AIAA JOURNAL
Vol. 48, No. 10, October 2010

Effects of Structural Tests on Aircraft Safety

Erdem Acar*
TOBB University of Economics and Technology, Ankara 06560, Turkey
and
Raphael T. Haftka† and Nam H. Kim‡
University of Florida, Gainesville, Florida 32611

DOI: 10.2514/1.J050202

This paper presents a methodology to investigate the effects of structural tests on aircraft safety. In particular, the paper focuses on the effect of the number of coupon tests and structural element tests on the final distribution of failure stress. The mean failure stress is assumed to be predicted by a failure criterion (e.g., Tsai-Wu), and the initial distribution of this mean failure stress reflects the uncertainty in the analysis procedure that uses coupon test data to predict structural failure. In addition to the uncertainty in the mean failure stress, there is also uncertainty in its variability due to the finite number of coupon tests. Bayesian updating is used to update the failure stress distribution based on results of the element tests. A Monte Carlo simulation of a large number of uncertainties and the possible test results are used to obtain the probability of structural failure in a certification test or in actual flight. Incorporating the Bayesian updating into the Monte Carlo simulation loop is computationally prohibitive; therefore, a surrogate procedure is devised to overcome the computational challenge. A structural design following the Federal Aviation Administration regulations is considered, and the tradeoffs between the number of tests and the weight and probability of failure in the certification test and in service are explored. To make this tradeoff analysis computationally affordable, response surface approximations are used to relate the knockdown factor to the probability of failure in service and in the certification test. It is found that it is possible to do a simultaneously probabilistic design and satisfy the Federal Aviation Administration regulations for deterministic design.

Nomenclature

A = load-carrying area of a small part of the overall structure
$A_{\text{built-av}}$ = fleet average value of the built area after element tests
$A_{\text{built-av-}c}$ = built value of the load-carrying area after coupon tests
A_{COV} = coefficient of variation of the load carrying area between different companies
b_t = bound of error in the design thickness e_t
b_w = bound of error in the design width e_w
c_{cf} = coefficient of variation of failure stress calculated from coupon tests
$(c_{ef})_{\text{calc}}$ = coefficient of variation of failure stresses calculated from coupon tests
$E()$ = expected value (i.e., mean value)
e_{ef} = error associated with failure criterion used while predicting failure in the structural element tests
e_f = error in predicting failure of the entire structure in certification or proof test
e_p = error in load calculation
e_t = error in the design thickness due to construction errors
e_{total} = total error
e_w = error in the design width due to construction errors
e_σ = error in stress calculation
k_B = tolerance limit factor
k_d = knockdown factor at coupon level due to use of conservative (B basis) material properties
k_f = additional knockdown factor at the structural level (nominal value is taken as 0.95 here)
k_{2d} = ratio between the unidirectional failure stress and the failure stress of a ply in a laminate under combined loading
n_c = number of coupon tests
n_e = number of element tests
P_{calc} = calculated design load
P_d = true design load based on the Federal Aviation Administration specifications (e.g., gust load specification)
P_f = probability of failure (in service)
S_F = Federal Aviation Administration load safety factor of 1.5
t = thickness of a small part of the overall structure
t_{design} = design thickness when all errors exist
v_t = variability in the built thickness
v_w = variability in the built width
w = width of a small part of the overall structure
w_{design} = design width when all errors exist
σ = stress in a small part of the overall structure
σ_a = allowable stress (B basis) of the entire structure
σ_{ca} = allowable stress (B basis) from coupon testing
σ_{cf} = failure stress from coupon testing
σ_{ea} = allowable stress (B basis) from element testing
σ_{ef} = failure stress of the structural element
$(\sigma_{ef})_{\text{calc}}$ = calculated (or predicted) element failure stress
$(\sigma_{ef})^{\text{test}}$ = element failure stress measured in tests
$(\sigma_{ef})^{\text{upd}}$ = updated value of the calculated (or predicted) element failure stress
$(\overline{\sigma}_{ef})^{\text{upd}}$ = most likely value of the updated distribution of mean failure stresses
σ_f = failure stress of the overall structure

Presented as Paper 2009-2265 at the 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Palm Springs, CA, 4–7 May 2009; received 10 September 2009; revision received 18 June 2010; accepted for publication 23 June 2010.

*Assistant Professor, Mechanical Engineering. Member AIAA.
†Distinguished Professor, Mechanical and Aerospace Engineering. Fellow AIAA.
‡Associate Professor, Mechanical and Aerospace Engineering. Member AIAA.

Subscripts

built-av = average built value that differs from the design value due to errors in construction
built-var = actual built value that differs from the average built value due to variabilities in construction
calc = calculated (or predicted) value that differs from the design value due to errors in design
design = design value
true = true value (error-free value)

I. Introduction

THE safety of aircraft structure can be achieved by designing the structure against uncertainty and by taking steps to reduce the uncertainty. Safety factors and knockdown factors are examples of measures used to compensate for uncertainty during the design process. For instance, uncertainty due to the limited number of coupon and element tests is often considered by a knockdown factor, which reduces the measured/calculated failure stress to compensate for the uncertainty. Uncertainty reduction measures (URMs), on the other hand, may be employed during the design process or later on throughout the operational lifetime. Examples of URMs for aircraft structural systems include structural testing, quality control, inspection, health monitoring, maintenance, and improved structural analysis and failure modeling.

In reliability-based design, all uncertainties that are available at the design stage are considered in calculating the reliability of the structure. However, the actual aircraft is much safer, because after design, it is customary to engage in vigorous uncertainty reduction activities using various URMs. However, so far, these URMs are heuristically applied without knowing their contributions to the structural reliability. It would be, therefore, beneficial to quantify the contribution of URMs to reliability and to include the effects of these planned URMs in the design process. It may even be advantageous to design the URMs together with the structure, trading off the cost of more weight against additional tests or more refined analytical simulations. To incorporate URMs into the design process, it is important to model their effect on the safety of the system, which is a challenging task, because many URMs will occur in the future. In this paper, the effect of structural tests (coupon tests and element tests) on structural reliability is investigated. In the building block testing framework [1], the former is performed before design, while the latter is performed after design. If a structural element fails the element test, redesign is required to satisfy the element test.

In reliability-based design, the contribution of each URM can be represented by a distribution. In the case of predesign tests (e.g., coupon tests) to estimate failure stress, since the tests have already been performed, the distribution of failure stress is known in terms of distribution parameters (e.g., mean and variance). In the case of future tests (e.g., element tests), however, since the tests are not yet performed, the distribution parameters are uncertain, and the same is true for other future URMs. In this case, these parameters should be considered as random variables. Then, a future URM can be modeled as a distribution of distributions.

There have been numerous studies [e.g., [2–16]] on computing safety of aircraft and their components. In these studies, first, all the uncertainties are quantified and assumed to be fixed. Then the reliability of the aircraft structure is computed on the basis of these fixed uncertainties. These studies did not consider the effects of URMs. Recently, several attempts have been made in considering the effect of URMs on safety and reliability. For instance, Dhillon et al. [17] noted for industrial robots that there has been substantial work on incorporating URMs into safety and reliability evaluation. Similarly, the authors showed that quality control [18], improved failure-prediction models [19,20] can lead to substantial reduction of failure probability of aerospace structures. More recently, Li et al. [21] considered systems that have interval uncertainty in their inputs. They developed a multiobjective optimization model to obtain optimal reduction of parameter uncertainty that provide the maximum improvement in system performance with the least amount of investment.

There are few papers in the literature that address the effect of tests on structural safety. Jiao and Moan [22] investigated the effect of proof tests on structural safety using Bayesian updating. They showed that the proof testing reduces the uncertainty in the strength of a structure, thereby leading to substantial reduction in probability of failure. Jiao and Eide [23] explored the effects of testing, inspection, and repair on the reliability of offshore structures. Beck and Katafygiotis [24] addressed the problem of updating a probabilistic structural model using dynamic test data from structure by using Bayesian updating. Similarly, Papadimitriou et al. [25] used Bayesian updating within a probabilistic structural analysis tool to compute the updated reliability of a structure using test data. They found that the reliabilities computed before and after updating were significantly different. We aim to extend the work of these earlier authors in simulating all possible outcomes of future tests, which would allow the designer to design the tests together with the structure.

The objectives of the present paper are 1) to explore modeling the effects of past and future tests on reducing the uncertainty and narrowing the probability distribution of uncertainty in structural failure predictions and 2) to study their effects on structural design. Since the distribution type of failure stress is unknown a priori, we use one of the most general distribution types, the Johnson distribution [26], which can be represented by four quantiles. As mentioned previously, since the distribution parameters of future tests are uncertain, the four quantiles of failure stress distribution are modeled as normal distributions. Then, the uncertainty of these four quantiles will depend on the number of tests; more tests will reduce the variance of quantile distributions. Then, the critical information for tradeoff analysis will be how much uncertainty can be reduced by a given number of future tests.

In the paper, we investigate in particular the effect of the number of coupon and future element tests on the final distribution of the failure stress. It is assumed that the mean value of the failure stress (mean over a large number of aircraft) is obtained from a failure criterion (e.g., Tsai-Wu theory [27]) using the results of coupon tests. The initial uncertainty in this mean failure stress reflects the confidence of the analytical model in this prediction. The Bayesian updating technique is then used to update the mean failure stress distribution from the possible results of the future element tests. In addition, there is the variability of the failure stress from one aircraft to another or from one structural component to another.

Finally, we consider structural design following the Federal Aviation Administration (FAA) regulations. The FAA regulations (FARs) state the use of a load safety factor of 1.5, conservative material properties (using *B*-basis allowables) and conservative design practice at each level (e.g., using knockdown factors smaller than one). We show tradeoffs between the number of tests and the weight of the structure for a given probability of failure. These could provide tradeoffs between additional tests and heavier weight, depending on the cost of testing and the cost of carrying the additional weight.

Our earlier research on this subject includes investigations of the effects of explicit and implicit safety measures [28,29] and effects of URMs [18–20] on aircraft structural safety. In these studies, the effects of coupon tests and the certification test are included in the analysis, while element tests are not considered. In addition, the effect of element tests on failure stress distribution is analyzed using Bayesian updating [30]. The main contributions of this paper are the following:

1) We propose a methodology to investigate the effects of future tests on safety.

2) We devise a Monte Carlo simulation (MCS) procedure to make it computationally affordable. For that purpose, Bayesian updating is not directly integrated to the main reliability assessment loop but, rather, performed aside. In addition, response surface approximations (RSAs) are used to relate the knockdown factor to the probability of failure in service and in the certification test.

The paper is organized as follows. Section II presents a list of assumptions, simplifications, and limitations of this work. Section III discusses the safety measures taken during aircraft structural design. Section IV presents a simple uncertainty classification that distinguishes uncertainties that affect an entire fleet (errors) from the uncertainties that vary from one aircraft to another in the same fleet (variability). Section V discusses the modeling of errors and variability throughout the design and testing of an aircraft. Section VI discusses probability of failure estimation via MCSs. Finally, the results and the concluding remarks are given in the last two sections of the paper, respectively.

II. Assumptions, Simplifications, and Limitations

The major assumptions, simplifications, and limitations of this study can be listed as follows:

1) A small region in a small structural part of an aircraft is considered for design. It is assumed that the small region can be characterized by a thickness and a width.

2) The small region is assumed to be designed against a static point stress failure. Other failure mechanisms (e.g., fatigue, corrosion, etc.) are not considered.

3) Safety measures for protection against uncertainties are restricted to the use of a load safety factor and conservative material properties, while other measures, such as redundancy, are left out.

4) Uncertainty analysis is simplified by classifying uncertainties into two parts: errors and variability.

5) It is very rare to have data on the probability distribution of errors. The errors are assumed to follow uniform probability distribution with known bounds based on experience. The uniform distribution is based on the principle of maximum entropy.

6) It is assumed that the aircraft companies can predict stresses very accurately so that error in stress prediction is taken as zero.

7) The probability distributions of variabilities are also selected based on previous work [19,20,28–30].

8) The quantiles of the mean failure stress distribution are assumed to follow normal distribution.

9) It is assumed that a large number of nominally identical aircraft are designed by many aircraft companies (e.g., Airbus, Boeing, Embraer, Bombardier, etc.), with the errors being fixed for each aircraft.

10) Aircraft companies are assumed to follow conservative design practices at each stage of the design process. We assumed that these conservative practices can be simulated by using an additional knockdown factor k_f over the FARs. The nominal value of k_f is taken as 0.95.

11) Every aircraft in a fleet is assumed to experience limit load throughout its service life. This assumption will lead to very conservative failure probability estimations.

12) The mean failure stress is assumed to be predicted by a failure criterion using the results of coupon tests.

13) The structural test pyramid that has many layers (e.g., coupon tests, element tests, part tests, subassembly tests, assembly tests, and the certification test) is simplified to a three-level test pyramid composed of coupon tests, element tests, and the certification test only.

14) The nominal value for the material coupon tests is assumed to be 50.

15) The nominal value for the structural element tests is assumed to be 3.

16) For the redesign of elements based on test results, we could not find published data, so we devised a common sense approach. We assumed that, if the *B*-basis value obtained after element tests σ_{ea} is more than 5% higher than the *B*-basis value obtained from coupon tests σ_{ca}, then the load-carrying area is reduced. If the *B*-basis value obtained after element tests is more than 2% lower than the *B*-basis value obtained from coupon tests, the load-carrying area is increased.

17) It is assumed that structural element tests are conducted for a specified combination of loads corresponding to critical loading such that the failure surface is boiled down to a single failure stress value. Note that the failure surface is a surface in the space of stresses defining the failure state. When the stress state lies on the failure surface, the material is assumed to fail.

18) The information from the element tests is assumed to be used to update the failure stress distribution by performing Bayesian updating. In practice, simpler procedures are used (such as selecting the lowest value of test results), so this assumption will tend to overestimate the beneficial effect of element tests.

III. Safety Measures

The safety of aircraft structures is achieved by designing these structures to operate well in the presence of uncertainties and taking steps to reduce the uncertainties. The following gives a brief description of these safety measures.

A. Safety Measures for Designing Structures Under Uncertainties

1. Load Safety Factor

In transport aircraft design, FARs state the use of a load safety factor of 1.5 (FAR 25.303 [31]). That is, aircraft structures are designed to withstand 1.5 times the limit load without failure.

2. Conservative Material Properties

To account for uncertainty in material properties, FARs state the use of conservative material properties (FAR 25.613 [32]). The conservative material properties are characterized as *A*-basis or *B*-basis material property values. Detailed information on these values is provided in volume 1, chapter 8 of the Composite Materials Handbook [1]. In this paper, we use *B*-basis values. The *B*-basis value is determined by calculating the value of a material property exceeded by 90% of the population with 95% confidence. The basis values are determined by testing a number of coupons selected randomly from a material batch. In this paper, the nominal number of coupon tests is taken to be 50. In Sec. VII, the effect of the number of coupon tests will be explored.

Other measures, such as redundancy, are not discussed in this paper.

B. Safety Measures for Reducing Uncertainties

Improvements in accuracy of structural analysis and failure prediction of aircraft structures reduce errors and enhance the level of safety. These improvements may be due to better modeling techniques developed by researchers, more detailed finite-element models made possible by faster computers, or more accurate failure theories. Similarly, the variability in material properties can be reduced through quality control and improved manufacturing processes. Variability reduction in damage and aging effects is accomplished through inspections and structural health monitoring. The reader is referred to the papers by Qu et al. [18] for effects of variability reduction, Acar et al. [19] for effects of error reduction, and Acar et al. [20] for effects of reduction of both error and variability.

In this paper, we focus on error reduction through aircraft structural tests, while the other URMs are left out for future studies. Structural tests are conducted in a building block procedure (volume I, chapter 2 of [1]). First, individual coupons are tested to estimate the probability distribution of failure stress. The mean structural failure is estimated based on failure criteria (such as Tsai-Wu), and this estimate is further improved using element tests. Then a subassembly is tested, followed by a full-scale test of the entire structure. In this paper, we use the simplified three-level test procedure depicted in Fig. 1. The coupon tests, the structural element tests, and the final certification test are included.

The first level is the coupon tests, where coupons (i.e., material samples) are tested to estimate failure stress. The FAR 25-613 requires aircraft companies to perform "enough" tests to establish design values of material strength properties (*A*-basis or *B*-basis value). As the number of coupon tests increases, the errors in the assessment of the material properties are reduced. However, since testing is costly, the number of coupon tests is limited to about 100 to 300 for *A*-basis calculation and at least 30 for *B*-basis value calculation.

At the second level of testing, structural elements are tested. The main target of element tests is to reduce errors related to failure theories (e.g., Tsai-Wu) used in assessing the failure load of the

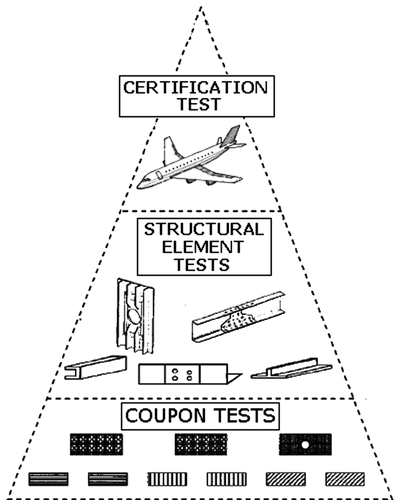


Fig. 1 Simplified three-level tests.

structural elements. In this paper, the nominal number of structural element tests is taken as three.

At the uppermost level, certification (or proof) testing of the overall structure is conducted (FAR 25-307 [33]). This final certification or proof testing is intended to reduce the chance of failure in flight due to errors in the structural analysis of the overall structure (e.g., errors in finite-element analysis and errors in failure mode prediction). While failure in flight often has fatal consequences, certification failure often has serious financial implications. So we measure the success of the URMs in terms of their effect (a reduction or an improvement) on the probability of failure in flight and in terms of their effect on the probability of failure in certification test (PFCT). For instance, structural tests lead to a narrower spread of the probability density function (PDF) of the mean failure stress, thereby reducing the probability of failure in service and in the certification test.

IV. Structural Uncertainties

A good analysis of different sources of uncertainty in engineering simulations is provided by Oberkampf et al. [34,35]. To simplify the analysis, we use a classification that distinguishes between errors (uncertainties that apply equally to the entire fleet of an aircraft model) and variability (uncertainties that vary for the individual aircraft) as we used in earlier studies [28,29]. The distinction, presented in Table 1, is important because safety measures usually target either errors or variability. While variabilities are random uncertainties that can be readily modeled probabilistically, errors are fixed for a given aircraft model (e.g., Boeing 737-400), but they are largely unknown. Since errors are epistemic, they are often modeled using fuzzy numbers or possibility analysis [36,37]. We model errors probabilistically by using uniform distributions, because these distributions correspond to minimum knowledge or maximum entropy.

Errors are uncertain at the time of the design, but they will not vary for a single structural component on a particular aircraft, while the variabilities vary for individual structural components. To model errors, we assume that we have a large number of nominally identical aircraft being designed (e.g., Airbus, Boeing, Embraer, Bombardier, etc.), with the errors being fixed for each aircraft.

V. Modeling Errors and Variability

To compute the probability of failure of aircraft structures, it is required to simulate coupon tests, element tests, and the certification test where the errors and variability must be carefully introduced. At the coupon level, we have errors in estimating material strength properties from coupon tests, due to limited number of coupon tests. At the element level, we have errors in structural element strength predictions due to the inaccuracy of the failure criterion used. At the full-scale structural level, we have errors in structural strength predictions, error in load calculation, and error in construction. Similarly, we have variability in loading, geometry, and failure stress. The following subsections describe modeling of these errors and variability in detail.

A. Errors in Estimating Material Strength Properties from Coupon Tests

Coupon tests are conducted to obtain the statistical characterization of material strength properties, such as failure stress, and their corresponding design values (A basis or B basis). With a finite number n_c of coupon tests, the statistical characterization involves errors. Therefore, the calculated values of the mean and the standard deviation of the failure stress will be uncertain. We assume that the failure stress follows a normal distribution, so the calculated mean also follows normal distribution. In addition, when n_c is larger than 25, the distribution of the calculated standard deviation tends to be normal. Then, the calculated failure stress can be expressed as

$$(\sigma_{cf})_{\text{calc}} = \text{normal}\ [(\bar{\sigma}_{cf})_{\text{calc}}; \qquad \text{std}(\sigma_{cf})_{\text{calc}}] \tag{1}$$

where the calculated mean and the calculated apparent standard deviation can be expressed using the first-order Taylor series as

$$(\bar{\sigma}_{cf})_{\text{calc}} = \text{normal}\ \left(\bar{\sigma}_f; \frac{\text{std}(\sigma_f)}{\sqrt{n_c}}\right) \tag{2}$$

$$\text{std}(\sigma_{cf})_{\text{calc}} = \text{normal}\ \left(\text{std}(\sigma_f)\sqrt{\frac{1+\sqrt{(n_c-3)/(n_c-1)}}{2}} \right. \\ \left. \text{std}(\sigma_f)\sqrt{\frac{1-\sqrt{(n_c-3)/(n_c-1)}}{2}}\right) \tag{3}$$

where $\bar{\sigma}_f$ and $\text{std}(\sigma_f)$ are, respectively, the true values of the mean and standard deviation of failure stress. Note that Eqs. (1–3) describe a random variable coming from a distribution (normal) for which the parameters are also random. In this paper, this will be referred to as a distribution of distributions.

The allowable stress at the coupon level, σ_{ca}, is computed from the mean failure stress calculated at the coupon level, $(\bar{\sigma}_{cf})_{\text{calc}}$, by using a knockdown factor k_d as

$$\sigma_{ca} = k_d(\bar{\sigma}_{cf})_{\text{calc}} \tag{4}$$

The knockdown factor k_d is specified by the FARs. For instance, for the B-basis value of the failure stress, 90% of the failure stresses (measured in coupon tests) must exceed the allowable stress with 95% confidence. The requirement of 90% probability and 95% confidence is responsible for the knockdown factor k_d in Eq. (4). For normal distribution, the knockdown factor depends on the number of coupon tests and the coefficient of variation (COV) of the failure stress as

Table 1 Uncertainty classification

Type of uncertainty	Spread	Cause	Remedies
Error (mostly epistemic)	Departure of the average fleet of an aircraft model (e.g., Boeing 737-400) from an ideal	Errors in predicting structural failure, construction errors, and deliberate changes	Testing and simulation to improve the mathematical model and the solution
Variability (aleatory)	Departure of an individual aircraft from fleet level average	Variability in tooling, manufacturing process, and flying environment	Improvement of tooling and construction, and quality control

$$k_d = 1 - k_B (c_{cf})_{\text{calc}} \tag{5}$$

where $(c_{cf})_{\text{calc}}$ is the COV of failure stress calculated from coupon tests, and k_B is called the tolerance limit factor. The tolerance limit factor k_B is a function of the number of coupon tests n_c, as given in [1] (volume 1, chapter 8, page 84) as

$$k_B \approx 1.282 + \exp\left(0.958 - 0.520 \ln(n_c) + \frac{3.19}{n_c}\right) \tag{6}$$

The variation of the tolerance limit factor with the number of coupon tests is depicted in Fig. 2. It is clear that the reduction in k_B (or increase in k_d) is relatively small for a large number of coupon tests, but the reduction of k_B is significant for a small number of coupon tests.

B. Errors in Structural Element Strength Predictions

The second level in the testing sequence is structural element testing, where structural elements are tested to validate the accuracy of the failure criterion used (e.g., Tsai-Wu). Here, we assume that structural element tests are conducted for a specified combination of loads corresponding to critical loading. For this load combination, the failure surface can be boiled down to a single mean failure stress $\bar{\sigma}_{ef}$, where the subscript e stands for structural element tests. The mean failure stress of the elements $\bar{\sigma}_{ef}$ can be predicted from the mean failure stress of the coupons $\bar{\sigma}_{cf}$ through

$$\bar{\sigma}_{ef} = k_{2d} \bar{\sigma}_{cf} \tag{7}$$

where k_{2d} is the ratio between the unidirectional failure stress and the failure stress of a ply in a laminate under combined loading. If the failure theory used to predict the failure was perfect, and we performed an infinite number of coupon tests, then we could predict k_{2d} exactly, and the actual value would vary only due to material variability. However, neither the failure theory is perfect nor the infinite tests performed, so the calculated value of k_{2d} will be

$$(k_{2d})_{\text{calc}} = (1 - e_{ef}) k_{2d} \tag{8}$$

where e_{ef} is the error in the failure theory. Note that the sign in front of the error term is negative, since we consistently formulate the error expressions such that a positive error implies a conservative decision. Then, the calculated value of the mean failure stress at the element level can be related to the calculated value of the mean failure stress at the coupon level via

$$(\bar{\sigma}_{ef})_{\text{calc}} = (k_{2d})_{\text{calc}} (\bar{\sigma}_{cf})_{\text{calc}} = (1 - e_{ef}) k_{2d} (\bar{\sigma}_{cf})_{\text{calc}} \tag{9}$$

Here, we take $k_{2d} = 1$ for simplicity. So we have

$$(\bar{\sigma}_{ef})_{\text{calc}} = (1 - e_{ef})(\bar{\sigma}_{cf})_{\text{calc}} \tag{10}$$

The initial distribution of $(\bar{\sigma}_{ef})_{\text{calc}}$ is obtained by estimating the distribution of the error e_{ef} and using the results of the coupon tests $(\bar{\sigma}_{cf})_{\text{calc}}$. The information from the element tests is used to update the failure stress distribution by performing Bayesian updating (see Appendix A for details). The initial distribution of $(\bar{\sigma}_{ef})_{\text{calc}}$ is uniform for a given set of coupon test results. In practice, simpler procedures are often used, such as selecting the lowest failure stress from element tests. Therefore, our assumption will tend to overestimate the beneficial effect of element tests.

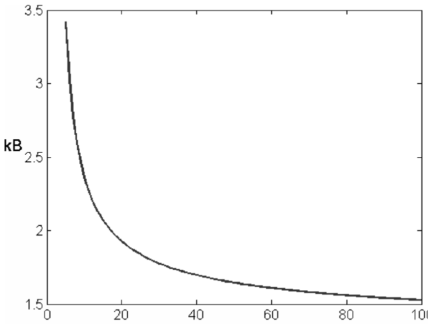


Fig. 2 Variation of the tolerance coefficient with the number of coupon tests.

If Bayesian updating was used directly within the main MCS loop for design load-carrying area determination, the computational cost would be very high. Instead, Bayesian updating is performed outside of the MCS loop for a range of possible test results. It is important to note that the error definition used in the Bayesian updating code is different from the error definition used in the MCS code. In the Bayesian updating code, the error is measured from the calculated values of the failure stress, $(\bar{\sigma}_f)_{\text{calc}}$, such that the true and the calculated values of the failure stress are related through $(\bar{\sigma}_f)_{\text{true}} = (1 + e_{ef-B})(\bar{\sigma}_f)_{\text{calc}}$. In the MCS code, on the other hand, the error is measured from the true value of the failure stress such that the true and the calculated values of the failure stress are related through $(\bar{\sigma}_f)_{\text{calc}} = (1 - e_{ef})(\bar{\sigma}_f)_{\text{true}}$. Therefore, while the Bayesian updating detailed in Appendix A is implemented, a random error e_{ef} generated in the main MCS code is transferred to $e_{ef-B} = [1/(1 - e_{ef})] - 1$ while running the Bayesian updating code. This complication reflects the fact that, in the MCS loop, we consider many possible element analysis and test results, while the engineer carrying the element tests has a unique set of computations and test results.

The allowable stress based on the element test is calculated from

$$\sigma_{ea} = k_d (\bar{\sigma}_{ef})_{\text{calc}}^{\text{updated}} \tag{11}$$

where $(\bar{\sigma}_{ef})_{\text{calc}}^{\text{updated}}$ is the most likely value (maximum PDF location) of the updated distribution of the mean failure stress, starting from the initial distribution in Eq. (10) and updated by n_e numbers of element tests. Recall that the initial distribution of $(\bar{\sigma}_{ef})_{\text{calc}}$ is obtained by the estimate of the error e_{ef} and using the results of coupon tests $(\bar{\sigma}_{cf})_{\text{calc}}$. Since the results of coupon tests are random, the initial distribution is also random, thereby the allowable stress σ_{ea} is also random.

Combining Eqs. (4), (10), and (11), we have

$$\sigma_{ea} = (1 - e_{ef}) \sigma_{ca} \tag{12}$$

Note that σ_{ea} is the updated value of the allowable stress. From this point on, updated distribution of the failure stress is used in our calculations. The superscript "updated" is dropped for convenience.

C. Errors in Structural Strength Predictions

Because of the complexity of the overall structural system, there will be additional errors in failure prediction of the overall structure, which is denoted by e_f. The calculated mean failure stress of the overall structure, $(\bar{\sigma}_f)_{\text{calc}}$, can be expressed in terms of the calculated mean failure stress of the structural element, $(\bar{\sigma}_{ef})_{\text{calc}}$, through

$$(\bar{\sigma}_f)_{\text{calc}} = (1 - e_f)(\bar{\sigma}_{ef})_{\text{calc}} \tag{13}$$

The allowable stress at the structural design level, σ_a, can be related to the allowable stress computed at the element level, σ_{ea}, through the following relation:

$$\sigma_a = k_f (1 - e_f) \sigma_{ea} \tag{14}$$

where k_f is an additional knockdown factor used at the structural level as an extra precaution. Here, the nominal value of k_f is taken 0.95. Combining Eqs. (12) and (14), we can obtain

$$\sigma_a = (1 - e_{ef})(1 - e_f) k_f \sigma_{ca} \tag{15}$$

D. Errors in Design

As noted earlier, along with the errors in failure stress predictions, there are also errors in design and construction. Before starting the structural design, aerodynamic analysis needs to be performed to determine the loads acting on the aircraft. However, the calculated design load value, P_{calc}, differs from the actual design load P_d under conditions corresponding to FAA design specifications (e.g., gust-strength specifications). Since each company has different design practices, the error in load calculation, e_p, is different from one company to another. The calculated design load P_{calc} is expressed in terms of the true design load P_d as

$$P_{\text{calc}} = (1 + e_P)P_d \tag{16}$$

Notice here that the sign in front of the load error term is positive while the sign in front of the failure stress error terms was negative. The reason for this choice is that we consistently formulate the error expressions such that a positive error implies a conservative decision.

Besides the error in load calculation, an aircraft company may also make errors in stress calculation. We consider a small region in a structural part, characterized by a thickness t and width w that resists the load in that region. The value of the stress in a structural part calculated by the stress analysis team, σ_{calc}, can be expressed in terms of the load values calculated by the load team P_{calc}, the design width w_{design}, and the thickness t of the structural part by introducing the term e_σ, representing error in the stress analysis:

$$\sigma_{\text{calc}} = (1 + e_\sigma)\frac{P_{\text{calc}}}{w_{\text{design}}t} \tag{17}$$

In this paper, we assume that the aircraft companies can predict stresses very accurately so that e_σ is negligible and is taken as zero. The calculated stress value is then used by a structural designer to calculate the design thickness t_{design}. That is, the design thickness can be formulated as

$$t_{\text{design}} = \frac{S_F P_{\text{calc}}}{w_{\text{design}}\sigma_a} = \frac{(1 + e_P)}{(1 - e_f)(1 - e_{ef})}\frac{S_F P_d}{w_{\text{design}}k_f\sigma_{ca}} \tag{18}$$

Then, the design value of the load-carrying area can be expressed as

$$A_{\text{design}} = t_{\text{design}}w_{\text{design}} = \frac{(1 + e_P)}{(1 - e_f)(1 - e_{ef})}\frac{S_F P_d}{k_f\sigma_{ca}} \tag{19}$$

E. Errors in Construction

In addition to the above errors, there will also be construction errors in the geometric parameters. These construction errors represent the difference between the values of these parameters in an average airplane (fleet average) built by an aircraft company and the design values of these parameters. The error in width e_w represents the deviation of the design width of the structural part, w_{design}, from the average value of the width of the structural part built by the company, $w_{\text{built-}av}$. Thus,

$$w_{\text{built-av}} = (1 + e_w)w_{\text{design}} \tag{20}$$

Similarly, the built thickness value will differ from its design value such that

$$t_{\text{built-av}} = (1 + e_t)t_{\text{design}} \tag{21}$$

Then, the built load-carrying area $A_{\text{built-av-}c}$ can be expressed using the first equality of Eq. (19) as

$$A_{\text{built-av-}c} = (1 + e_t)(1 + e_w)A_{\text{design}} \tag{22}$$

Note that the built load-carrying area computed in Eq. (22) is related to coupon tests through A_{design}, which is computed using the allowable stress value obtained from coupon tests [see Eq. (19)].

Table 2 presents nominal values for the errors assumed here. It is very rare to have data on the probability distribution of errors. Instead, analysts typically estimate error ranges based on experience. Consequently, the errors here are modeled by uniform distributions, following the principle of maximum entropy when only the range of the error is known. For instance, the error in the built thickness of a structural part (e_t) is defined in terms of the error bound $(b_t)_{\text{built}}$ using

Table 2 Distribution of error terms and their bounds

Error factors	Distribution type	Mean	Bounds, %
Error in load calculation, e_P	Uniform	0.0	±10
Error in width, e_w	Uniform	0.0	±1
Error in thickness, e_t	Uniform	0.0	±3
Error in failure prediction, e_f	Uniform	0.0	±10
Error in failure prediction, e_{ef}	Uniform	0.0	±10

$$e_t = \text{uniform}\ [-(b_t)_{\text{built-av}}, \quad +(b_t)_{\text{built-av}}] \tag{23}$$

It is seen that the error e_t has uniform distribution and the error bound is $(b_t)_{\text{built-av}} = 0.03$. Hence, the lower bound for the thickness value is the average value minus 3% of the average, and the upper bound for the thickness value is the average value plus 3% of the average.

F. Total Error e_{total}

The expression for the built load-carrying area of a structural part computed based on coupon test results, $A_{\text{built-av-}c}$, can be reformulated by combining Eqs. (19) and (22) as

$$A_{\text{built-av-}c} = (1 + e_{\text{total}})\frac{S_F P_d}{k_f\sigma_{ca}} \tag{24}$$

where

$$e_{\text{total}} = \frac{(1 + e_P)(1 + e_t)(1 + e_w)}{(1 - e_f)(1 - e_{ef})} - 1 \tag{25}$$

Here, e_{total} represents the cumulative effect of the individual errors on the load-carrying capacity of the structural part.

G. Redesign Based on Element Tests

Besides updating the failure stress distribution, element tests have an important role of leading to design changes if the design is unsafe or overly conservative. That is, if very large or very small failure stress values are obtained from the element tests, the company may want to increase or reduce the load-carrying area of the elements. We did not find published data on redesign practices, and so we devised a common sense approach. We assumed that if the B-basis value obtained after element tests, σ_{ea}, is more than 5% higher than the B-basis value obtained from coupon tests, σ_{ca}, then the load-carrying area is reduced. If the B-basis value obtained after element tests is more than 2% lower than the B-basis value obtained from coupon tests, the load-carrying area is increased. This lower tolerance reflects the need for safety. In both cases, the area is scaled up or down by σ_{ca}/σ_{ea}. Otherwise, no redesign is performed. The built load-carrying area can be revised by multiplying Eq. (24) by a redesign correction factor c_r as

$$A_{\text{built-av}} = c_r A_{\text{built-av-}c} = (1 + e_{\text{total}})c_r\frac{S_F P_d}{k_f\sigma_{ca}} \tag{26}$$

where

$$c_r = 1 \quad \text{(no redesign)}$$

$$c_r = \frac{1.01}{\text{CF}}, \qquad \text{CF} = \frac{\sigma_{ea}}{\sigma_{ca}} \quad \text{(redesign)} \tag{27}$$

Since redesign requires new elements to be built and tested, it is costly. Therefore, we do not model a second round of redesign. To protect against uncertainties in the test of the redesigned element, we have an additional 1% reduction in the calculated allowable value [see term 1.01 in Eq. (27)].

H. Variability

In the previous sections, we analyzed the errors made in the design and construction stages, representing the differences between the fleet-average values of geometry, material and loading parameters, and their corresponding design values. For a given design, these parameters vary from one aircraft to another in the fleet due to variability in tooling, construction, flying environment, etc. For instance, the actual value of the thickness of a structural part, $t_{\text{built-var}}$, is defined in terms of its fleet-average built value, $t_{\text{built-av}}$, by

$$t_{\text{built-var}} = (1 + v_t)t_{\text{built-av}} \tag{28}$$

We assume that v_t has a uniform distribution with 3% bounds (see Table 3). Then, the actual load-carrying area $A_{\text{built-var}}$ can be defined as

$$A_{\text{built-var}} = t_{\text{built-var}} w_{\text{built-var}} = (1 + v_t)(1 + v_w)A_{\text{built-av}} \tag{29}$$

where v_w represents the effect of the variability on the fleet-average built width.

Table 3 presents the assumed distributions for variabilities. Note that the thickness error in Table 2 is uniformly distributed with bounds of $\pm 3\%$. Thus, the maximum difference between all thicknesses over the fleets of all companies is $\pm 6\%$. However, the combined effect of the uniformly distributed error and variability is not uniformly distributed. Note here that, for the loading, we make a very conservative assumption that every aircraft in a fleet will experience limit load throughout its service life. This will lead to very conservative failure probability estimations, as will be seen in the Sec. VII.

I. Certification Test

After a structural part has been built with random errors in stress, load, width, allowable stress, and thickness, it may fail in certification testing. Recall that the structural part will not be manufactured with complete fidelity to the design due to variability in the geometric properties. That is, the actual values of these parameters $w_{\text{built-var}}$ and $t_{\text{built-var}}$ will be different from their fleet-average values $w_{\text{built-av}}$ and $t_{\text{built-av}}$. The structural part is then loaded with the design axial force of S_F times P_{calc}, and if the stress exceeds the failure stress of the structure σ_f, then the structure fails and the design is rejected; otherwise, it is certified for use. That is, the structural part is certified if the following inequality is satisfied:

$$\sigma - \sigma_f = \frac{S_F P_{\text{calc}}}{(1 + v_t)(1 + v_w)A_{\text{built-av}}} - \sigma_f \leq 0 \tag{30}$$

Table 3 Distribution of random variables having variability

Variables	Distribution type	Mean	Scatter
Actual service load, P_{act}	Lognormal	$P_d = 2/3$	10% COV
Actual built width, $w_{\text{built-var}}$	Uniform	$w_{\text{built-av}}$	1% bounds
Actual built thickness, $t_{\text{built-var}}$	Uniform	$t_{\text{built-av}}$	3% bounds
Failure stress, σ_f	Normal	1.0	8% COV
v_w	Uniform	0	1% bounds
v_t	Uniform	0	3% bounds

VI. Probability of Failure Calculation

To calculate the probability of failure, we first incorporate the statistical distributions of errors and variability in a MCS. Errors are uncertain at the time of design, but they do not change for individual realizations (in actual service) of a particular design. On the other hand, all individual realizations of a particular design are different from each other due to variability. The simulation of error and variability can be easily implemented through a two-level separable MCS [29]. At the upper level, different aircraft companies can be simulated by assigning random errors to each and, at the lower level, we simulated variability in dimensions, material properties, and loads related to manufacturing variability, and variability in service conditions can be simulated. The details of the separable MCS are provided in Appendix B.

The effect of element tests on failure stress distribution is modeled using Bayesian updating. If Bayesian updating was used directly within an MCS loop of probability of failure calculation, the computational cost would be very high. In this paper, instead, Bayesian updating is performed aside in a separate MCS before starting with the MCS loop (the details of which are provided in Table 4). The procedure followed for Bayesian updating can be described briefly as follows. First, the four quantiles of the mean failure stress are modeled as normal distributions. Then, these quantiles are used to fit a Johnson distribution to the mean failure stress. That is, the mean failure stress is represented as a Johnson distribution, for which the parameters are themselves distributions that depend on the number of element tests as well as the error in failure stress prediction of the elements, e_{ef}. Finally, Bayesian updating is used to update the mean failure stress distribution. Details of this procedure are provided in Appendix A.

VII. Results

In this section, the effects of the number of coupon tests, the number of element tests, the redesign of element tests, and the certification test are analyzed. The tradeoffs between the number of tests, weight, and probabilities of failure in certification tests and in service are explored. To make this tradeoff analysis computationally affordable, RSAs are used to relate the knockdown factor to weight and probabilities of failure in service and in the certification test. For any combination of the number of coupon and element tests, response surfaces (RSs) are constructed that take input as k_f and provide prediction for $A_{\text{built-av}}$ or the reliability index of P_f or the

Table 4 MCS procedure for probability of failure calculation

1. Compute the allowable stress based on coupon tests, σ_{ca}.
2. Calculate the built average load-carrying area using the results of coupon tests, $A_{\text{built-av-}c} = (1 + e_{\text{total}})[(S_F P_d/(k_f w_{\text{design}})](1/\sigma_{ca})$.
3. Generate random numbers for the quantiles of the updated mean failure stress (see Appendix A).
4. Calculate the B-basis value using the quantiles, σ_{ea}.
   a. Compute the bounds for mean failure stress $lb = 0.9(1 - e_{ef})$ and $ub = 1.1(1 - e_{ef})$.
   b. Compute the PDF of the mean failure stress having Johnson distribution within the bounds, and select the mean failure stress value with the highest PDF.
   c. Compute the B-basis value, $\sigma_{ea} = [1 - k_B(c_{cf})_{\text{calc}}](\bar{\sigma}_{ef})^{\text{updated}}_{\text{calc}}$.
5. Compute a correction factor (CF) for the B-basis value, $\text{CF} = \frac{\sigma_{ea}}{\sigma_{ca}}$. Limit the value of CF to the interval [0.9, 1.1]. That is, if $\text{CF} < 0.9$, then $\text{CF} = 0.9$. If $\text{CF} > 1.1$, then $\text{CF} = 1.1$.
6. Revise the built average load-carrying area based on the value of CF.
   a. If $\text{CF} < 0.98$, then redesign is needed. We will increase the load-carrying area by CF. Hence, the new load-carrying area is $A_{\text{built-av}} = \frac{1.01}{\text{CF}} A_{\text{built-av-}c}$. Here, the factor 1.01 is used to avoid a second redesign of elements.
   b. If $0.98 \leq \text{CF} \leq 1.05$, then no redesign is needed. So, the load-carrying area is $A_{\text{built-av}} = A_{\text{built-av-}c}$.
   c. If $\text{CF} > 1.05$, then redesign is needed. We will decrease the load-carrying area by CF. Hence, the new load-carrying area is $A_{\text{built-av}} = \frac{1.01}{\text{CF}} A_{\text{built-av-}c}$. Here again, the factor 1.01 is used to avoid a second redesign of elements.
7. Using $A_{\text{built-av}}$, compute the probability of failure in service (P_f) and PFCT. See Appendix A.

reliability index of PFCT. It should also be noted that probability distributions, the element test redesign procedure, and some selected parameters are based on our engineering judgment rather than published data. These assumptions have certain effects on the computed probabilities of failure in service and in the certification test.

A. Effect of the Number of Coupon Tests

The effects of the number of coupon tests on the built average load-carrying area and the probability of failure are presented in Table 5. For each aircraft company, we have a distribution for the load carrying area. $A_{\text{built-av}}$ is the mean of that distribution, and A_{COV} is the coefficient of variation of that distribution. Since we model multiple aircraft companies, we have distributions for each. The area values provided in Table 5 are based on the load and material property values given in Table 3. As the number of coupon tests increases, both the mean area ($A_{\text{built-av}}$) is reduced (since the B-basis knockdown factor k_d is increased), and the COV of the area (A_{COV}) is reduced (since the COV of the B-basis value is reduced). These two reductions have an opposing effect on the probability of failure and the PFCT. However, the net effect is that both probabilities of failure increase. This reflects the fact that the knockdown factor used by the FAA to compensate for a small number of coupon tests [Eq. (6)] is conservative, so performing more tests actually makes the average aircraft less safe. The COV of the area represents the variation between companies, each having different errors in their designs At over 9%, it is substantial, indicating that the variation of the probability of failure between companies can be large.

To provide an indication of the accuracy of the numbers in Table 5, the numbers in the parentheses show the COV of the corresponding value computed from five MC simulations with different seeds. It is seen that the mean built area calculations are accurate to the fourth digit, while the probability estimations are only accurate to the second digit.

Since $A_{\text{built-av}}$ reduces as the number of coupon tests increases, the aircraft builder may decide to keep $A_{\text{built-av}}$ constant. This can be achieved by adjusting the knockdown factor k_f in Eq. (14) so as to have the same $A_{\text{built-av}}$ for a different number of coupon tests. First, the knockdown factor k_f is varied by −10, −5, 5, and 10% of its nominal value, and simulations are performed. Then, as noted earlier, RSs are constructed for the built average area ($A_{\text{built-av}}$), the reliability index of the probability of failure (P_f), and the reliability index of PFCT. That is, to obtain the results in Tables 6–9, the number of element tests is set to three (its nominal value). Three different values are used for the number of coupon tests (n_c) as 30, 50, and 80. For each value of n_c, three different RSs are constructed that take input as k_f and provide prediction for $A_{\text{built-av}}$ or the reliability index of P_f or the reliability index of PFCT. Finally, P_f and PFCT values corresponding to $A_{\text{built-av}} = 1.238$ are computed. This practice also reduces the numerical noise in simulation results.

Table 6 shows that increasing the number of coupon tests from 50 to 80 leads to a 3.5% reduction in the probability of failure, whereas reducing the number of coupon tests to 30 increases the probability of failure by 5.7%. It can also be concluded that increasing the number of coupon tests reduces the probability of failure for the same weight (i.e., the same $A_{\text{built-av}}$), but the rate of reduction diminishes with the number of tests. The second column of Table 6 shows that k_f values are all smaller than 1.0, so the FAA deterministic design regulations are not violated. Note that the P_f and PFCT results in Table 6 are RS predictions rather than simulation results. To evaluate the accuracy of the RSs, the RS predictions of $A_{\text{built-av}}$, P_f, and PFCT are compared with the MCS results for these responses. MCS are repeated by five times with different seeds, and the average values are used in comparison. Table 7 shows that the RS predictions for all the responses are quite accurate, the most accurate being the prediction of $A_{\text{built-av}}$. In addition, Table 7 provides R^2 values for the constructed RSs as another measure of accuracy.

Table 8 shows the change of the built average area with the number of coupon tests for the same probability of failure. If the number of coupon tests is reduced from 50 to 30, the built average load-carrying area is increased by 0.23%. On the other hand, if the number of coupon tests is increased from 50 to 80, the built average load-carrying area decreases by 0.15%. The second column of Table 8 shows that k_f values are all smaller than 1.0, so the FAA deterministic design regulations are not violated.

Table 9 shows the change of the built average area with the number of coupon tests for the same PFCTs. If the number of coupon tests is

Table 5 Effects of the number of coupon tests[a]

n_c	$A_{\text{built-av}}$	A_{COV}	P_f ($\times 10^{-4}$)[b]	PFCT
30	1.252 (0.03%)	0.0945 (0.2%)	0.507 (0.9%)	0.0347 (2.3%)
50	1.238 (0.03%)	0.0931 (0.2%)	0.634 (1.7%)	0.0414 (2.1%)
80	1.229 (0.04%)	0.0922 (0.2%)	0.722 (1.4%)	0.0464 (1.2%)

[a]The numbers in parentheses show the COV of the corresponding value with repeated MCSs. The number of element tests, n_e, is three. The redesign of the element tests and the certification test are included in the analysis.
[b]P_f is the mean value of the probabilities of failure over multiple aircraft companies.

Table 6 Effects of the number of coupon tests for the same weight[a]

n_c	k_f	$A_{\text{built-av}}$	P_f ($\times 10^{-4}$)[b]	PFCT
30	0.961	1.238	0.666	0.0430
50	0.950	1.238	0.630	0.0412
80	0.942	1.238	0.608	0.0402

[a]The number of element tests, n_e, is three. The redesign of the element tests and the certification test are included in the analysis.
[b]P_f is the mean value of the probabilities of failure over multiple aircraft companies.

Table 7 Evaluating the accuracy of RSs for designs in Table 6

n_c	k_f	Result	$A_{\text{built-av}}$	P_f ($\times 10^{-4}$)[a]	PFCT
30	0.961	RS prediction	1.238	0.666	0.0430
		MCS	1.238	0.665	0.0430
		R^2	0.9995	0.9982	0.9912
50	0.950	RS prediction	1.238	0.630	0.0412
		MCS	1.238	0.634	0.0414
		R^2	0.9997	0.9994	0.9984
80	0.942	RS prediction	1.238	0.608	0.0402
		MCS	1.238	0.611	0.0405
		R^2	0.9989	0.9981	0.9973

[a]P_f is the mean value of the probabilities of failure over multiple aircraft companies.

Table 8 Effects of the number of coupon tests for the same probability of failure[a]

n_c	k_f	$A_{\text{built-av}}$	Change in area	P_f ($\times 10^{-4}$)[b]	PFCT
30	0.958	1.241	0.23	0.630	0.0412
50	0.950	1.238	——	0.630	0.0412
80	0.945	1.236	−0.15	0.630	0.0414

[a]The number of element tests, n_e, is three. The redesign of the element tests and the certification test are included in the analysis.
[b]P_f is the mean value of the probabilities of failure over multiple aircraft companies.

Table 9 Effects of the number of coupon tests for the same PFCT[a]

n_c	k_f	$A_{\text{built-av}}$	Change in area	P_f ($\times 10^{-4}$)[b]	PFCT
30	0.958	1.241	0.23	0.630	0.0412
50	0.950	1.238	——	0.630	0.0412
80	0.945	1.236	−0.13	0.627	0.0412

[a]The number of element tests, n_e, is three. The redesign of the element tests and the certification test are included in the analysis.
[b]P_f is the mean value of the probabilities of failure over multiple aircraft companies.

reduced from 50 to 30, the built average load-carrying area is increased by 0.23%. On the other hand, if the number of coupon tests is increased from 50 to 80, the built average load-carrying area decreases by 0.13%. Overall, it appears that increasing the number of coupon tests has only a small effect on the probability of failure in service or on the PFCT. The second column of Table 9 shows that k_f values are all smaller than 1.0, so the FAA deterministic design regulations are not violated. Comparing Tables 8 and 9, it can be seen that designing to a constant probability of failure or a constant PFCT are essentially the same, with the difference being mostly due to MCS noise.

B. Effect of the Number of Element Tests

The effects of increasing the number of element tests on the load-carrying area and probability of failure are presented in Table 10. If we employ redesign after the element tests, the mean load-carrying area is increased, since we introduced an additional 1% reduction in the calculated allowable value [see term 1.01 in Eq. (27)]. Increasing the number of element tests reduces both the average built load-carrying area as well as its COV. The effect on the COV is more significant because of the reduction of the error term e_{ef}.

To provide an indication of the accuracy of the numbers in Table 10, the numbers in the parentheses show the COV of the corresponding value computed from five MC simulations with different seeds. It is seen that the mean built area calculations are accurate to the fourth digit, while the probability estimations are only accurate to the second digit.

The effect of the number of element tests on the probability of failure in service and in the certification test is shown in Table 11. We see that increasing the number of element tests from three to five leads to a 12% reduction in the probability of failure, while reducing the number of element tests to one causes a 34% increase in probability of failure. Similar effects are observed on the probability of failure in certification. It appears that three element tests (typical of present practice) are a reasonable choice. The second column of Table 11 shows that k_f values are all smaller than 1.0, so the FAA deterministic design regulations are not violated.

To analyze the probability of failure and weight tradeoffs, the probability of failure can be fixed to a value, and the variation of the built average load-carrying area with number of element tests can be explored. Here, the probability of failure is fixed to 0.63×10^{-4}, which corresponds to performing three element tests and 50 coupon tests (the nominal values). Table 12 shows that, if we want to do away with element tests, then we will need to put 1.67% extra weight to achieve the same probability of failure. On the other hand, if we increase the number of element tests from three to five, we can save around 0.5% weight. The second column of Table 12 shows that k_f values are all smaller than 1.0, so the FAA deterministic design regulations are not violated.

The probability of failure in certification tests is high and likely a big motivator for the aircraft companies, hence we also investigate how much extra weight would be needed to maintain the PFCT if the company intends to eliminate the element tests. Table 13 shows that, if a company aims to eliminate the element tests, the structural weight must be increased by 1.5% to achieve to the same PFCTs. The second column of Table 13 shows that k_f values are all smaller than 1.0, so the FAA deterministic design regulations are not violated.

Section VII.A investigates the effects of the number of coupon tests on P_f and PFCT when the number of element tests is fixed to its nominal value of three. It is found that, if the number of coupon tests is increased to 80, the weight can be decreased by 0.15% for the same P_f (see Table 6). Similarly, this section investigates the effects of the number of element tests when the number of coupon tests is fixed to its nominal value of 50. It is found that, if the number of element tests is increased to five, the weight can be decreased by 0.52% for the same P_f (see Table 12). To provide a brief investigation for the interaction between the number of coupon tests and the number of element tests, we compute the value of k_f that leads to $P_f = 0.63 \times 10^{-4}$ (the P_f value for the nominal case). It is found that $k_f = 0.957$ results in $A_{\text{built-av}} = 1.2302$ leading to a weight reduction of 0.62%. It is seen that the combined effect of the coupon test and the element tests is smaller than the sum of the individual effects of the coupon tests and the element tests. A detailed analysis of the interaction between the coupon tests and the element tests is an interesting topic, which will be addressed is a future work.

C. Effect of the Certification Test

Finally, the effect of the certification test on the mean area and reliability are explored. Table 14 shows that if certification is not performed, then the average value of the built area is reduced by a small amount while its COV is increased significantly. Therefore, the probability of failure is increased by 22%. If the average

Table 10 Effect of the number of element tests[a]

n_e	$A_{\text{built-av}}$	A_{COV}	P_f ($\times 10^{-4}$)	PFCT
0	1.229 (0.01%)	0.1015 (0.3%)	1.111 (1.2%)	0.0628 (1.2%)
1	1.242 (0.03%)	0.0996 (0.2%)	0.792 (0.8%)	0.0484 (1.7%)
2	1.239 (0.04%)	0.0947 (0.2%)	0.670 (1.8%)	0.0435 (1.6%)
3	1.238 (0.03%)	0.0931 (0.2%)	0.630 (1.7%)	0.0412 (2.2%)
4	1.236 (0.02%)	0.0917 (0.3%)	0.601 (0.7%)	0.0404 (1.4%)
5	1.236 (0.04%)	0.0904 (0.07%)	0.575 (1.1%)	0.0390 (2.5%)

[a]The numbers in parentheses show the COV of the corresponding value with repeated MCSs. The number of coupon tests, n_c, is 50. The redesign of the element tests and the certification test are included in the analysis.

Table 11 Effects of the number of element tests for the same weight[a]

n_e	k_f	$A_{\text{built-av}}$	P_f ($\times 10^{-4}$)	PFCT
0	0.943	1.238	0.934	0.0549
1	0.953	1.238	0.846	0.0511
2	0.951	1.238	0.688	0.0444
3	0.950	1.238	0.630	0.0412
4	0.948	1.238	0.583	0.0395
5	0.948	1.238	0.556	0.0380

[a]The number of coupon tests, n_c, is 50. The redesign of the element tests and the certification test are included in the analysis.

Table 12 Effects of the number of element tests for the same probability of failure[a]

n_e	k_f	$A_{\text{built-av}}$	% change in area	P_f ($\times 10^{-4}$)	PFCT
0	0.928	1.258	1.67	0.630	0.0402
1	0.941	1.254	1.29	0.630	0.0399
2	0.947	1.243	0.38	0.630	0.0414
3	0.950	1.238	——	0.630	0.0412
4	0.952	1.234	−0.33	0.630	0.0420
5	0.954	1.231	−0.52	0.630	0.0420

[a]The number of coupon tests, n_c, is 50. The redesign of the element tests and the certification test are included in the analysis.

Table 13 Effects of the number of element tests for the same PFCT[a]

n_e	k_f	$A_{\text{built-av}}$	% change in area	P_f ($\times 10^{-4}$)	PFCT
0	0.929	1.257	1.53	0.651	0.0412
1	0.942	1.252	1.11	0.656	0.0412
2	0.947	1.243	0.41	0.626	0.0412
3	0.950	1.238	——	0.630	0.0412
4	0.951	1.235	−0.23	0.616	0.0412
5	0.952	1.233	−0.42	0.615	0.0412

[a]The number of coupon tests, n_c, is 50. The redesign of the element tests and the certification test are included in the analysis.

Table 14 Effects of certification test[a]

	$A_{\text{built-av}}$	A_{COV}	P_f ($\times 10^{-4}$)
Certification	1.238	0.0931	0.630
No certification	1.233	0.0948	0.766
No certification with adjusted mean area	1.238	0.0948	0.692

[a]The number of coupon tests is 50, and the number of element tests is three.

load-carrying area is adjusted to its nominal value, the probability of failure is 9.8% larger.

VIII. Conclusions

The effects of aircraft structural tests on aircraft structural safety were explored. In particular, the effects of the number of coupon tests and the number of structural element tests on the final distribution of the failure stress were investigated. We simulated a structural design following the FARs and explored the tradeoffs between the number of tests, the weight, and the probability of failure. From the results obtained in this study, the following conclusions can be drawn:

1) As the number of coupon tests is increased, the mean allowable stress increases, so the mean load-carrying area reduces. While the standard deviation of the area decreases, the probability of failure increases, as does the probability of failure in certification. This indicates that the FAA knockdown factor for compensating for a small number of coupon tests is conservative (as intended).

2) As the number of coupon tests is increased, maintaining the same weight as the nominal case, the probability of failure reduces, but the rate of the reduction diminishes with the number of coupon tests. Overall, the number of coupon tests has only a marginal effect on the probability of failure.

3) If the number of element tests is increased, the probability of failure reduces for the same weight, and the rate of this reduction decreases with the number of tests.

4) If we want to dispense with element tests, then we will need to put about 1.5% extra structural weight to achieve the same probability of failure.

5) If the certification test is not performed, the probability of failure is increased by 10% for the same weight.

As noted earlier, probability distributions, the element test redesign procedure, and some selected parameters are based on our engineering judgment rather than published data. These assumptions have certain effects on the probabilities of failure in service and in the certification test. Nevertheless, a structural design practice following the FARs is performed, and it is concluded that it is possible to do simultaneously probabilistic design and satisfy the FARs for deterministic design by having an additional safety factor (k_f in this paper) at the structural level. This conclusion is firm and independent of the assumptions mentioned.

Appendix A: Bayesian Updating of the Failure Stress Distribution from the Results of Element Tests

The initial distribution of the element failure stress is obtained by using a failure criterion (e.g., Tsai-Wu theory) using the results of coupon tests. There will be two sources of error in this prediction. First, since a finite number of coupon tests are performed, the mean and standard deviation of the failure stress obtained through the coupon tests will be different from the actual mean and standard deviation.

We consider a typical situation relating to updating analytical predictions of strength based on tests. We assume that the analytical prediction of the failure stress, $(\sigma_f)_{\text{calc}}$, applies to the average failure stress $(\bar{\sigma}_f)_{\text{true}}$ of an infinite number of nominally identical structures. The error e_f of our analytical prediction is defined by Eq. (A1):

$$(\bar{\sigma}_f)_{\text{true}} = (1 + e_f)(\sigma_f)_{\text{calc}} \tag{A1}$$

Here, we assume that the designer can estimate the bounds b_e (possibly conservative) on the magnitude of the error, and we further assume that the errors have a uniform distribution between the bounds. Note here that it is more convenient to define the error to be measured from the calculated values of the failure stress as shown in Fig. A1.

As in earlier work [30], we neglect the effect of coupon tests and assume the initial distribution of the mean failure stress $f^{\text{ini}}(\bar{\sigma}_f)$ is uniform within the bounds b_e as

$$f^{\text{ini}}(\bar{\sigma}_f) = \begin{cases} \frac{1}{2b_e(\sigma_f)_{\text{calc}}} & \text{if } \left|\frac{\bar{\sigma}_f}{(\sigma_f)_{\text{calc}}} - 1\right| \leq b_e \\ 0 & \text{otherwise} \end{cases} \tag{A2}$$

Then, the distribution of the mean failure stress is updated using the Bayesian updating with a given $(\sigma_f)_{1,\text{test}}$ as

$$f^{\text{upd}}(\bar{\sigma}_f) = \frac{f_{1,\text{test}}(\bar{\sigma}_f) f^{\text{ini}}(\bar{\sigma}_f)}{\int_{-\infty}^{\infty} f_{1,\text{test}}(\bar{\sigma}_f) f^{\text{ini}}(\bar{\sigma}_f)\, \mathrm{d}\bar{\sigma}_f} \tag{A3}$$

where

$$f_{1,\text{test}}(\bar{\sigma}_f) = \text{normal}\,[(\sigma_f)_{1,\text{test}}; \quad \bar{\sigma}_f, \quad \text{std}(\sigma_f)]$$

is the likelihood function reflecting possible variability of the first test result $(\sigma_f)_{1,\text{test}}$. Note that $f_{1,\text{test}}(\bar{\sigma}_f)$ is not a probability distribution in $\bar{\sigma}_f$; it is the conditional probability density of obtaining test result $(\sigma_f)_{1,\text{test}}$, given that the mean value of the failure stress is $\bar{\sigma}_f$. Subsequent tests are handled by the same equations, using the updated distribution as the initial one.

If the Bayesian updating procedure defined previously is used directly within an MCS loop for design load-carrying area determination, the computational cost will be very high. In this paper, instead, the Bayesian updating is performed aside from the MCS loop. In this separate loop, we first simulate the coupon tests by

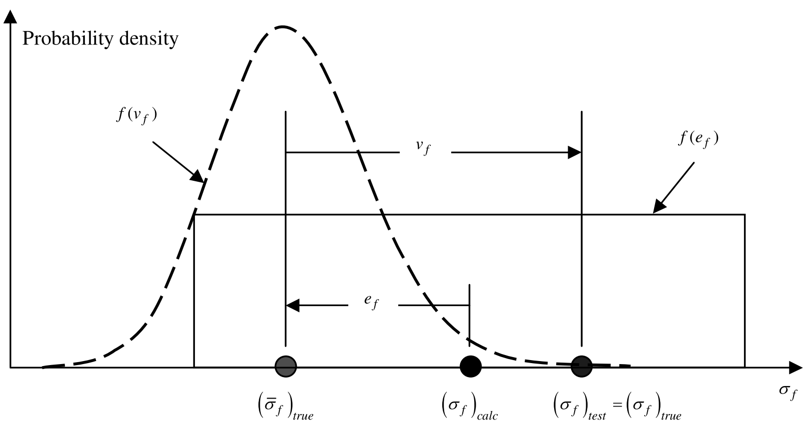


Fig. A1 Error and variability in failure stress. The error is centered around the computed value, and it is assumed to be uniformly distributed here. The variability distribution, on the other hand, is lognormal, with the mean equal to the true average failure stress.

drawing random samples for the mean and standard deviation of the calculated failure stress $\bar{\sigma}_{cf}$ and $\text{std}(\sigma_{cf})$. Then, we simulate n_e number of element tests, $(\sigma_{ef})_{\text{test}}$. The element test results along with the mean and the standard deviation are used to define the likelihood function as

$$f_{1,\text{test}}(\bar{\sigma}_f) = \text{normal}\,[(\sigma_{ef})_{1,\text{test}}; \quad \bar{\sigma}_{cf}, \quad \text{std}(\sigma_{cf})]$$

in Eq. (A3). The initial distribution $f^{\text{ini}}(\bar{\sigma}_f)$ in Eq. (A3) is uniformly distributed within some bounds, as given in Eq. (A4):

$$f^{\text{ini}}(\bar{\sigma}_f) = \begin{cases} \frac{1}{2b_e\bar{\sigma}_{cf}} & \text{if } \left|\frac{\bar{\sigma}_f}{\bar{\sigma}_{cf}} - 1\right| \le b_e \\ 0 & \text{otherwise} \end{cases} \tag{A4}$$

We found that applying the error bounds b_e before the Bayesian updating or after the updating do not matter. Applying the error bounds before Bayesian updating means calculating the initial distribution $f^{\text{ini}}(\bar{\sigma}_f)$ from Eq. (A4) and then using Eq. (A3). To apply the error bounds after the Bayesian updating, however, we first assume very large error bounds b_e, calculate the initial distribution $f^{\text{ini}}(\bar{\sigma}_f)$ from Eq. (A4), and finally apply the error bounds b_e to the distribution obtained using Eq. (A3).

Applying the error bounds after the Bayesian updating is more useful when we want to fit distributions (e.g., Johnson distribution) to the mean failure stress obtained through Bayesian updating. If we apply the error bounds at the beginning, the distribution after Eq. (A3) will be a truncated one, and it will be difficult to fit a distribution with good fidelity. However, if we apply the error bounds at the end, the distribution after Eq. (A3) will be a continuous one, and we will high likely fit a good distribution.

So the overall procedure is as follows. Within an MCS loop, we generate random mean and standard deviation values for the failure stress to be obtained through coupon tests. Then, we assume large error bounds to be used in Eq. (A4), simulate element tests, and use Eq. (A3) to obtain the distribution of the mean failure stress. Then, we compute the four quantiles of the mean failure stress distribution. Finally, we compute the mean and standard deviations of the quantiles, and we model these quantiles as normal distributions. Note that the quantiles are the values of failure stress for CDF values of [0.067, 0.309, 0.691, 0.933].

Table A1 The mean and standard deviation of the quantiles of the mean failure stress after element tests if 30 coupon tests are performed

	Mean values of the quantiles ($Q_{1–4}$)				Standard deviation of the quantiles ($Q_{1–4}$)			
	$\bar{Q}_1$	$\bar{Q}_2$	$\bar{Q}_3$	$\bar{Q}_4$	std(Q_1)	std(Q_2)	std(Q_3)	std(Q_4)
Test1	0.899	0.968	1.049	1.145	0.073	0.077	0.084	0.094
Test2	0.925	0.975	1.032	1.096	0.053	0.055	0.058	0.063
Test3	0.937	0.979	1.025	1.076	0.044	0.045	0.047	0.051
Test4	0.945	0.982	1.021	1.065	0.038	0.039	0.041	0.043
Test5	0.950	0.983	1.019	1.057	0.035	0.036	0.037	0.039

Table A2 The mean and standard deviation of the quantiles of the mean failure stress after element tests if 50 coupon tests are performed

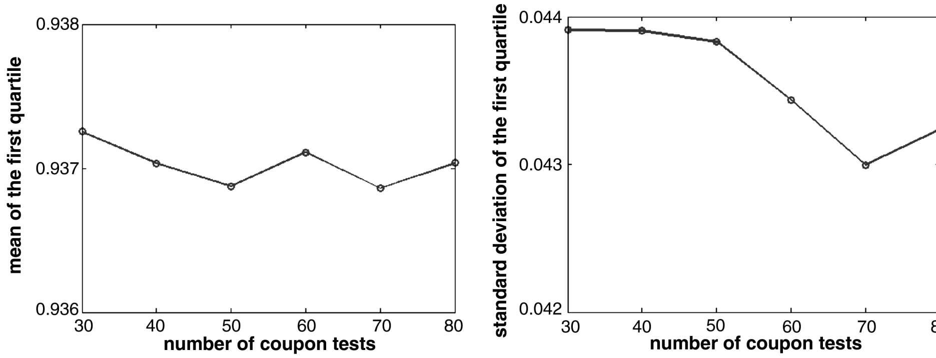
	Mean values of the quantiles ($Q_{1–4}$)				Standard deviation of the quantiles ($Q_{1–4}$)			
	$\bar{Q}_1$	$\bar{Q}_2$	$\bar{Q}_3$	$\bar{Q}_4$	std(Q_1)	std(Q_2)	std(Q_3)	std(Q_4)
Test1	0.897	0.966	1.047	1.143	0.073	0.078	0.084	0.093
Test2	0.924	0.975	1.032	1.095	0.053	0.055	0.058	0.063
Test3	0.937	0.979	1.025	1.075	0.044	0.045	0.047	0.050
Test4	0.944	0.981	1.021	1.064	0.038	0.039	0.041	0.043
Test5	0.950	0.983	1.019	1.057	0.035	0.035	0.037	0.039

Table A3 The mean and standard deviation of the quantiles of the mean failure stress after element tests if 80 coupon tests are performed

	Mean values of the quantiles ($Q_{1–4}$)				Standard deviation of the quantiles ($Q_{1–4}$)			
	$\bar{Q}_1$	$\bar{Q}_2$	$\bar{Q}_3$	$\bar{Q}_4$	std(Q_1)	std(Q_2)	std(Q_3)	std(Q_4)
Test1	0.898	0.967	1.049	1.144	0.071	0.076	0.083	0.091
Test2	0.924	0.975	1.032	1.096	0.052	0.055	0.058	0.062
Test3	0.937	0.979	1.025	1.076	0.043	0.045	0.047	0.050
Test4	0.944	0.982	1.021	1.065	0.038	0.039	0.040	0.042
Test5	0.950	0.983	1.019	1.057	0.034	0.035	0.036	0.038

Table A4 The variation of the mean and standard deviation of the quantiles of the mean failure stress with the error in failure stress prediction, e_{fe}

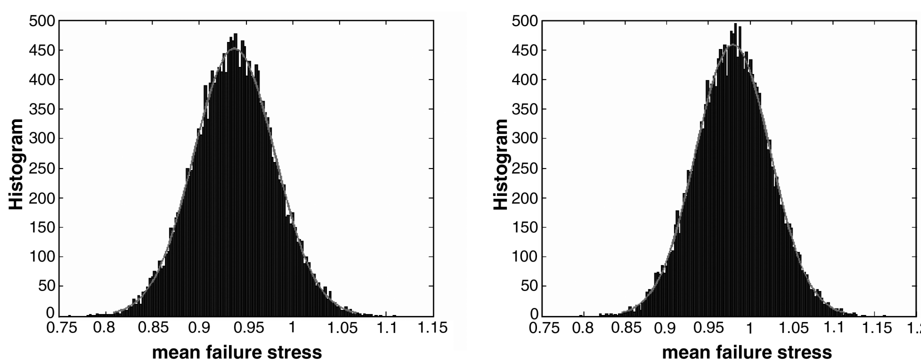
	Mean values of the quantiles ($Q_{1–4}$)				Standard deviation of the quantiles ($Q_{1–4}$)			
e_f	$\bar{Q}_1$	$\bar{Q}_2$	$\bar{Q}_3$	$\bar{Q}_4$	std(Q_1)	std(Q_2)	std(Q_3)	std(Q_4)
−.10	0.835	0.881	0.923	0.968	0.039	0.041	0.043	0.045
−0.05	0.890	0.930	0.974	1.022	0.042	0.043	0.045	0.048
0	0.937	0.979	1.025	1.076	0.044	0.045	0.047	0.050
0.05	0.983	1.027	1.075	1.128	0.045	0.047	0.049	0.052
0.10	1.031	1.077	1.128	1.183	0.048	0.050	0.052	0.055



a) Mean value of the first quartile

b) Standard deviation of the first quartile

Fig. A2 Variation of the mean and standard deviation of the first quantile of the mean failure stress with number of coupon tests (after the third element test).



a) Histogram of the first quartile

b) Histogram of the second quartile

Fig. A3 Histograms of the first and the second quantiles of the mean failure stress (after the third element test). The continuous lines show the normal fits.

The quantiles are functions of the number of coupon tests (n_c), the number of element tests (n_e), and the error in failure stress prediction (e_{ef}). At first, we wanted to build RSA for the mean and standard deviation of the quantiles in terms of n_c and e_{ef} after each element test, and so we would have 10 RSAs (five for the mean and five for the standard deviation) in terms of n_c and e_{ef}. Our numerical analysis revealed, on the other hand, that n_c does not have a noticeable effect on quantiles (see Tables A1–A3 alongside Fig. A2), and the effect of the error can be represented by just multiplying the quantiles with the $(1 - e_{ef})$ term (see Table A4).

Figure A3 show the histograms of the first and second quantiles of the mean failure stress (for 50 coupon tests after the third element test when $e_f = 0$) obtained through MCS with 20,000 samples. We see that the quantiles do not exactly follow normal distributions.

The results obtained in this separate MCS loop are used in the main MCS loop for determining the built average load-carrying area. The mean and standard deviations of the quantiles are used to fit a Johnson distribution to the mean failure stress. The error bounds b_e are then applied to the Johnson distribution, and random values from this distribution are drawn whenever element tests are simulated. Note also that the quantiles are strongly correlated to each other, so this correlation is also included in our analysis while random quantiles are generated in the main MCS loop using Gaussian copula. The reader is referred to the work of Noh et al. [38] for further details of reliability estimation of problems with correlated input variables using a Gaussian Copula.

Appendix B: Separable Monte Carlo Simulations

The prediction of probability of failure via conventional MCS requires trillions of simulations for the level of 10^{-7} failure probability. To address the computational burden, the separable Monte Carlo procedure can be used [29]. The reader is referred to Smarslok et al. [39] for more information on the separable Monte Carlo procedure. This procedure applies when the failure condition can be expressed as $g_1(x_1) > g_2(x_2)$, where x_1 and x_2 are two disjoint sets of random variables. To take advantage of this procedure, we need to formulate the failure condition in a separable form, so that g_1 will depend only on variabilities, and g_2 will depend only on errors. The common formulation of the structural failure condition is in the form of a stress exceeding the material limit. This form, however, does not satisfy the separability requirement. For example, the stress depends on variability in material properties as well as design area, which reflects errors in the analysis process. To bring the failure condition to the right form, we instead formulate it as the required cross-sectional area A'_{req} being larger than the built area $A_{\text{built-av}}$. In view of Eq. (30), the relation between the built area and the required area becomes

$$A_{\text{built-av}} < \frac{A_{\text{req}}}{(1 + v_t)(1 + v_w)} \equiv A'_{\text{req}} \tag{B1}$$

where A_{req} is the cross-sectional area required to carry the actual loading conditions for a particular copy of an aircraft model, and A'_{req} is what the built area (fleet average) needs to be in order for the particular copy to have the required area after allowing for variability in width and thickness:

$$A_{\text{req}} = P_{\text{act}}/\sigma_f \tag{B2}$$

The required area depends only on variability, while the built area depends only on errors. When certification testing is taken into account, the built area $A_{\text{built-av}}$ is replaced by the certified area A_{cert}, which is the same as the built area for companies that pass

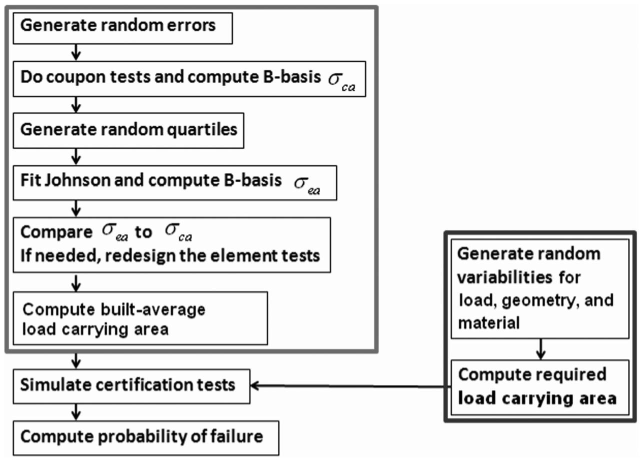


Fig. B1 Flowchart for separable MCSs.

certification. However, companies that fail are not included. That is, the failure condition is written as failure without certification tests,

$$A_{\text{built-av}} - A'_{\text{req}} < 0 \tag{B3a}$$

and failure with certification tests,

$$A_{\text{cert}} - A'_{\text{req}} < 0 \tag{B3b}$$

The separable MCS procedure is summarized in Fig. B1.

Acknowledgments

The authors gratefully acknowledge the support of the research by the NASA center and the U.S. Air Force, U.S. Air Force Research Laboratory, under award FA9550-07-1-0018 (Victor Giurgiutiu, Program Manager). The authors also wish to thank graduate students Deepti Buchi and Sriram Pattabhiraman of the University of Florida for their work on the Johnson distribution and the quantiles.

References

[1] "Guidelines for Property Testing of Composites," *Composite Materials Handbook MIL-HDBK-17*, U.S. Dept. of Defense, Washington, D.C., 2002.
[2] Lincoln, J. W., "Method for Computation of Structural Failure Probability for an Aircraft," Aeronautical Systems Div. TR 80-5035, Wright–Patterson AFB, OH, July 1980.
[3] Lincoln, J. W., "Risk Assessment of an Aging Aircraft," *Journal of Aircraft*, Vol. 22, No. 8, Aug. 1985, pp. 687–691. doi:10.2514/3.45187
[4] Shiao, M. C., Nagpal, V. K., and Chamis, C. C., "Probabilistic Structural Analysis of Aerospace Components Using NESSUS," AIAA Paper 1988-2373, 1988.
[5] Wirsching, P. H., "Literature Review on Mechanical Reliability and Probabilistic Design," *Probabilistic Structural Analysis Methods for Select Space Propulsion System Components (PSAM)*, NASA CR 189159, Vol. III, 1992.
[6] Lykins, C., Thomson, D., and Pomfret, C., "The Air Force's Application of Probabilistics to Gas Turbine Engines," AIAA Paper 1994-1440, 1994.
[7] Ebberle, D. H., Newlin, L. E., Sutharshana, S., and Moore, N. R., "Alternative Computational Approaches for Probabilistic Fatigue Analysis," AIAA Paper 1995-1359, 1995.
[8] Ushakov, A., Kuznetsov, A. A., Stewart, A., and Mishulin, I. B., "Probabilistic Design of Damage Tolerant Composite Aircraft Structures," Final Rept. Under Annex 1 to Memorandum of Cooperation AIA/CA-71 Between the FAA and Central Aero-Hydrodynamic Institute (TsAGI), 1996.
[9] Mavris, D. N., Macsotai, N. I., and Roth, B., "A Probabilistic Design Methodology for Commercial Aircraft Engine Cycle Selection," Society of Automotive Engineers Paper 985510, 1998.
[10] Long, M. W., and Narciso, J. D., "Probabilistic Design Methodology for Composite Aircraft Structures," U.S. Department of Transportation Final Rept. DOD/FAA/AR-99/2, June 1999.
[11] Ushakov, A., Stewart, A., Mishulin, I., and Pankov, A., "Probabilistic Design of Damage Tolerant Composite Aircraft Structures," U.S. Department of Transportation Final Rept. DOD/FAA/AR-01/55, Jan. 2002.
[12] Rusk, D. T., Lin, K. Y., Swartz, D. D., and Ridgeway, G. K., "Bayesian Updating of Damage Size Probabilities for Aircraft Structural Life-Cycle Management," *Journal of Aircraft*, Vol. 39, No. 4, July 2002, pp. 689–696. doi:10.2514/2.2983
[13] Allen, M., and Maute, K., "Reliability-Based Design Optimization of Aeroelastic Structures," *Structural and Multidisciplinary Optimization*, Vol. 27, No. 4, 2004, pp. 228–242. doi:10.1007/s00158-004-0384-1
[14] Huang, C. K., and Lin, K. Y., "A Method for Reliability Assessment of Aircraft Structures Subject to Accidental Damage," AIAA Paper 2005-1830, 2005.
[15] Nam, T., Soban, D. S., and Mavris, D. N., "A Non-Deterministic Aircraft Sizing Method under Probabilistic Design Constraints," AIAA Paper 2006-2062, 2006.
[16] Lin, K. Y., and Styuart, A. V., "Probabilistic Approach to Damage Tolerance Design of Aircraft Composite Structures," *Journal of Aircraft*, Vol. 44, No. 4, July–Aug. 2007, pp. 1309–1317. doi:10.2514/1.26913
[17] Dhillon, B. S., Pashandi, A. R. M., and Liu, K. L., "Robot Systems Reliability and Safety: A Review," *Journal of Quality in Maintenance Engineering*, Vol. 8, No. 3, 2002, pp. 170–212. doi:10.1108/13552510210439784
[18] Qu, X., Haftka, R. T., Venkataraman, S., and Johnson, T. F., "Deterministic and Reliability-Based Optimization of Composite Laminates for Cryogenic Environments," *AIAA Journal*, Vol. 41, No. 10, 2003, pp. 2029–2036. doi:10.2514/2.1893
[19] Acar, E., Haftka, R. T., Sankar, B. V., and Qui, X., "Increasing Allowable Flight Loads by Improved Structural Modeling," *AIAA Journal*, Vol. 44, No. 2, 2006, pp. 376–381. doi:10.2514/1.17804
[20] Acar, E., Haftka, R. T., and Johnson, T. F., "Tradeoff of Uncertainty Reduction Mechanisms for Reducing Structural Weight," *Journal of Mechanical Design*, Vol. 129, No. 3, 2007, pp. 266–274. doi:10.1115/1.2406097.
[21] Li, M., Williams, N., and Azarm, S., "Interval Uncertainty Reduction and Sensitivity Analysis with Multi-Objective Design Optimization," *Journal of Mechanical Design*, Vol. 131, No. 3, 2009, Paper 031007. doi:10.1115/1.3066736
[22] Jiao, G., and Moan, T., "Methods of Reliability Model Updating Through Additional Events," *Structural Safety*, Vol. 9, No. 2, 1990, pp. 139–153. doi:10.1016/0167-4730(90)90005-A.
[23] Jiao, G., and Eide, O. I., "Effects of Testing, Inspection and Repair on the Reliability of Offshore Structures," *Seventh Speciality Conference on Probabilistic Mechanics and Structural Reliability*, American Society of Civil Engineers, Reston, VA, Aug. 1996, pp. 154–157.
[24] Beck, J. L., and Katafygiotis, L. S., "Updating models and their uncertainties: Bayesian Statistical Framework," *Journal of Engineering Mechanics*, Vol. 124, No. 4, 1998, pp. 455–461. doi:10.1061/(ASCE)0733-9399(1998)124:4(455)
[25] Papadimitriou, C., Beck, J. L., and Katafygiotis, L. S., "Updating Robust Reliability Using Structural Test Data," *Probabilistic Engineering Mechanics*, Vol. 16, No. 2, 2001, pp. 103–113. doi:10.1016/S0266-8920(00)00012-6
[26] Johnson, N. L., "Systems of Frequency Curves Generated by Methods of Translation," *Biometrika*, Vol. 36, Nos. 1–2, 1949, pp. 149–176. doi:10.1093/biomet/36.1-2.149
[27] Zhu, H., Sankar, B. V., and Marrey, R. V., "Evaluation of Failure Criteria for Fiber Composites Using Finite Element Micromechanics," *Journal of Composite Materials*, Vol. 32, No. 8, 1998, pp. 766–782. doi:10.1177/002199839803200804
[28] Acar, E., Kale, A., Haftka, R. T., and Stroud, W. J., "Structural Safety Measures for Airplanes," *Journal of Aircraft*, Vol. 43, No. 1, 2006, pp. 30–38. doi:10.2514/1.14381
[29] Acar, E., Kale, A., and Haftka, R. T., "Comparing Effectiveness of Measures that Improve Aircraft Structural Safety," *Journal of Aerospace Engineering*, Vol. 20, No. 3, 2007, pp. 186–199. doi:10.1061/(ASCE)0893-1321(2007)20:3(186)
[30] An, J., Acar, E., Haftka, R. T., Kim, N. H., Ifju, P. G., and Johnson, T. F., "Being Conservative with a Limited Number of Test Results," *Journal*

of Aircraft, Vol. 45, No. 6, 2008, pp. 1969–1975.
doi:10.2514/1.35551

[31] "Factor of Safety," *Airworthiness Standards: Transport Category Airplanes*, Federal Aviation Administration Regulation Part 25, Sec. 25.303, 1970.

[32] "Material Strength Properties and Material Design Values," *Airworthiness Standards: Transport Category Airplanes*, Federal Aviation Administration Regulation Part 25, Sec. 25.613, 2003.

[33] "Proof of Structure," *Airworthiness Standards: Transport Category Airplanes*, Federal Aviation Administration Regulation Part 25, Sec. 25.307, 1990.

[34] Oberkampf, W. L., Deland, S. M., Rutherford, B. M., Diegert, K. V., and Alvin, K. F., "Estimation of Total Uncertainty in Modeling and Simulation," Sandia Rept. SAND 2000-0824, Albuquerque, NM, 2000.

[35] Oberkampf, W. L., Deland, S. M., Rutherford, B. M., Diegert, K. V., and Alvin, K. F., "Error and Uncertainty in Modeling and Simulation," *Reliability Engineering and System Safety*, Vol. 75, No. 3, 2002, pp. 333–357.
doi:10.1016/S0951-8320(01)00120-X

[36] Antonsson, E. K., and Otto, K. N., "Imprecision in Engineering Design," *Journal of Mechanical Design*, Vol. 117, No. B, 1995, pp. 25–32.
doi:10.1115/1.2836465

[37] Nikolaidis, E., Chen, S., Cudney, H., Haftka, R. T., and Rosca, R., "Comparison of Probability and Possibility for Design Against Catastrophic Failure Under Uncertainty," *Journal of Mechanical Design*, Vol. 126, No. 3, 2004, pp. 386–394.
doi:10.1115/1.1701878

[38] Noh, Y.,·Choi, K. K., and Du, L., "Reliability-Based Design Optimization of Problems with Correlated Input Variables Using a Gaussian Copula," *Structural and Multidisciplinary Optimization*, Vol. 38, No. 1, 2009, pp. 1–16.
doi:10.1007/s00158-008-0277-9

[39] Smarslok, B., Haftka, R. T., and Kim, N-H., "Comparison and Efficiency Analysis of Crude and Separable Monte Carlo Simulation Methods," 47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, AIAA Paper 2006-1632, April 2006.

A. Pelegri
Associate Editor