Numerical integration schemes and parallel computation for wear prediction using finite element method

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ABSTRACT

In this paper, numerical integration schemes and parallel computation methodologies for wear occurring in bodies that experience oscillatory contact are proposed. The methodologies build upon a widely used iterative wear prediction procedure in which the contact pressure and the incremental sliding distance are calculated using nonlinear finite element analysis, and the geometry of the contact interface is progressively changed according to the wear model. It is well known that the discretization in space and time causes errors and instabilities during the wear integration process. In this paper, two approaches are proposed to minimize the computational costs while maintaining the accuracy and stability of wear integration. In the first approach, an extrapolation scheme that optimizes the use of resources while maintaining simulation stability is used based on the variation of contact pressure. The second approach involves the parallel computation of the wear prediction methodology. The effect of geometry update intervals on the stability and efficiency of wear integration is studied. The proposed methodologies are used to predict the wear on an oscillatory pin joint and the predicted results are validated against those from actual experiments.

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1. Introduction

Mechanical systems employ mechanisms to convert one type of motion into another. These mechanisms consist of connections, such as joints, where component parts have mating surfaces and undergo relative motion. The contact and relative motion between parts at connections introduce wear which after a period of time may alter the joint kinematics and cause the mechanisms to fail. This existence of wear presents a challenge in the design of such systems because the performance is deteriorated and the kinematics of the mechanism is changed.

A common practice, to incorporate wear into design, is to perform tests to predict the amount of wear that may occur on a given design based on specified operating conditions. This technique has proven to be time consuming and expensive due to its destructive nature. In addition, it is difficult to take into account the change in the system kinematics due to the evolution of the joint geometry. It is critically important to develop a design tool that can efficiently predict the fundamental physics of wear and estimate its influence on the system kinematics.

Recently, a significant amount of resources and efforts have been placed into developing techniques that utilize computer simulations in predicting wear. A number of papers dealing with this subject have also been published [1–22]. A general trend, in wear prediction, that has emerged is the use of Archard's wear model which requires knowledge of the contact conditions (i.e. contact pressure and sliding distance) and tribological data such as the wear coefficient of the material in contact. Earlier prediction procedures employed Archard's model to estimate worn geometry based on initial contact conditions. The procedures assumed that the geometry and thus contact pressure did not evolve. Linear extrapolations were then applied to determine the final geometry. This procedure has been found to produce erroneous results [21,22].

In later procedures, wear predictions have been based on evolving contact conditions. The procedures allow the contact geometry to vary gradual and thus resulting in an iterative procedure in which the contact pressure and the sliding distance are commuted at each iteration. The geometry is also updated at each iteration to reflect the evolution caused by the wear. Various methods have been employed in computing the contact pressure and sliding distance. These include the finite element method [1,2,4–7,9,11,13], boundary element method [10], Winkler model [3,14,16], Hertz contact

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Nomenclature

A	contact area
A_j	extrapolation factor at jth cycle
α	constant for geometric effect
δ_{max}	maximum deformation in contact interface
F_N	normal force in the contact interface
H	Brinell hardness
h	wear depth
K	dimensionless wear coefficient
k	dimensioned wear coefficient
p	contact pressure
s	sliding distance
V	volume lost by wear

model [17,18] each having its pros and cons with regard to accuracy and computational expense. Procedures for geometry update have also been developed which generally involve moving the contact boundary in the normal direction by an amount equivalent to the wear [1].

Depending on the effectiveness of the geometry update and the accuracy of the contact pressure, the iterative procedure has been found to yield relatively reasonable results [1–5]. This has allowed the procedure to be used in various application such as in the prediction of gear wear [16–18], cam-follower wear [19–21,23], knee joint wear [24], and hip joint wear [25,26].

Despite the reported success from the iterative procedure, it has been found to be quite computationally expensive. This is primarily due to the iterative process that is required to capture the evolving geometry. Several ideas have been implemented in an attempt to reduce computational costs associated with the wear simulation process. Põdra and Andersson [3] attempted to minimize the computational cost by using the Winkler model to determine the contact pressure distribution. The Winkler model was used as an alternative to the more expensive but relatively accurate FEM. Although the method was found to be less expensive it can be argued that the benefit of using more accurate results from the finite element technique outweigh the gains in computational efficiency when complicated geometries are considered. Põdra and Andersson [5] also employed a scaling approach to tackle the problem of computational costs. In this approach the incremental wear at any particular cycle of the simulation was scaled based on a predefined maximum allowable wear increment. The scaling factor was obtained as a ratio between the maximum allowable wear increment and the current maximum wear increment (maximum wear increment of entire geometry). They found that this procedure was more computationally effective. Kim et al. [1] used a constant extrapolation technique to reduce the computational costs for the oscillatory wear problem. In their technique one finite element analysis was made to represent a number of wear cycles. Through this extrapolation, they were able to reduce the total number of analyses needed to estimate the final wear profile. A similar procedure was done by McColl et al. [6] as well as Dickrell et al. [27]. In another paper [7], the computational costs of simulating a pin on a rotating disc was reduced by approximating the state of strain on the center of the wear track as plain strain. A less costly two-dimensional idealization was then used in place of the more expensive three-dimensional problem.

The purpose of the present paper is to develop a prediction procedure for wear resulting in bodies that experience oscillatory contact (such as pin–pivot joints). A practical challenge is on the edge of contact region where the curvature of the boundary changes suddenly. This difficulty was not observed in contact between continuously rotating bodies. The focus of the development is on the reduction of computational costs involved in the prediction while maintaining reasonably accurate predictions. In the first part of the paper a wear prediction methodology, similar to those found in the literature but specific to bodies undergoing oscillatory contact is presented. Next, techniques that are incorporated into the methodology in order to reduce the computational costs are discussed. These techniques are further improved by performing a cycle updating procedure instead of the widely used step updating procedure and a parallel processing implementation of the methodology. Finally the simulation procedure is employed to predict the wear on a pin joint and the result is compared against the experimental counterpart as a validation step.

2. Computational prediction method

2.1. Wear model

In developing a wear prediction methodology, it is assumed that all the wear cases to be predicted fall within the plastically dominated wear regime, where sliding velocities are small and surface heating can be considered negligible. Archard's wear law [28] would thus serve as the appropriate wear model to describe the wear as discussed by Lim and Ashby [8] as well as Cantizano et al. [9]. In that model, first published by Holm [29], the worn out volume, during the process of wear, is considered to be proportional to the normal load. The model is expressed mathematically as follows:

$$\frac{V}{s} = K\frac{F_N}{H}, \tag{1}$$

where V is the volume lost, s the sliding distance, K the dimensionless wear coefficient, H the Brinell hardness of the softer material, and F_N the applied normal force. Since the wear depth is the quantity of interest, as opposed to the volume lost, Eq. (1) is often written in the following form:

$$\frac{hA}{s} = kF_N, \tag{2}$$

where h is the wear depth and A is the contact area such that $V = hA$. The non-dimensioned wear coefficient K and the hardness are bundled up into a single dimensioned wear coefficient k (Pa^{-1}). Note that the wear coefficient is not an intrinsic material property. The value of k for a specific operating condition and given pair of materials may be obtained by experiments [1]. Eq. (2) can further be simplified by noting that the contact pressure may be expressed with the relation $p = F_N/A$ so that the wear model is expressed as

$$\frac{h}{s} = kp. \tag{3}$$

In this work the contact pressure p is computed using commercial finite element analysis software, ANSYS.

The wear process is generally considered to be a time-dependent process (rate of change of the wear depth with respect to sliding distance) so that the first order differential form of Eq. (3) can be expressed as

$$\frac{dh}{ds} = kp(s), \tag{4}$$

where the sliding distance is considered as a time in the dynamic process. The contact pressure varies during the wear process.

2.2. Simulation procedure (step-update)

A numerical solution for the wear depth may be obtained by estimating the differential form in Eq. (4) with a finite divide difference to yield the following updating formula for wear depth:

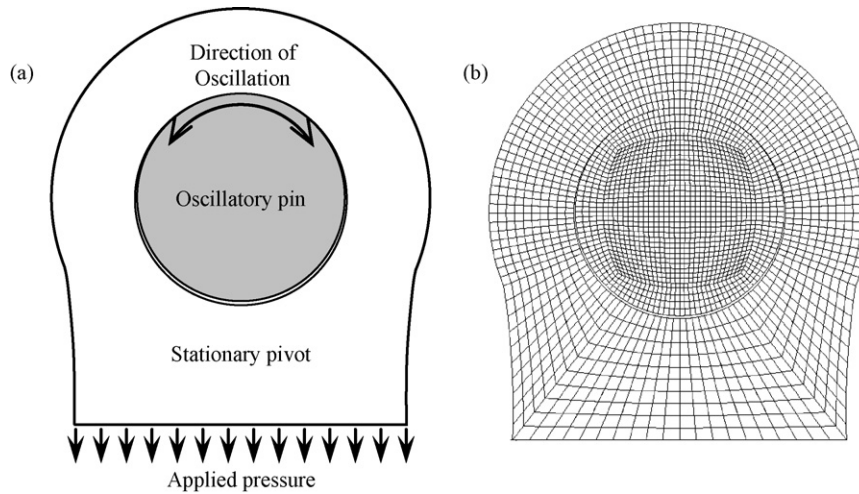$$h_i = h_{i-1} + kp_i \, \Delta s_i. \tag{5}$$

Fig. 1. A pin-and-pivot joint that exhibits oscillatory relative motion: (a) problem definition and (b) finite element model.

In Eq. (5), h_i refers to the wear depth at the ith step while h_{i-1} represents the wear depth at the previous step. The last term in Eq. (5) is the incremental wear depth which is a function of the contact pressure (p_i) and the incremental sliding distance (Δs_i) at the corresponding cycle. Note that Eq. (5) is forward Euler integration, in which the accuracy and stability depend on the incremental sliding distance (Δs_i) of the step.

The most widely used procedure to simulate wear occurring at a contact interface is the numerical integration in Eq. (5). A number of papers [1–6], that demonstrate the implementation of Eq. (5) in predicting wear, have been published. Although the details of the various procedures differ, three main steps are common to all of them. These include the following:

• Computation of the contact pressure resulting from the contact of bodies.
• Determination of the incremental wear amount based on the wear model.
• Update of geometry to reflect the wear and to provide the new geometry for the next step and hence allowing for a more accurate prediction of the wear process.

In this paper, the abovementioned steps are employed in predicting wear on oscillatory contacts, which is common in many mechanical joints. Fig. 1 shows a representative geometry of a common pin-and-pivot joint that experiences oscillatory relative motion and the corresponding finite element model.

In general, the wear process is continuous. However, in order to predict the wear process numerically, two different types of discretizations are introduced: (1) the continuous geometry is discretized by finite elements and (2) the continuous material removal in time is approximated at a discrete set of times. For the given wear model, these two discretizations claim for instability and inaccuracy of numerical wear prediction.

The goal is to develop a procedure that can predict the wear over several thousand cycles. In this work a cycle is defined as a rotation of the pin from one extreme angle to the other (e.g., $\pm\theta°$). The simulation of the oscillating pin is achieved by discretizing every cycle into a number of steps (or incremental angles) so that a complete rotation (from one extreme to the other) is decomposed into a number of incremental rotations. At each step a finite element analysis is performed to determine the contact pressure which in turn is used to calculate the incremental wear depth according to the following equation:

$$h_{i,j}^n = h_{i-1,j}^n + kp_i \, \Delta s_i, \tag{6}$$

where n refers to surface nodes (of the finite element model) that can establish contact with the opposing surface. The subscripts i and j indicate the current step and cycle, respectively. All other terms are as defined previously.

The geometry is then updated to prepare the model for the next step. The oscillating pin then moves to a new position (in the next step) by rotating through an angle corresponding to the incremental angle. Once all the steps in a particular cycle have been finished the direction of rotation is reversed and the next cycle commences. The term 'step-update' is adopted for this procedure since the geometry is updated after every step. This procedure is depicted in Fig. 2.
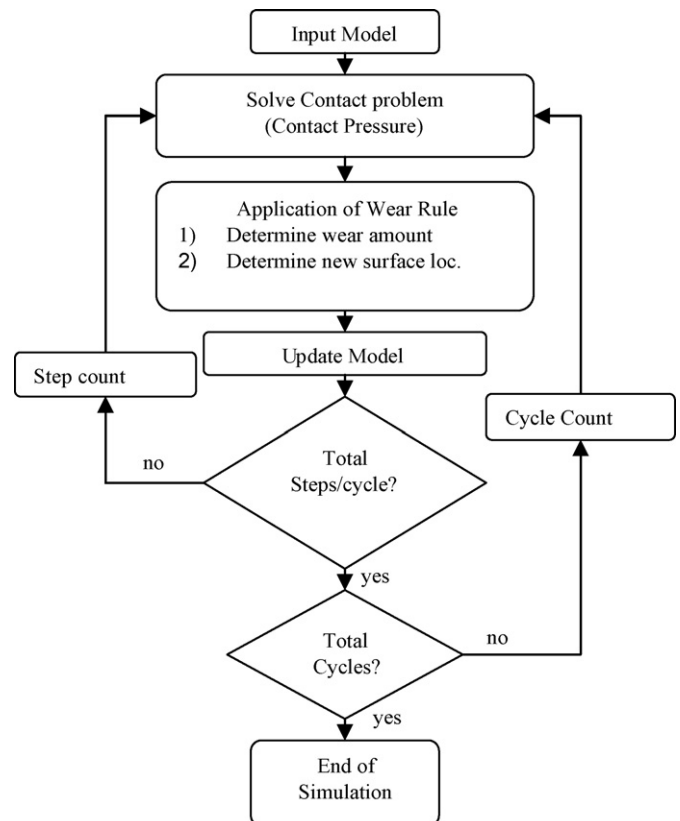

Fig. 2. Wear simulation flow chart for the 'step update' procedure.

Although the procedure discussed provides a way to simulate the wear resulting from oscillatory contacts, the process can be quite expensive. For instance, if one desires to simulate 100,000 oscillatory cycles for a case in which each cycle is discretized into 10 steps then 1,000,000 finite element analyses (nonlinear) as well as geometry updates would be required to complete the simulation process. Clearly this may not be practically feasible and the need for techniques to combat the computational cost becomes immediately apparent.

3. Extrapolation schemes

As mentioned earlier, extrapolations have been used in various forms with the goal of reducing computational costs. In this work an extrapolation factor (A_j) is used to project the wear depth at a particular cycle to that of several hundreds of cycles. A modification to Eq. (6) to incorporate an extrapolation factor would result in the following equation:

$$h_{i,j}^n = h_{i-1,j}^n + kA_j p_i \ \Delta s_i. \tag{7}$$

The condition placed on the selection of the extrapolation size is that the selected size would not severely affect the overall smoothness of the pressure distribution. The contact pressure distribution (obtained from the finite element analysis) over the mating surfaces is generally not perfectly smooth. The use of an extrapolation factor magnifies this imperfection and hence causing the updated surface to be non-smooth, which directly affects the smoothness of the contact pressure distribution of the following cycle. A smooth contact surface is critical for two reasons. The first reason is that a smooth contact surface is consistent with the actual case that is being simulated, and the second is that a non-smooth surface would affect the solution of the finite element problem. This is especially important for the oscillatory motion because at the end of contact region the curvature changes suddenly causing high contact pressure values to be observed.

Extrapolation provides a solution to the computational cost problem but its use may introduce other problems. The accuracy and stability of the simulation may be lost by using extrapolation sizes that are too large. In particular, the high-pressure values observed at the contact edges will result in an oscillation of the contact pressure. This oscillation, if left unchecked will propagate into the interior contact region and eventually compromising the simulation results. On the other hand using too small extrapolation sizes will result in a less than optimum use of resources. Even if an appropriate extrapolation size was selected at the beginning of the simulation it may be that at a different stage of the simulation a different extrapolation size would be required to provide optimum use of the available resources. In the following a procedure is described that seeks for the optimum extrapolation sized during the entire simulation process.

The adaptive extrapolation technique that is proposed here is an alternative to the constant extrapolation scheme. The idea behind it is to seek for the largest extrapolation size while maintaining a state of stability (overall smooth pressure distribution) throughout the simulation. The scheme is a three-step process. In the first part an initial extrapolation size (A_0) is selected. This was originally determined from experience. It was, however, observed that a general formula can be stated to determine the initial extrapolation size for a similar geometry with different dimensions and different material properties. The proposed formula is shown in the following equation:

$$A_0 \le \frac{\alpha \delta_{\max}}{k p_{\max} \ \Delta s}. \tag{8}$$

In this equation $\delta_{\max}$ and $p_{\max}$ are, respectively, the maximum deformation and the corresponding pressure in the first analysis of

the simulation, and α is a dimensionless constant ranging between zero and one. This constant is attributed to geometry effects and is determined through simulation experiments. Its value is the same for a specific geometry and works for different dimensions and material properties. Physically, Eq. (8) states that the maximum incremental wear depth ($A_0 k p_{\max} \ \Delta s$) must be less than the maximum deformation scaled down by a factor of α.

In the second part of the adaptive extrapolation scheme, a stability check is performed. A single check, preferably at the center of the cycle, is sufficient for an entire cycle. The stability check involves monitoring the contact pressure distribution within an element for all elements on the contact surface except the elements at the contact edges. This essentially translates to monitoring the local pressure variation in the interior contact region. If the contact pressure difference within an element is found to exceed a stated critical pressure difference Δp_{crit} then a state of instability is noted. In the final step of the adaptive scheme, the extrapolation size is altered based on the result of the stability check; i.e., an increase in the extrapolation size for the stable case and a decrease for the unstable case. This process can be summarized as

$$A_j = \begin{cases} A_{j-1} + \Delta A_{\mathrm{inc}} & \text{if } \Delta p_{\mathrm{ele}} < \Delta p_{\mathrm{crit}} \\ A_{j-1} - \Delta A_{\mathrm{dec}} & \text{if } \Delta p_{\mathrm{ele}} > \Delta p_{\mathrm{crit}} \end{cases}. \tag{9}$$

It must be noted that in order to maintain consistency in the geometry update as well as in the 'bookkeeping' of the number of cycles simulated, a single extrapolation size should be maintained throughout a cycle. That is, every step in a cycle will have the same extrapolation size while different cycles may have different extrapolation sizes. Fig. 3 shows a graph of the extrapolation history for the oscillating pin–pivot assembly. From the graph, it can be seen that the extrapolation took on a conservative initial value of about 3900 and increased steadily up to the 12th cycle (actual computer cycles not considering the extrapolations). Thereafter the extrapolation size oscillated about a mean of about 6000. The area under the curve is the total number of simulation cycles.

It was mentioned that the elements at the contact edges were ignored during the stability check. This is because the results of a stability check on these elements would almost always be a state of instability since these elements report high contact pressure variation. An important idea of the paper is that the extrapolation size is adaptively controlled such that the pressure variation on the edge does not propagate into the contact region. It was also mentioned
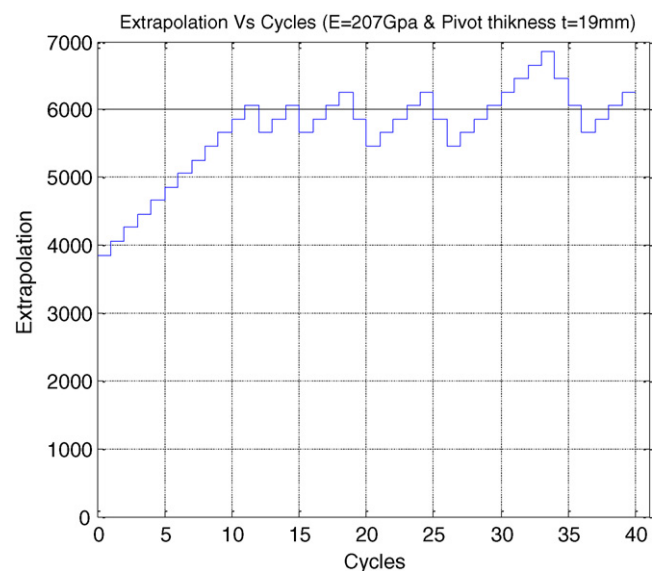

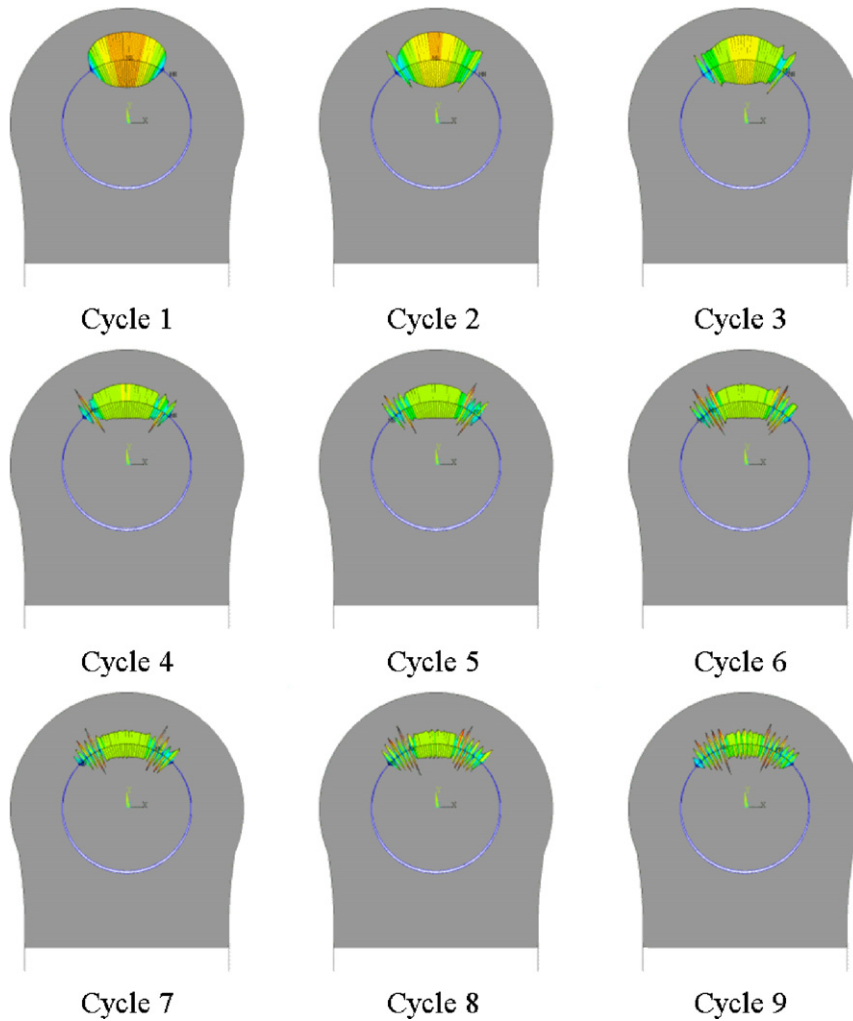**Fig. 3.** Graph showing the extrapolation history for a pin joint.

Fig. 4. Evolution of contact pressure at first nine cycles when a fixed extrapolation size is used.

that if a large extrapolation size is used, the high contact pressure at the contact edges would oscillate and propagate into the interior contact region and thus compromising the accuracy as well as stability of the simulation. This problem is, however, dealt with indirectly through the adaptive scheme. As long as the pressure oscillations at the edges do not propagate to the interior contact region, the simulation will be reasonably accurate. Any propagation of the oscillating pressure distribution from the contact edges into the interior contact region will automatically be detected during the stability check and the extrapolation size adjusted accordingly. In doing so a smooth distribution of the contact pressure can be maintained at all times of the simulation. Fig. 4 shows the evolution of contact pressure at different cycles when a constant extrapolation size is used. In early cycles, pressure oscillation is observed only at the contact edges. However, in later cycles the pressure oscillations propagate to the interior contact region. On the other hand, Fig. 5 shows the evolution of contact pressure when the adaptive extrapolation scheme is used. The contact pressure is observed to oscillate at the contact edges but does not propagate into the interior contact region.

4. Parallel computation

Although the use of extrapolations is probably the most effective way to reduce the computational costs, other ways are also available. A parallel computation of the simulation procedure is proposed as an additional way to remedy the problem. It is, however, befitting to introduce the concept of 'cycle-update and intermediate cycle-update' which is a central idea in the parallel computation procedure.

4.1. Cycle- and intermediate cycle-update

The wear simulation procedure that was discussed earlier was termed as the 'step-update' for the reason that geometry updates were performed after every step. An alternative to the step-update would be to exclude all geometry updates during the entire steps in a cycle and perform a single update at the end of the cycle. We term this procedure as the 'cycle-update'. For the cycle-update, information from each analysis performed at each step is saved and is later used to update the model at the end of the cycle. The modified equation for the cycle-update becomes

$$h_j^n = h_{j-1}^n + kA_j \sum_{i=1}^{n\_\text{step}} p_i \ \Delta s_i, \tag{10}$$

where $n\_$step is the total number of steps in a cycle. All other terms are as defined previously. The cycle-update procedure can be summarized in the flowchart shown in Fig. 6.

Although the objective of the step-update and the cycle updated procedures are identical, the difference in the implementation of the geometry update in each of the procedures has an implica-
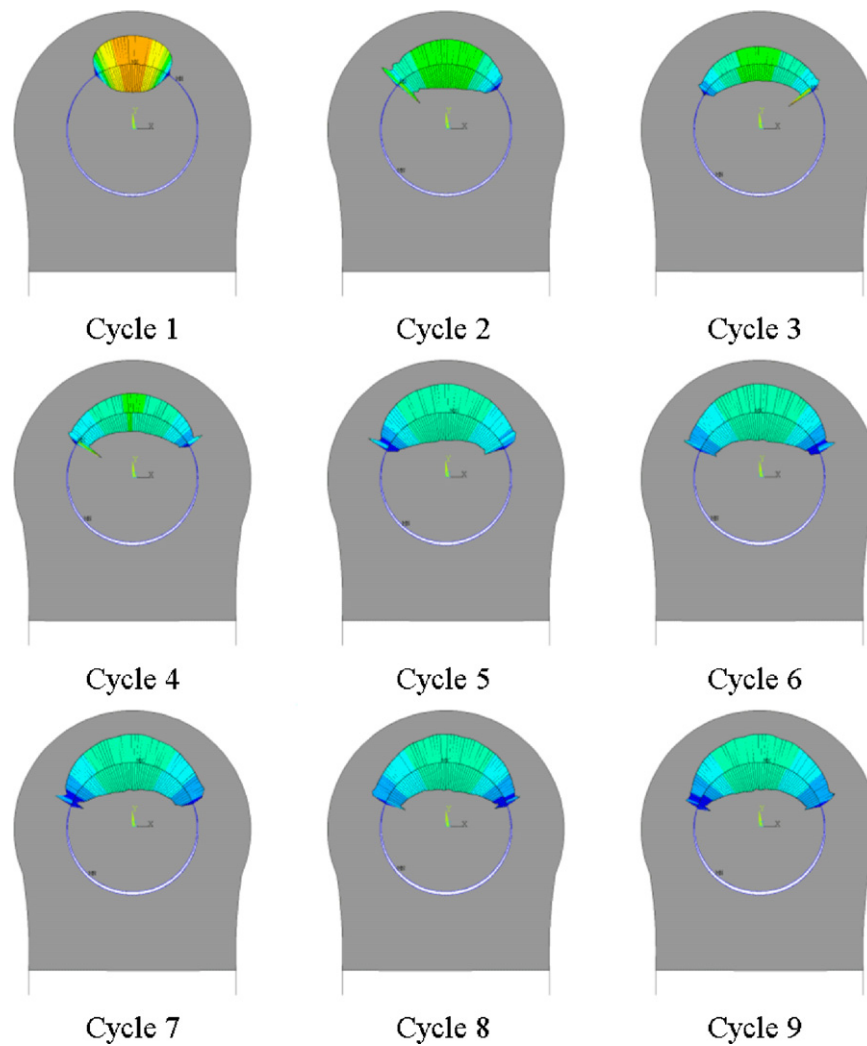
Fig. 5. Evolution of contact pressure at first nine cycles when an adaptive extrapolation scheme is used.

tion in the stability of the simulation process. In the step-update material is removed at each step of the cycle which is an approximation of the actual wear process in which material is continuously removed/worn out during the entire cycle. In essence the step-update procedure discretizes the continuous material removal process (for the cycle) into several discrete steps. In the case of the cycle-update, material removal is delayed and performed at the end of the cycle. Thus the discretization in the cycle-update only serves as a means to collect the contact pressure information necessary for the wear depth calculations. No discretization is done for the material removal process.

The step-update therefore has a closer resemblance to the actual wear process. It would thus be expected that the use of the cycle-update procedure in wear simulations would yield less reliable results in comparison to the step-update. Indeed, this is what is observed when the procedure is tested. More specifically, the smoothness of contact pressure distribution during the simulation is severely affected by the cycle-update than is by the step-update. A simplified explanation for this phenomenon is that the step-update, performed at each step, closely captures intermediate geometry changes within a cycle and hence the contact between two mating surface remains conforming throughout the simulation. The result is that the pressure distribution remains smooth. In the case of the cycle-update the geometry is updated once in an entire cycle. This does not allow for the contacting surface to evolve smoothly throughout the cycle and hence resulting in a less conforming contact between the mating surfaces. In this case the pressure distribution would be less smooth.

Even though it was mentioned in the previous discussion that the cycle update technique may yield less than accurate results, the technique may still be used with caution. A general observation can be made regarding the accuracy when using the cycle-update: for a fixed extrapolation size, as the total sliding distance (which is a function of both rotation radius and total rotation angle for a complete cycle) covered through a complete cycle increases, the smoothness of the pressure distribution is affected and hence affecting the stability and accuracy of the simulation. A critical total sliding distance s_{crit} is defined beyond which, if exceeded during sliding, a geometry update must be performed. At this point the approach used to determine s_{crit} involves several simulation tests. It is concluded that the cycle update is best suited for cases in which the total oscillation angle is smaller than s_{crit}. In the event that the total sliding distance for a complete cycle is larger than s_{crit} we may still take advantage of the idea behind cycle-update procedure. Instead of performing a single update at the end of the cycle we may perform several equally spaced updates within the cycle, a hybrid of the step- and cycle-update procedure. We term this as the 'intermediate cycle-update' procedure. The advantage of this is that we are able to reduce the number of updates in the cycle while still maintaining reasonable accuracy in the simula-
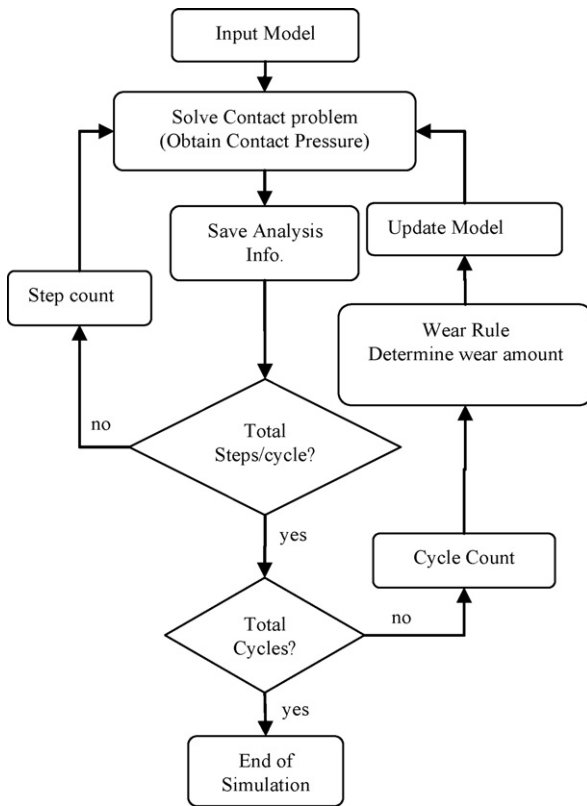
Fig. 6. Wear simulation flow chart for the 'cycle-update' procedure.

tion. The intermediate cycle-update procedure is summarized in Fig. 7.

4.2. Parallel computation

In recent years computer technology has advanced significantly at relatively low cost. Because of this parallel computation is becoming a more feasible and readily used option for reducing computation time. The idea proposed is a direct parallel implementation of the cycle update and the intermediate cycle-update procedures. For the sake of brevity only the parallel implementation of the cycle-update is discussed.

As noted above, the cycle update procedure is centered on the idea that no update is performed on the geometry during the entire cycle. This means that all the analysis performed at each step within a cycle is done on the same geometry. The difference between any two analyses within a cycle is the angle at which the two bodies contact. This information may be exploited to construct the parallel computation equivalent of the wear simulation procedure.

The parallel implement works as follows. Several processors are dedicated to the wear analysis simulation. One of these processors is assigned the duty of a master processor. This will be the processor responsible for distribution of tasks to other processors as well as consolidating the results of other processors. The remaining processors will be the slave processors. Each of the processors, both slave and master processors, will represent a particular step within a cycle. In the beginning of any cycle, the appropriate model of the assembly to be analyzed for wear is fed into the master processor. The master processor then distributes the same model to the remaining processors. In addition to distributing the model, the master also allocates different contact angles (each contact angle corresponds to a specific step in the cycle) and corresponding analysis conditions to each of the slave processors. At this point the master processor instructs the slave processor to solve the contact
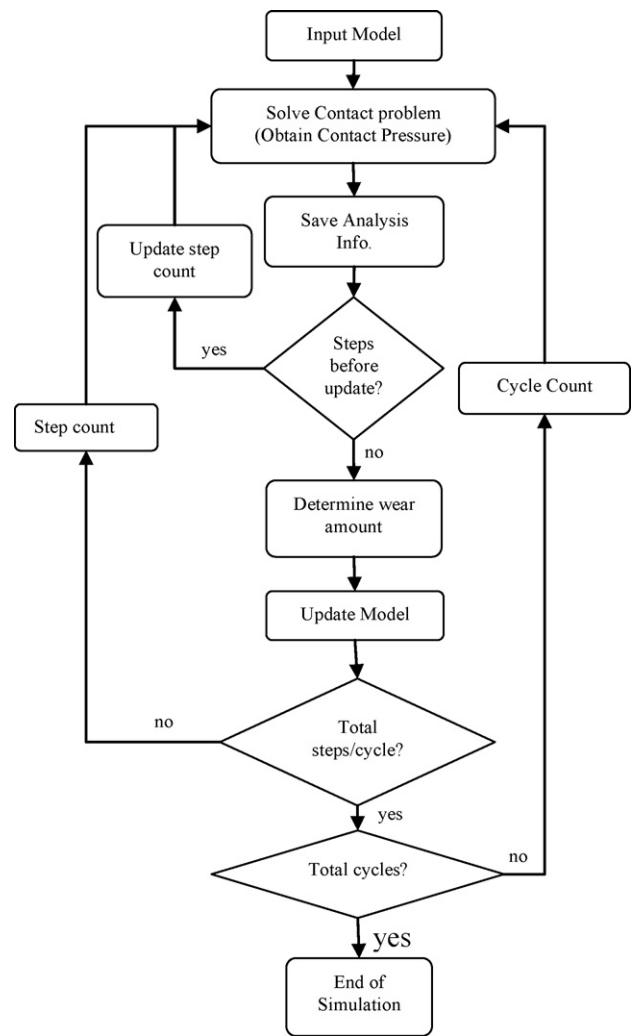

Fig. 7. Wear simulation flow chart for the 'intermediate cycle update' procedure.

problem with different contact angles. Once the analysis in the different slave processors is done the master node collects the results and computes the wear amount for that cycle. The model geometry is then updated and thereafter a new cycle commences. The parallel implementation of the cycle update procedure is summarized in the flowchart shown in Fig. 8. From the flowchart it can be seen that considerable amount of time is saved by using the parallel computation of cycle-updating procedures. If the number of processors available is equivalent to the number of steps selected for a cycle, then the time required to complete a single cycle while using the parallel procedure is approximately equal to the time required to complete a single step in the step and cycle updating procedures.

5. Numerical results

Probably the most convincing way to validate the results of a simulation is to compare them against those from an actual experiment. In this paper the simulation results were validated by comparing them with the wear tests performed on an oscillating pin-and-pivot joint. The test consisted of a fixed steel pin inside an un-lubricated oscillating steel pivot. The pin was set to oscillate with amplitude of 6° and was loaded in the direction of its shoulder (see Fig. 1) to produce 150 kN tension. The force was kept approximately constant throughout the test. A total number of 400,000 cycles were completed during the test to yield a maximum wear depth of about 2 mm. It should be noted that the definition of the
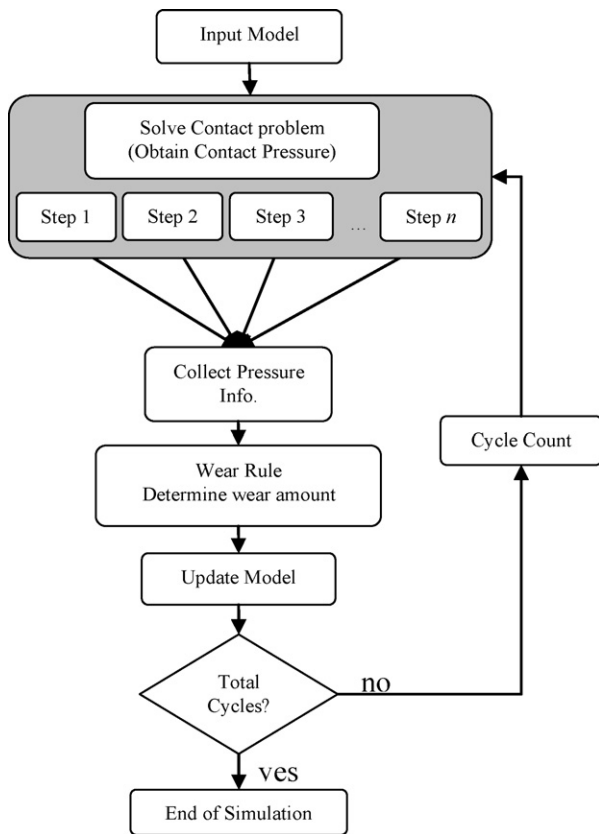
Fig. 8. Wear simulation flow chart for the parallel implementation of the 'cycle-update' procedure.

test cycles is different from that of the simulation cycles. Here a test cycle is defined as a complete rotation from one extreme to the other and then back to the starting position (in this case from −3° to 3° and back to −3°). The test information is summarized in Table 1 for convenience.

Three different simulations were performed to mimic the actual tests performed on the pin-and-pivot joint. The three simulations experiments were as follows:

- Step-updating procedure.
- Intermediate cycle-update procedure
- Parallel implementation of the intermediate cycle-update procedure

All three simulation tests were performed with the model shown in Fig. 1. From the pin-on-plate wear test in Kim et al. [1], a wear coefficient of 1.0×10^{-5} mm^3/Nm was used. In all three cases the cycles were discretized into 10 steps. Both the step- and intermediate cycle-updating simulation tests were performed on the same computer (for time comparison), however, the parallel implementation was performed on parallel clusters. The following is a brief discussion of these simulation tests and the corresponding results.

Table 1
Wear test and simulation information for the pin joint.

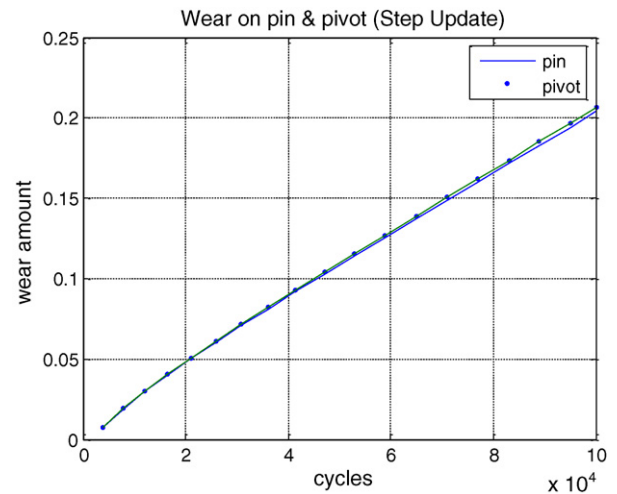|	
Oscillation amplitude	±3°
Applied load	150 kN tension
Test condition	Un-lubricated steel on steel
Total test cycles	408,000
Max. wear depth on pin	~2.00 mm
Wear coefficient (k)	1.0×10^{-5} mm^3/Nm
Total simulation cycles	100,000
Steps per cycle	10

Fig. 9. Maximum wear depth on pin and pivot.

5.1. Step-update simulation test

The step-updating simulation test was performed with oscillation amplitude and loading identical to that of the actual wear test. The simulation test was run for 100,000 cycles (considering the extrapolation size). The simulation test parameters are summarized in Table 1.

In Fig. 9, the history of wear depth for the pin and pivot nodes that experienced the most wear is shown. From the figure, a transient and steady state wear regime can be identified as discussed by Yang [12].

The transient wear regime corresponds to the beginning of the simulation until the contact between the pin and the pivot is conforming. Thereafter the wear transitions to the steady state wear regime. The steady state wear regime is marked by an interesting phenomenon where by the contact pressure distribution is observed to be approximately constant over the region of contact. This is in contrast to the transient wear regime during which a range of contact pressure values is observed over the contact region. The pressure distributions are compared in Fig. 10.

Within the steady state wear regime, the wear is approximately linear with respect to the cycles as can be seen in Fig. 9. This information may be extrapolated to determine the wear on the maximum wear nodes after 408,000 cycles. Noting that one test cycle has twice the sliding distance in comparison to that of the simulation test, the following equation may be used to predict the wear depth at the 408,000th cycle:

$$h = \frac{2 \times n_{\text{test}}}{n_{\text{simulation}}} h_{\text{FEM}} \tag{11}$$

In Eq. (11), h is the predicted wear depth where as n_{test} and $n_{\text{simulation}}$ are the total number of test and simulation cycles, respectively. The wear depth from the simulation corresponding to $n_{\text{simulation}}$ is represented by h_{FEM}.

A value of 1.867 mm was predicted as the maximum wear depth on the pin. Although this value underestimates the wear depth it is reasonable considering that the wear phenomenon is a complex process. The variation of the extrapolation size is depicted in Fig. 11. The simulation took approximately 206 min.

5.2. Intermediate cycle-update—parallel computation

The intermediate cycle-update procedure and its parallel implementation were performed with the same parameter values as were used in the step-updating procedure (see Table 1). However, in this
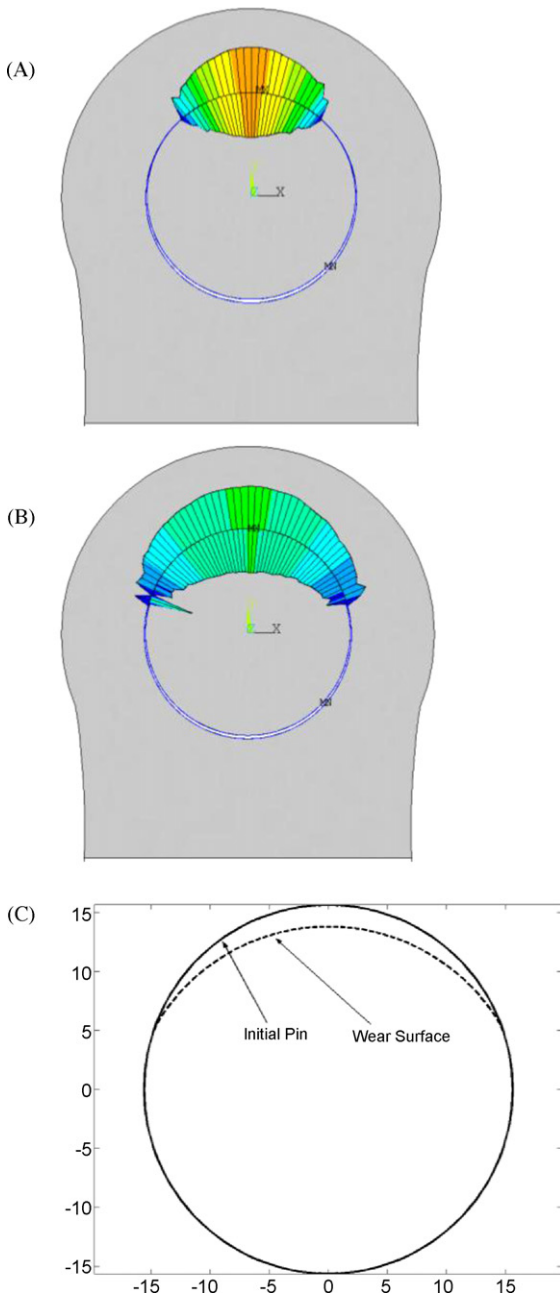
Fig. 10. Contact pressure distribution on the pin joint during wear analysis. (A) Contact pressure distribution in the transient wear regime. A range of pressure values is observed; (B) Contact pressure distribution within the steady wear regime. The pressure distribution is approximately constant over the region of contact. (C) Geometry change of the pin.

Fig. 11. Extrapolation history plot for the step updating simulation procedure.

Fig. 12. Cumulative maximum wear on pin and pivot for the intermediate cycle updating procedure and the parallel implementation.

Fig. 13. Extrapolation history plot for the intermediate cycle update procedure and its parallel implementation.

procedure, the update was performed after every 3 steps so that 3 updates were done in each cycle. This is in contrast to the step-update procedure where 10 updates were performed, one at the end of every step. The result for the intermediate cycle-update and the corresponding parallel implementation are identical. The plot of the wear on the pin and pivot nodes that experience the most wear is shown in Fig. 12.

A maximum wear depth (on the pin) of 1.854 mm was obtained from the intermediate cycle-update procedure and its parallel implementation. A plot of the extrapolation during the analysis is shown in Fig. 13. A simulation time of 450 min was required for the intermediate cycle update procedure. This is slightly more than twice the time it took to complete the step-update simulation test.
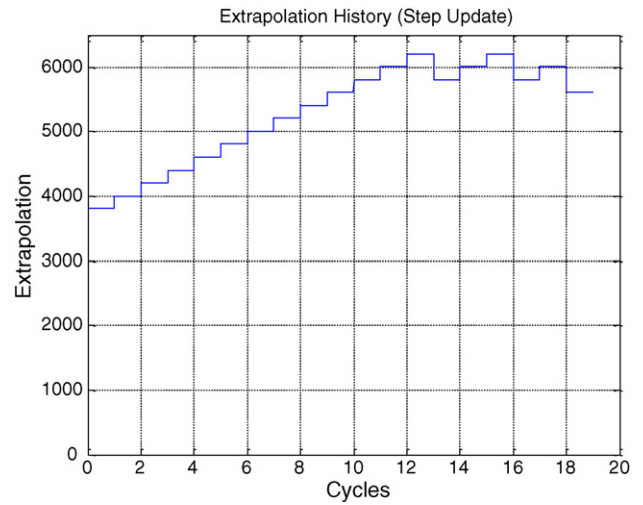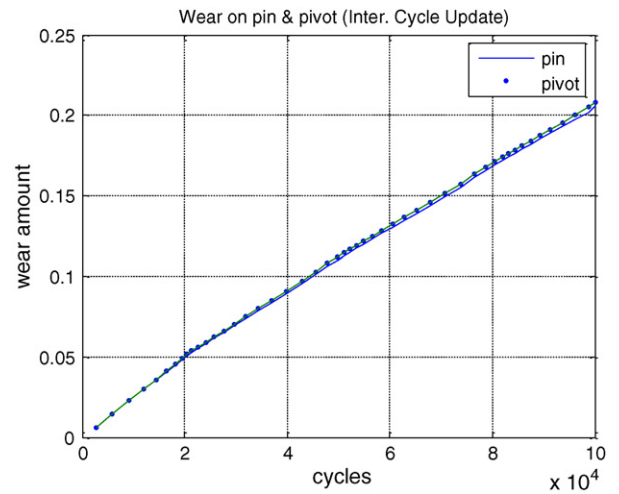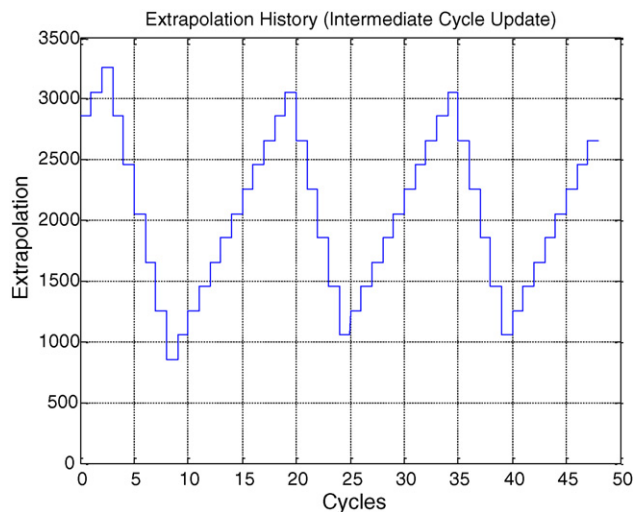
Table 2
Comparison of results from the simulation tests and actual wear tests for the pin and pivot assembly.

	Max. wear depth (pin) (mm)	Simulation time (min)
Actual test	2.000	–
Step update	1.867	206
Inter. cycle Update	1.854	450
Parallel	1.854	135

This time difference can be explained by examining the extrapolation history plots (Figs. 11 and 13) for the two procedures. The average extrapolation for the step update is slightly greater than twice that of the intermediate cycle update procedure. As a result, only 19 cycles were required to complete the step-update simulation wear test, while 49 cycles were required to complete the intermediate cycle update simulation test. On the other hand, the parallel implementation of the intermediate update procedure only took approximately 135 min to complete. Clearly this procedure provides a time advantage. A comparison of the results from the simulation tests and the actual tests is shown in Table 2. A 2.0-GHz Intel Pentium computer with 2.0 GB of RAM is used for all the simulations.

6. Discussions and concluding remarks

In this work, procedures to predict wear on bodies experiencing oscillatory contact have been presented. The first of these was called the step-update procedure owing to the fact that geometry updates were performed at the end of each step. Two ideas were proposed to minimize the computational costs of the simulation. The first idea involved incorporating an adaptive extrapolation scheme, whereas the second was a parallel implementation of the simulation procedure. The adaptive extrapolation was incorporated to optimize the selection of the extrapolation factor while ensuring stability in the simulation. Two additional procedures were also examined. One of these was called the cycle-update procedure, and the other was referred to as the intermediate cycle-update procedure. It was found that the step-update approach was computationally cheaper than the intermediate cycle-update procedure. The reason for this is that the intermediate cycle-update procedure is a less stable procedure (due to the reduced number of geometry updates in a cycle) and thus required the use of smaller extrapolation sizes. This resulted in a longer simulation time. The parallel implementation of intermediate cycle-update procedure proved to be the cheapest in terms of computational cost. It may be deduced that in the absence of parallel computing resources the most reasonable simulation procedure to use would be the step-update procedure with the adaptive extrapolation.

Although the wear depth on the pin predicted by the simulation procedures was not far off from the true value, they were under predicted. A possible reason for this is the inaccuracy in the wear coefficient that was used. The wear model used is a phenomenological model in which the wear coefficient is determined through experiments. Hence an inaccuracy in this coefficient has a great effect on the prediction process. Based on the results it is concluded that the procedure is a reasonable way to predict wear on bodies experiencing oscillatory contact.

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