hz, which is the first acoustic resonant mode under 100 Hz. Figure 4 shows the sound pressure level distribution inside the cabin compartment at this frequency. The sound pressure level at the driver's ear position is 77.78 dB. Design modification is carried out mainly focusing on reducing the peak noise level in the neighborhood of this frequency.

The vehicle structure is divided into forty different panels, whose thicknesses are selected as design variables in this example. In order to carry out DSA, the acoustic adjoint problem in (8) and the structural adjoint problem in (12) are solved to obtain the adjoint response  $\lambda^*$ . Using the original velocity response  $v$  and the adjoint response  $\lambda^*$ , the numerical integration of (14) is carried out to calculate the sensitivity for each structural panel, as shown in Table 2. The sensitivity contributions from all panels are normalized in order to compare the relative magnitude of the design sensitivity. The results indicate that a thickness change in the chassis component has the greatest potential for achieving reduction in the sound pressure level. Since the numerical integration process is carried out on each finite element, the element sensitivity information can be obtained without any additional effort. Figure 5 plots the sensitivity contribution from each element to the sound pressure level. Such graphic-based sensitivity information is very helpful for the design engineer to determine a desirable direction of design modification.

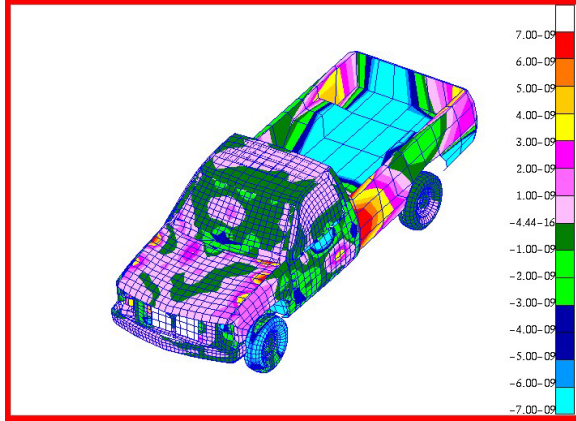


Figure 5. Element Design Sensitivity Results with respect to Panel Thickness

Table 2. Normalized Sound Pressure Sensitivity

Component	Sensitivity
Chassis	-1.00
Left Wheelhouse	-0.82
Right Door	0.73
Cabin	-0.35
Right Wheelhouse	-0.25
Bed	-0.19
Chassis MTG	-0.11
Chassis Connectors	-0.10
Right Fender	-0.07
Left Door	-0.06
Bumper	-0.03
Rear Glass	0.03

## 5.2. Design Optimization of the Vehicle Model

The design optimization problem is to search a design with minimum weight, while the noise level at the driver's ear position can be controlled to a desirable level. The weight (mass) of the vehicle is chosen as the objective function and the sound pressure level at the driver's ear position is chosen as a design constraint. The sound pressure level of 65.0 dB, which is 12.78 db less than the maximum sound pressure level (SPL) at the initial design and equivalent to more than 75% noise reduction, is used as design constraints.

Although, the maximum sound pressure occurs at 93.6 Hz at the initial design, the frequency where the maximum pressure occurs may shift during design process. However, it is difficult to constrain all continuous frequency ranges. Thus, a fixed set of discrete frequencies is chosen in order to evaluate the sound pressure level. Since the most significant acoustic reso-

nance occurs around 93.6 Hz, a total of eleven equally distributed frequencies in the neighborhood of 93.6 Hz are chosen to evaluate the sound pressure level during design optimization.

Although forty design variables are used to calculate the design sensitivity information, ten design variables are selected to change during design optimization because some of panel thicknesses are difficult to change for design purposes and some of them are related to the vehicle's dynamic performance. Ten selected design variables are panel thicknesses of Chassis, Fender-Left, Fender-Right, Wheelhouse-Left, Wheelhouse-Right, Cabin, Door-Left, Door-Right, Chassis-Conn, and Chassis-MTG, which significantly contribute to the sound and vibration level inside the cabin. The design space is chosen such that each design variable can change up to  $\pm 50\%$ . Accordingly, the design optimization problem is formulated as

$$\begin{aligned}
 &\text{Minimize} && c(\mathbf{u}) = \text{mass} \\
 &\text{Subject to} && g_i = p(\mathbf{u}, f_i) - 65.0 \leq 0 \\
 &&& 0.5\mathbf{u}_0 \leq \mathbf{u} \leq 1.5\mathbf{u}_0 \\
 &&& f_i = (93.2 + 0.1i) \text{ Hz} \\
 &&& i = 1, \dots, 11 \\
 &&& \mathbf{u} = [h_1, h_2, \dots, h_{10}]^T
 \end{aligned} \tag{16}$$

Table 3. Design Variables at the Optimum Design

Design Variable	Initial Value	Optimum Value
$u_1$ (Chassis)	3.137	1.568500
$u_2$ (Fender-Left)	0.800	0.40020
$u_3$ (Fender-Right)	0.800	0.40020
$u_4$ (Wheelhouse-Left)	0.696	0.348000
$u_5$ (Wheelhouse-Right)	0.696	0.368218
$u_6$ (Cabin)	2.500	1.250080
$u_7$ (Door-Left)	1.240	1.859970
$u_8$ (Door-Right)	1.240	0.620000
$u_9$ (Chassis-Con)	3.611	1.805500
$u_{10}$ (Chassis-MTG)	3.000	1.500000

The design optimization procedure illustrated in Figure 1 is carried out. A seamless integration between FEA, BEA, sensitivity module, and optimization module is critical in an automated design process. MSC/NASTRAN is used for frequency response FEA, while COMET/Acoustic is used for the acoustic BEA. The design sensitivity information is calculated from The Design Sensitivity Analysis and Optimization (DSO) Tool [20]. A sequential quadratic programming algorithm in DOT is used for design optimization. The

optimization problem is converged after five design iterations. A total of 15 response analyses and five design sensitivity analyses have been performed during design optimization. Table 3 compares the design variables between the initial and optimum designs.

Figure 6 illustrates the design variable history during optimization. It is observed that all the design variables are decreased to reach the lower bound except two of them, the right wheelhouse and left door. Table 4 and Figure 7 show the history of the cost function. The total mass of the vehicle is reduced from 1,705.834 Kg to 1,527.182 Kg, which is 178.652 Kg reducing, while the design constraints are satisfied. Figure 8 shows the sound pressure distribution inside the cabin before and after optimization.

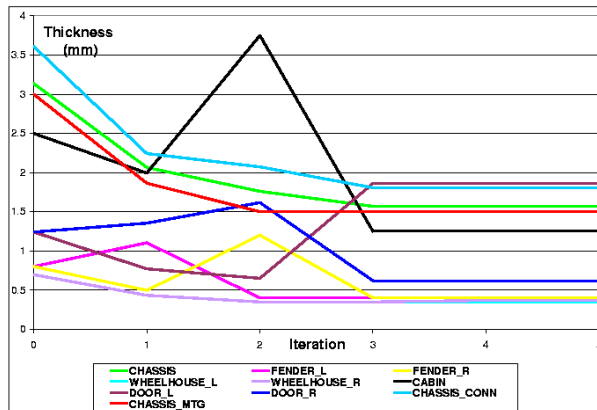


Figure 6. Design Variable History

Table 4. Cost and Constraint Function History

Iteration	Cost Function (Mass, ton)	Constraint (dB), $f=93.6$ Hz	$g_4$
0 (Initial)	1.705834	77.780	
1	1.610563	80.473	
2	1.751358	71.247	
3	1.527082	66.185	
4	1.527182	62.586	
5 (Optimum)	1.527182	62.586	

Figure 9 plots the change of the sound pressure level at the driver's ear position for the frequency range from 93.3 to 94.3 Hz during the optimization process. At the optimum design, the peak noise level is reduced to 65.0 dB, a total amount of 12.78 dB reduction. Note that the frequency, at which the maximum sound pressure appears, is shifted from 93.6 Hz to 93.7 Hz. However, this is not due to the shift of the acoustic resonant mode. The acoustic resonant mode still remains unchanged because of the same geometry of the acoustic space.

This indicates that the correct acoustic resonant frequency should be in-between 93.6 and 93.7 Hz. However, the selected design constraints are broad enough to cover the frequency range where the maximum noise level would occur.

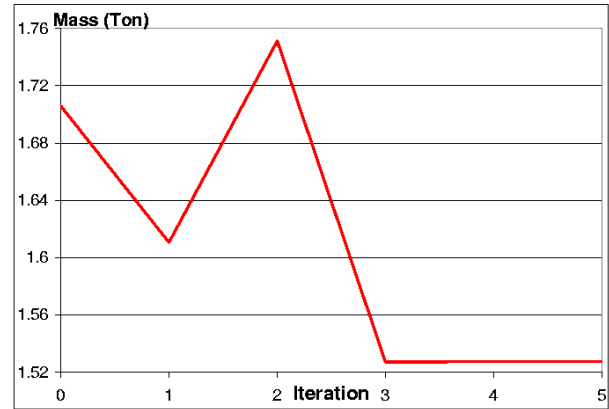
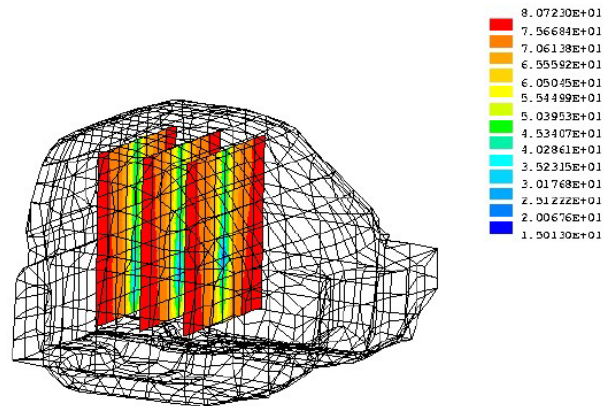
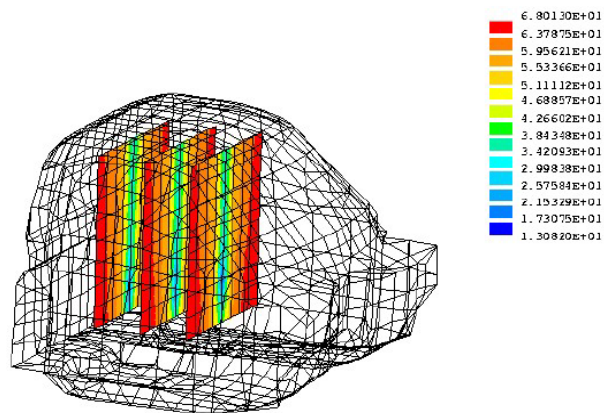


Figure 7. Cost Function History



(a) Initial Design (77.78 dB)



(b) Optimum Design (65.0 dB)

Figure 8. Acoustic Pressure Distribution of the Cabin Compartment

however, since the selected design constraints do not cover the entire frequency range, concern may arise that, while the peak noise level around 93.6 Hz is reduced, some other undesirable noises may occur at other frequencies. In order to check, the sound pressure levels at all frequencies below 100 Hz at the optimum design are computed and compared with initial design as plotted in Figure 10. The result indicates that the noise level at all frequency range is maintained under the constraint value. Although the sound pressure levels at some other frequencies increased, the value is still within the constraint limit. On the other hand, proper selection of constraints is effective and computationally efficient.

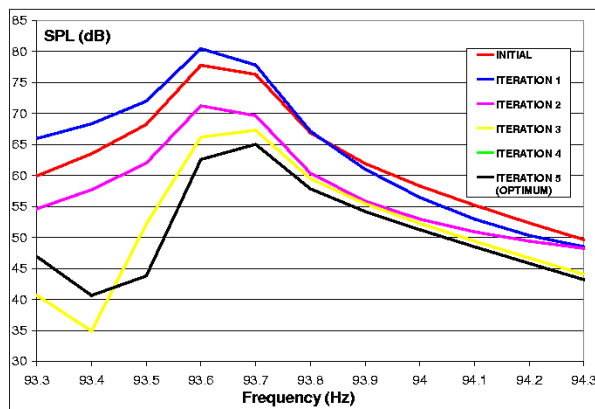


Figure 9. Design Constraints History in the Frequency Range between 93.3 and 94.3 Hz

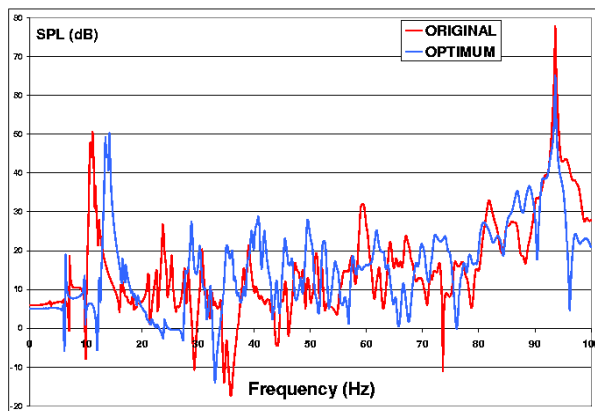


Figure 10. Frequency Response of Sound Pressure Levels at Driver's Ear Position (Initial and Optimum Designs)

## 7. CONCLUSION

Under the assumption that acoustic behavior does not influence structural behavior, design sensitivity analysis and optimization of a sequential structural-acoustic problem is presented using FEA-BEA. In the adjoint

variable method, a reverse sequential adjoint problem is formulated, in which the adjoint load is calculated by solving a boundary adjoint problem and the adjoint solution is calculated from a structural adjoint problem. Design optimization based on the sequential FEA-BEA analysis and reverse adjoint variable DSA method is carried out on a concept vehicle structure with satisfactory results, by reducing the noise level at the driver's ear position significantly while lowering the weight considerably.

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