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Doubly weighted moving least squares and its application to structural reliability analysis

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Abstract In this paper, we proposed a two-stage hybrid reliability analysis framework based on the surrogate model, which combines the first-order reliability method and Monte Carlo simulation with a doubly-weighted moving least squares (DWMLS) method. The first stage consists of constructing a surrogate model based on DWMLS. The weight system of DWMLS considers not only the normal weight factor of moving least squares, but also the distance from the most probable failure point (MPFP), which accounts for reliability problems. An adaptive experimental design scheme is proposed, during which the MPFP is progressively updated. The approximate values and sensitivity information of DWMLS are chosen to determine the number and location of the experimental design points in the next iteration, until a convergence criterion is satisfied. In the second stage, MCS on the surrogate model is then used to calculate the probability of failure. The proposed method is applied to five benchmark examples to validate its accuracy and efficiency. Results show that the proposed surrogate model with DWMLS can estimate the failure probability accurately, while requiring fewer original model simulations.

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Keywords Doubly weighted moving least squares · Surrogate model · Two-stage hybrid method · Adaptive experimental design · Structural reliability analysis · Latin hypercube design

Nomenclature

n	number of input random variables
N	number of experimental points
$\mathbf{x}$	vector of input random variables
β	reliability index
β_{HL}	reliability index by Hasofer–Lind algorithm
μ	mean
σ	standard deviation
$g(\mathbf{X})$	limit state function
$\hat{g}(\mathbf{X})$	approximate limit state function/ response surface function
$\mathbf{X}_{add}$	new added experimental points in any iteration
MLS	moving least square
DWMLS	doubly weighted moving least square
SVR	support vector regression
ANN	artificial neural networks
MPFP	most probable failure point
MCS	Monte Carlo Simulation
FORM	first order reliability method
SORM	second order reliability method
H–L	Hasofer–Lind algorithm
RSM	response surface method
LHD	Latin hypercube design
FEA	finite element analysis
CFD	computational fluid dynamics
COV	coefficient of variation
UDR	Univariate Dimension-Reduction
MPP-UDR	Most probable point based UDR
SGI	Sparse Grid Interpolation

1 Introduction

It has been well recognized that uncertainties in engineering systems (e.g., applied loads, material properties and geometric tolerances) can result in catastrophic failure and should be managed appropriately. The traditional factor-of-safety approach to compensate for uncertainties often leads to either un-conservative or too-conservative designs. Reliability analysis takes into account these uncertainties in evaluating system's safety, which has become an important part of recent engineering design. There has been a growing interest in the use of reliability methods for structural design and safety assessment (see, e.g. Ditlevsen and Madsen 1996; Hurtado 2004; Bucher and Bourgund 1990; Rachwitz 2001; Zou et al. 2002; Youn and Choi 2004).

However, with the development of advanced numerical simulation methods, which commonly take several hours to perform a single evaluation, classical reliability methods can easily become impractical. Monte Carlo Simulation (MCS) and its variants demand tremendous computational resources that prohibit its practicality. On the other hand, approximation methods, such as first-order reliability method (FORM) and second-order reliability method (SORM), have issues with relatively poor performance in accuracy. Therefore, it seems reasonable to use a response surface or surrogate model to approximate the performance function and apply either MCS to calculate reliability.

However, reliability analysis is different from approximation problems, from which the surrogate model originated (see, e.g. Queipo et al. 2005; Viana et al. 2010b). In conventional approximation problems, the general criterion of a surrogate model is to minimize the error between the true function and the surrogate model in the entire domain of interest. In reliability analysis, however, it is important to identify a limit state, which is the boundary between safe and failed regions, especially near the most probable failure point (MPFP). In this paper, a doubly weighted moving least squares (DWMLS) method is proposed, which takes into account the characteristic feature of reliability analysis. The proposed DWMLS method consists of two weighting schemes—the first weight considers the distance between the sampling point and the prediction point, while the second considers the distance between the sampling points and the MPFP. A hybrid, two-stage reliability analysis framework is proposed, which takes full advantage of the FORM/SORM, MCS, adaptive experiment design and DWMLS to achieve both accuracy and efficiency.

The remainder of this paper is organized as follows. In Section 2, a brief literature review of various surrogate models in reliability analysis is presented. In Section 3, a general principle of the moving least squares method is stated, after which the newly added weighting system of DWMLS is detailed. In Section 4, a new adaptive experimental design procedure is illustrated in detail. A complete flowchart of the proposed hybrid reliability analysis framework is then presented in Section 5. Five numerical examples are presented in Section 6, which highlight the capabilities of the proposed method and demonstrate its accuracy and efficiency, followed by a summary and conclusions in Section 7.

2 Review on surrogate models in reliability analysis

The increased complexity of simulations on real systems stimulates the development of surrogate models that approximate the behavior of complex systems, improve their validation process, and aid optimization of the system. Surrogate models are developed in order to analyze experimental data and to build empirical models based on observations. These models were first introduced in design optimization and applied to reliability analysis and design because of their merits in efficiency.

Wong (1985) first proposed a complete, quadratic form polynomial and applied it to reliability analysis. In his work, the number of polynomials and the required sampling points increase rapidly with the number of random variables. In order to reduce the number of sampling points, Bucher and Bourgund (1990) proposed a two-iteration quadratic polynomial without cross-terms. Rajashkhar and Ellingwood (1993), and Liu and Moses (1994) improved this approach by updating the surrogate model parameters until a convergence criterion was satisfied. Kim and Na (1997) proposed a sequential approach to the surrogate model where the gradient projection method is used to ensure that the sampling points are located near the failure surface. Das and Zheng (2000) proposed an improved surrogate model and applied it to reliability analysis of a stiffened plate structure. Guan and Melchers (2001) evaluated the effect of surrogate model parameter variation on reliability. Kaymaz and McMahon (2005) suggested a new surrogate model, in which a weighted regression method was applied instead of normal regression. Inspired by Kaymanz and McMahon's method, Nguyen et al. (2009) improved the weighted regression method where the fitting points were weighted according to their distance from the true failure surface and the estimated design point.

However, when the limit state function is highly nonlinear, polynomial-based surrogate models can perform poorly because they try to approximate the performance function globally. Thus, advanced surrogate modeling methods were introduced to replace traditional global polynomial-based models. Choi et al. (2003), and Kim et al. (2006) introduced polynomial chaos expansion for reliability analysis and design. Papadrakakis et al. (1996), Elhewy et al. 2006, and Hurtado and Alvarez (2001) used Neural Networks to reliability analysis in conjunction with Monte Carlo Simulation. Gomes and Awruch (2004) compared Neural Networks with FORM, MCS, and the Importance Sampling technique. Kaymaz (2005), and Panda and Manohar (2008) proposed the Kriging method to reliability analysis and compared it with the most common surrogate models. Echard et al. (2011) presented a Kriging enhanced MCS method in reliability analysis and the test problems have demonstrated its efficiency and accuracy. Most (2007) presented an efficient adaptive response surface approach for reliability analysis, where support vector machines were used to classify the failure and safe domain. Guo and Bai (2009) proposed a least squares support vector machine for regression into reliability analysis and the results demonstrate excellent accuracy and smaller computational cost than the reliability method based on support vector machines.

Moving least squares (MLS) is a local weighted least squares method, originally introduced by Lancaster and Salkauskas (1981) for smoothing and interpolating data. After that, it is widely used to obtain approximations in meshfree methods and structural optimization because of its 'localized' approximation property (Zadeh et al. 2005). Unlike other surrogate models, the MLS has no expensive inner parameter optimization during the adaptive modeling process. Therefore, Krishnamurthy (2005) compared the MLS method with other local methods, such as Kriging, and found it to be more accurate and computationally effective for the examples considered. Bucher and Most (2008) carried out research aimed at comparing the performance of these response surfaces and its application in reliability analysis. In Kang et al. (2010), MLS has made it possible to derive the approximation function closer to the limit state function and exhibited improved performance in terms of significant reduction of the number of structural analyses and sensitivity accuracy of the reliability index to the random variables. Youn and Choi (2004), and Song et al. (2011) have applied the moving least squares method to reliability based design optimization and MLS performed well in uncertainty design.

3 Doubly weighted moving least squares (DWMLS)

The basic idea of surrogate modeling in reliability analysis is to replace the performance function with an approximate function, whose value can be computed easily. Compared with the global polynomial regression method, the moving least squares (MLS) method is a local regression and a relatively new surrogate modeling technique. The MLS method can be found extensively in the element-free Galerkin method and computer graphics. In this section, the basic principle of the MLS method is presented first, followed by an introduction to a new weighting scheme that works better for reliability analysis.

3.1 Basic principle of moving least squares

We consider a performance function $g(\mathbf{x})$ in an n-dimensional space of random variables, in which the vector of random variables is defined as $\mathbf{x} = [x_1, x_2, \ldots, x_3]^{\mathrm{T}}$. In the MLS method, the performance function is approximated by

$$\hat{g}(\mathbf{x}) = \mathbf{p}(\mathbf{x})^{\mathrm{T}}\mathbf{a}(\mathbf{x}) \tag{1}$$

where $\mathbf{p}(\mathbf{x}) = [p_1(\mathbf{x}), p_2(\mathbf{x}), ..., p_m(\mathbf{x})]^{\mathrm{T}}$ is a vector of m polynomial basis functions, and $\mathbf{a}(\mathbf{x}) = [a_1(\mathbf{x}), a_2(\mathbf{x}), ..., a_m(\mathbf{x})]^{\mathrm{T}}$ is a vector of corresponding coefficients. It is noted that in global regression methods, the coefficients are constant, while they are functions of $\mathbf{x}$ in MLS. In this paper, the basis function $\mathbf{p}(\mathbf{x})$ is defined using polynomials up to the second order without cross terms, as

$$\mathbf{p}(\mathbf{x}) = \left[1, x_1, x_2, ..., x_n, x_1^2, x_2^2, ..., x_n^2\right]^{\mathrm{T}} \tag{2}$$

where the dimension of $\mathbf{p}(\mathbf{x})$ is $m = 2n + 1$. However, it is possible that the basis function can include cross-terms, as well as higher order terms.

The unknown coefficients in (1) can be calculated by minimizing the error between the performance function and its approximation at discrete points. In order to do that, N sample points are first selected from the input space; these samples are denoted by $\mathbf{x}_I, I = 1, \ldots, N$. Then, the performance functions, $g(\mathbf{x}_I)$, are calculated at these sample points. This process may involve numerical simulations, such as finite element analysis or computational fluid dynamics. At a given prediction point $\mathbf{x}$, the MLS technique determines the unknown coefficients by minimizing the error between actual and approximated values of the performance function with weights, as

$$R(\mathbf{x}) = \sum_{I=1}^{N} w(\mathbf{x} - \mathbf{x}_I)\left[g(\mathbf{x}_I) - \mathbf{p}(\mathbf{x}_I)^{\mathrm{T}}\mathbf{a}(\mathbf{x})\right]^2 \tag{3}$$

The above formula can also be written in a matrix form as:

$$R(\mathbf{x}) = [\mathbf{Pa}(\mathbf{x}) - \mathbf{g}]^{\mathrm{T}}\mathbf{W}[\mathbf{Pa}(\mathbf{x}) - \mathbf{g}] \tag{4}$$

where **g**, **P** and **W** are defined as:

$$\mathbf{g} = [g(\mathbf{x}_1), g(\mathbf{x}_2), \ldots, g(\mathbf{x}_N)]^{\mathrm{T}} \tag{5}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}^{\mathrm{T}}(\mathbf{x}_1) \\ \mathbf{p}^{\mathrm{T}}(\mathbf{x}_2) \\ \vdots \\ \mathbf{p}^{\mathrm{T}}(\mathbf{x}_N) \end{bmatrix}_{N*m} \tag{6}$$

$$\mathbf{W}(\mathbf{x}) = \begin{bmatrix} w(\mathbf{x}-\mathbf{x_1}) & 0 & \ldots & 0 \\ 0 & w(\mathbf{x}-\mathbf{x_2}) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & w(\mathbf{x}-\mathbf{x_N}) \end{bmatrix}_{N*N} \tag{7}$$

In (7), the weight $w(\mathbf{x}\text{-}\mathbf{x}_I)$ takes the following Gaussian form:

$$w(\mathbf{x}-\mathbf{x_I}) = \begin{cases} \dfrac{\left(e^{-(\alpha||\mathbf{x}-\mathbf{x_I}||/D_I)^2} - e^{-\alpha^2}\right)}{\left(1-e^{-\alpha^2}\right)} & \text{if } ||\mathbf{x}-\mathbf{x_I}|| \le D_I \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where the parameter α is used to control the weight function curve, D_I defines the domain of the influence of point $\mathbf{x}_I$, $||\mathbf{x}-\mathbf{x}_I||$ is the Euclid distance between sampling point $\mathbf{x}_I$ and prediction point $\mathbf{x}$, and $r = \|\mathbf{x}-\mathbf{x}_I\| / D_I$ is a normalized distance. In Fig. 1, the shape of Gaussian weight function $w(r)$ with different values of α is demonstrated.

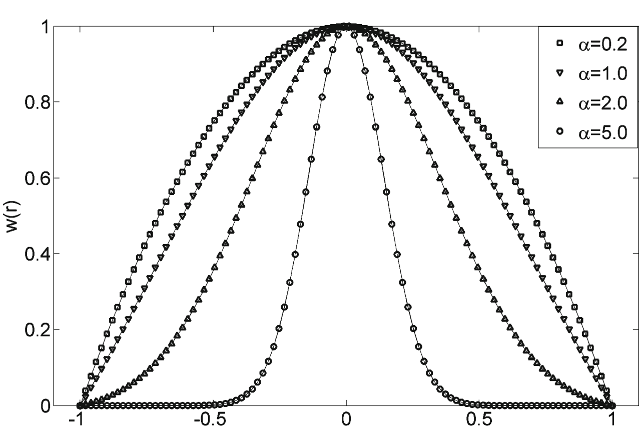

Fig. 1 Shape of the Gaussian weight function with different control parameter α

The minimum of the square error $R(\mathbf{x})$ can be achieved by vanishing the partial derivatives with respect to unknown coefficients, as

$$\frac{\partial R(\mathbf{x})}{\partial a_i} = 0, \quad i = 1, \ldots, m \tag{9}$$

The conditions in (9) yield the following system of linear equations:

$$\mathbf{A}(\mathbf{x})\mathbf{a}(\mathbf{x}) = \mathbf{b}(\mathbf{x}) \tag{10}$$

where $\mathbf{A}(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$ are defined by:

$$\mathbf{A}(\mathbf{x}) = \mathbf{P}^{\mathrm{T}}\mathbf{W}(\mathbf{x})\mathbf{P} \tag{11}$$

$$\mathbf{b}(\mathbf{x}) = \mathbf{P}^{\mathrm{T}}\mathbf{W}(\mathbf{x})\mathbf{g} \tag{12}$$

Once the unknown coefficients, $\mathbf{a}(\mathbf{x})$, are calculated by solving (10), (1) is used to approximate the performance function.

As can be seen from the weight scheme in (8), the weight w_I, which is associated with sampling point $\mathbf{x}_I$, decreases as $\mathbf{x}$ moves away from $\mathbf{x}_I$. The contribution of those points whose distance from $\mathbf{x}$ is greater than D_I vanishes, and thus, there is no need to include them in the regression process. Therefore, the dimension of the matrices in the MLS process is much smaller than the total number of sample points N. It is important to note that at a given prediction point $\mathbf{x}$, there must be enough sample points $\mathbf{x}_I$ such that the coefficient matrix $\mathbf{A}(\mathbf{x})$ should be non-singular. Also, it should be noted that the coefficients, $\mathbf{a}(\mathbf{x})$, must be calculated at every prediction point.

3.2 Doubly weighted moving least squares

In MLS, it is rational to impose a heavy weight to the points that are close to the prediction point and a light weight for more distant points in order to better approximate the performance function. In reliability analysis, however, the most important region is around the MPFP, because it contributes most to the probability of failure. Thus, we introduce an additional weighting scheme into MLS where the distances of sampling points to the prediction point $\mathbf{x}$ and to the MPFP are considered simultaneously. As discussed in the previous section, the first weighting scheme is based on the distance of sampling points $\mathbf{x}_I$ to the prediction point $\mathbf{x}$:

$$w_I(\mathbf{x}, \mathbf{x}_I) = w(\|\mathbf{x}-\mathbf{x}_I\|) \tag{13}$$

The second weighting scheme takes into account the distance between the sampling points and the current MPFP. It aims to penalize points located far from the current MPFP. The second weight factor is expressed as:

$$w_{II}(\mathbf{x}^*, \mathbf{x}_I) = \exp\left(-d_I^2\right) \tag{14}$$

Table 1 Distribution parameters of basic variables in test example

Random variables	Distribution type	Mean	Standard deviation
Q	Normal	10,000	400
E	Normal	2.0E10	0.5E10
I	Normal	8.0E-4	1.5E-4

where d_I is the distance between the *I-th* sampling point and the current MPFP. It is noted that the value of probability density can also be used instead of distance. It is also noted that since there is no information on the position of MPFP in the first stage, the conventional MLS is applied to the first surrogate model.

Then, through the advantage of the above-mentioned two weighting schemes, we find the following expression suitable to obtain the weight for each sampling point:

$$w(\mathbf{x}, \mathbf{x}_I) = w_I(\mathbf{x}, \mathbf{x}_I)^* w_{II}(\mathbf{x}_I) \tag{15}$$

In this paper, the weight matrix in (4) is replaced with this double weight matrix and the new surrogate model considers both the distance from the prediction point and the MPFP.

3.3 Test problem

In order to assess the accuracy of DWMLS on representing the limit state surface, the following example is presented with DWMLS, ordinary MLS, and the second-order polynomial response surface (Guo and Bai 2009):

$$g(q, E, I) = \frac{L}{360} - 0.0069\frac{qL^4}{EI} \tag{16}$$

where the distributions of three random variables are presented in Table 1. A full factorial design with 27 points and the MPFP estimated by FORM is added to assist the DWMLS. Based on 28 samples, DWMLS, ordinary MLS, and the second-order polynomial response surface are constructed. Table 2 shows the probability of failures estimated with 2E7 samples using MCS. The third column in Table 2 shows the error in each method with respect to the original MCS.

Table 2 Failure probability of MCS estimates with 2E7 samples

Method	P_f	ε_{pf}
MCS	8.63E-4 (COV=0.76%)	–
Polynomial RS + MCS	8.88E-4	2.97%
Ordinary MLS + MCS	8.54E-4	1.02%
DWMLS + MCS	8.58E-4	0.60%

The results in Table 2 show that DWMLS-based MCS provides the most accurate failure probability compared with other two models. However, the difference is relative small, partly because the nonlinearity of the performance function is not severe. In the following section, an adaptive experimental design method is proposed and combined with DWMLS.

4 Adaptive design of experiments (DOE)

4.1 Discussion on DOE in reliability analysis

The quality of probability estimates using a surrogate model depends on not only the surrogate model itself, but also the location of the points chosen to build the surrogate model (design of experiments). In the context of reliability analysis, there exist two kinds of DOE strategy: single design and adaptive design. For the former, Wong (1985) and Faravelli (1989) employed single factorial experimental design containing 2^n points to fit a quadratic function and to estimate the failure probability. Cheng et al. (2008) proposed a surrogate model based on the artificial neural network and using uniform experimental design in predicting failure probability. However, in the case of a black-box computer model, single experimental design cannot guarantee the accuracy of approximation, especially in domain around the limit state surface. Hence, it would be better to start with an initial set of samples and gradually add more samples based on the information provided by previous samples. Such a process is commonly referred as the adaptive design of experiments and has received more attentions than the former strategy (see, e.g. Picheny et al. 2010; Duprat and Sellier 2006).

In reliability analysis, Bucher and Bourgund (1990) first applied a surrogate model in which experimental points are chosen around the mean values of random variables, which form a matrix called the design matrix. Quadratic polynomials without cross terms are then used to fit these experimental points. Among various sampling methods, a common approach is to evaluate $g(\mathbf{X})$ at a $2n + 1$ combination of μ_i and $\mu_i \pm f\sigma_i$, where μ_i and σ_i are the mean and standard deviation of random variables, X_i, and f is a factor that defines the sampling range. The number of unknown coefficients in the performance function is $2n+1$, given as

$$\hat{y}(\mathbf{x}) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} a_{ii} x_i^2 \tag{17}$$

A FORM algorithm was applied to estimate the MPFP based on the above quadratic polynomial function, and the next design matrix was constructed on the new center point determined by the following expression:

$$\mathbf{x}_m = \boldsymbol{\mu} - g(\boldsymbol{\mu})\frac{\boldsymbol{\mu} - \mathbf{x}_D}{g(\boldsymbol{\mu}) - g(\mathbf{x}_D)} \tag{18}$$

where $\mathbf{x}_m$ and $\mathbf{x}_D$ indicate the new center point and the interim MPFP obtained in the previous stage, respectively. This second design matrix is used to form another quadratic polynomial just as the first matrix and the final failure probability was estimated by it. Thus, this procedure requires a $4n + 3$ evaluation of $g(\mathbf{X})$. Rajashkhar and Ellingwood (1993) questioned if a single cycle of updating is adequate, and they proposed to improve it by using more iterations until a convergence criterion is satisfied.

The above surrogate model in structural reliability analysis has some disadvantages: (1) with the increase of random variables, the total $g(\mathbf{X})$ evaluation number increases excessively fast, particularly while the convergence process is slow; (2) the result obtained by the above procedures has been shown to be sensitive to the parameter f and may not always give an acceptable approximation to the true failure probability (it is possible that the sequential center points during iteration may oscillate in the domain around the true design point and not converge); and (3) at different stages of surrogate modeling, only a part of available information on all previous $g(\mathbf{X})$ is directly used. Thus, it is considered that the accuracy of above iterative algorithm depends mainly on the characteristics of the nonlinear performance function, thereby limiting its application.

Simpson et al. (2001) concluded that a recommended experimental design should have a space-filling property. In the current study, an optimal uniform Latin hypercube design is utilized as the initial experimental design. In the following adaptive experiment design process, the distance from the center experimental point to other half-star shape experimental points are determined not by the constant parameter f, but by the location of limit state estimated using all the previous information of $g(\mathbf{X})$.

4.2 Adaptive experimental design

4.2.1 *Initial experimental design–Latin hypercube design (LHD)*

The statistical method of the Latin hypercube design (LHD) was developed to generate a set of samples from a multidimensional distribution. The technique was first described by McKay et al. (1979), and further elaborated by Iman and Conover (1980). LHD is popular in design and analysis of computer experiments. The location of LHD points is determined through a random procedure and a complete theory can be found in Forrester's work (2008). The goal of a good LHD is to make the selected sampling points as uniform as possible to cover the entire design space. In this paper, the φ_p-criterion is selected as a uniformity measure and the translational propagation algorithm proposed by Viana et al. (2010a) is used to obtain the optimal uniform LHD. In this algorithm, sampling points are determined by minimizing the following criterion:

$$\varphi_p = \left[\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} d_{ij}^{-p}\right]^{1/p} \tag{19}$$

where d_{ij} is the distance between two sample points, $\mathbf{x}_i$ and $\mathbf{x}_j$,

$$d_{ij} = d(\mathbf{x}_i, \mathbf{x}_j) = \left[\sum_{k=1}^{n} \left|x_{ik} - x_{jk}\right|^t\right]^{1/t} \tag{20}$$

and $p = 50$ and $t = 1$ were recommended by Jin et al. (2005). The space-filling property of optimal uniform LHD ensures that no two sampling points are too close to each other; uniformly distributed experimental points would enhance the approximation capacity of surrogate models. In this paper, LHD is used as an initial set of experimental designs.

4.2.2 *Adaptive experimental design process*

Based on the developed DWMLS model, the FORM algorithm can determine an interim MPFP, which is presumably located close to the true MPFP step by step. The iterative process adds new experimental points to improve the accuracy of the surrogate model near the MPFP. In this paper, an additional sampling point is added in the location of the limit state surface in each variable direction, starting from the current MPFP.

Let us assume that $\mathbf{x}_k^*$ is the MPFP at kth DWMLS model. Then, the performance function $g(\mathbf{X})$ is evaluated at $\mathbf{x}_k^*$, and additional sampling points will be added. Firstly, it is necessary to check the magnitude of $g(\mathbf{x}_k^*)$, which helps judge whether $\mathbf{x}_k^*$ is close enough to the true MPFP. A ratio factor C_r is introduced to measure the closeness of $\mathbf{x}_k^*$ to limit state surface, as

$$C_r = \frac{g\left(\mathbf{x}_k^*\right)}{g(\boldsymbol{\mu})} \tag{21}$$

where $\boldsymbol{\mu}$ is the mean of random variables. If C_r is smaller than the threshold C_r^0 (always set as 0.05), then $\mathbf{x}_k^*$ is considered to be close enough to the limit state surface, and the following single point is added to the existing sampling points:

$$\mathbf{x}_{add}^k = \boldsymbol{\mu} - g(\boldsymbol{\mu})\frac{\boldsymbol{\mu} - \mathbf{x}_k^*}{g(\boldsymbol{\mu}) - g\left(\mathbf{x}_k^*\right)} \tag{22}$$

where $\mathbf{x}_{add}^{k}$ is the newly added experimental design in $k+1$ th iteration.

On the other hand, when C_r is larger than the threshold, it means that $\mathbf{x}_k^*$ is relatively far away from the limit state surface. In such a case, sampling points are added, starting from $\mathbf{x}_k^*$, close to the limit state surface in the direction of each variable (see Fig. 4). By doing this, the accuracy of the surrogate model is improved locally near the current MPFP and the limit state surface. This adaptive experimental design process can be applicable to other surrogate models. Unlike other surrogate models, the MLS has no expensive inner parameter optimization during the adaptive modeling process, so a temporary DWMLS model $\tilde{g}_{N_k+1}(\mathbf{x})$ can be established quickly based on existing N_k+1 points. The distance between $\mathbf{x}_k^*$ and the limit state surface in the *i-th* variable direction, Δ_i, is calculated using the first-order Taylor series expansion, as

$$\Delta_i = -\frac{g(\mathbf{x}_k^*)}{\left.\frac{\partial \tilde{g}_{N_k+1}(\mathbf{x})}{\partial x_i}\right|_{\mathbf{x}_k^*}} \tag{23}$$

Therefore, the following n + 1 samples are added at the $k+1$ th experimental design:

$$\begin{array}{l} \mathbf{x}_k^* \\ \mathbf{x}_k^* + [\Delta_1, 0, \ldots, 0] \\ \mathbf{x}_k^* + [0, \Delta_2, \ldots, 0] \\ \vdots \\ \mathbf{x}_k^* + [0, 0, \ldots, \Delta_n] \end{array} \tag{24}$$

The first-order approximation in (23) can have a large error when the performance function is highly nonlinear and the partial derivative becomes too small. Therefore, in order to prevent Δ_i from being too large, a threshold $\Delta_i^c = t \times \sigma_i$ is proposed, where σ_i is the standard deviation of i-th random variable and t is a constant between 2.0 and 3.0. Hence the adjusted sampling points become:

$$\mathbf{x}_{add_i}^{k} = \begin{cases} \mathbf{x}_k^* + [0, ..., \Delta_i, ..., 0] & \Delta_i \le \Delta_i^c \\ \mathbf{x}_k^* + [0, ..., \mathrm{sgn}(\Delta_i)\Delta_i^c, ..., 0] & \Delta_i > \Delta_i^c \end{cases} \tag{25}$$

After n + 1 additional experimental design points are determined, the performance functions are evaluated at these locations. After updating the DWMLS with more points, FORM is used to determine a new $\mathbf{x}_{k+1}^*$.

The adaptive experimental design process repeats until the limit state surface can be approximated accurately, particularly in the region near the MPFP. The following two convergence criteria are used:

$$\begin{cases} |\beta_k - \beta_{k+1}| \le \varepsilon_\beta \\ \left\|\mathbf{x}_k^* - \mathbf{x}_{k+1}^*\right\| \le \varepsilon_{MPFP} \end{cases} \tag{26}$$

where the two tolerances, ε_β and ε_{MPFP}, are fixed are 10^{-3}.

5 Procedure of the proposed method

The proposed two-stage hybrid reliability analysis method can be divided into (1) DWMLS enhanced FORM iterations and (2) MCS to calculate the failure probability. In the first stage, the DWMLS surrogate model is constructed and updated by adaptively adding sampling points near the limit state surface. In the second stage, MCS is performed on the final DWMLS to calculate the probability of failure. Figure 2 summarizes the step-by-step procedures of the proposed algorithm.

(1) Determine the initial experimental designs, $\mathbf{X}_{\text{initial}}$: The initial number of samples is determined by $N_1 = 3n$ where n is the number of random variables. The initial optimal LHD is generated in standard normal space U of random variables, so it is necessary to define the sampling range in U space. The sampling range on each dimension is selected by $[\mu - f\sigma_i,\ \mu + f\sigma_i]$ where f is fixed at 4.0. Then, an optimal LHD U_{initial} is generated through the SURROGATES Toolbox (Viana 2010). The independent experimental points in U_{initial} are transferred into the physical space of mutually correlated non-normal random variables by Nataf's rule, and the initial experimental design $\mathbf{X}_{\text{initial}}$ is determined. Since the mean point $\bar{\mathbf{x}}$ is not included in $\mathbf{X}_{\text{initial}}$, $\bar{\mathbf{x}}$ is added to $\mathbf{X}_{\text{initial}}$; thus, the number of initial samples is $N_1 + 1$.

(2) Compute the value of the performance function at each point of $\mathbf{X}_{\text{initial}}$:

$$g_k = g(\mathbf{x}_k), \quad k = \{1, 2, ..., N_1 + 1\} \tag{27}$$

(3) Calculate weight factors assigned to each point according to (8), and fit the first conventional MLS. Apply FORM based on the first surrogate model and determine the first reliability index β_1 and MPFP $\mathbf{x}_1^*$.

(4) Calculate the values of the performance function at the current MPFP and check the closeness ratio factor C_r by (21). If C_r is smaller than the threshold value, obtain the new adding point $\mathbf{X}_{add}$ using (22) and skip to step (7).

(5) Add the new MPFP observation evaluated from the above step into existing sampling points and constructing a temporary DWMLS to estimate the partial derivatives to each random variables on MPFP.

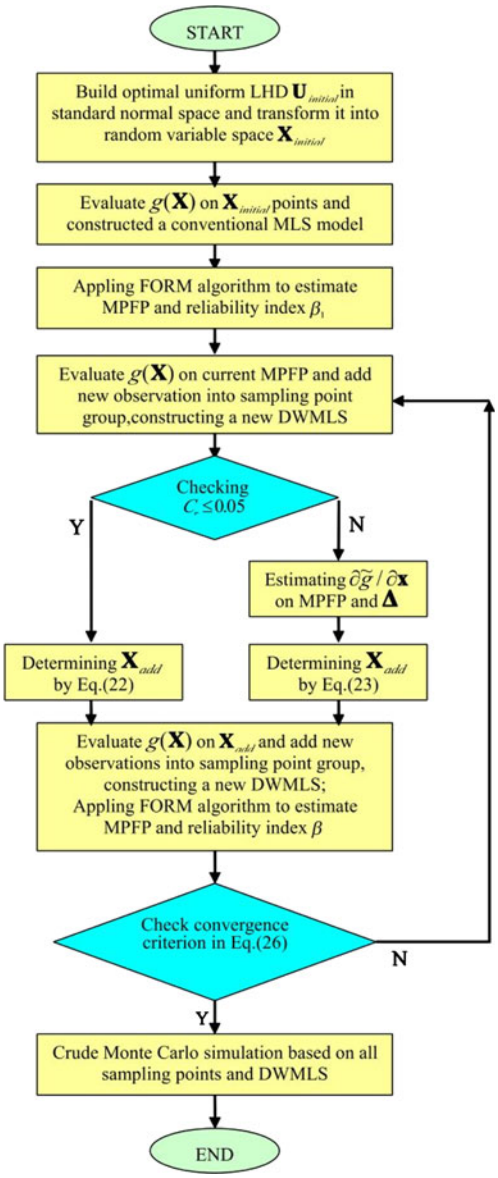

Fig. 2 Flow chart for the proposed doubly weighted moving least square method for reliability analysis

(6) Calculate the length Δ_i by (23) on each direction, compare with Δ_i^c and then determine $\mathbf{x}_{add_i}$ based on (25), and add to $\mathbf{X}_{add}$.
(7) Calculate the values of the performance function at given $\mathbf{X}_{add}$ and add these points into the existing sampling points.
(8) Calculate the weight factors assigned to each point according to (15) and build a DWMLS model.
(9) Apply the FORM algorithm to the DWMLS model and determine the reliability index β_{HL} and MPFP.
(10) Repeat step (4) $\sim$ step (9) until the convergence criteria in (26) is satisfied.
(11) Perform MCS using DWMLS to calculate the probability of failure.

6 Numerical examples

In this section, five typical examples involving explicit and implicit functions from structural applications are presented to illustrate the efficiency and accuracy of the proposed DWMLS method. Since it is assumed that numerical simulation is far more expensive than developing surrogate models, the number of numerical simulations N_s is used as a measure of efficiency. The proposed DWMLS method is a hybrid reliability analysis approach where first, adaptive experimental design combined with FORM algorithm is mainly aimed at positioning the MPFP, and then, subsequent Monte Carlo simulation is used to estimate the failure probability p_f. Therefore, the accuracy of the proposed method would not only depend on MCS for p_f, but also the interim FORM. In numerical examples, comparisons have been made with the other reliability analysis methods presented in literature to evaluate the performance of the proposed method.

6.1 Example 1: a nonlinear limit state function

In the first numerical example, the following two-dimensional performance function is used:

$$g(\mathbf{x}) = \exp[0.4(x_1 + 2) + 6.2] - \exp[0.3x_2 + 5] - 200 \tag{28}$$

where x_1 and x_2 are independent, standard normal random variables. This example is widely used in reliability analysis methods (e.g., Kang et al. 2010; Kaymaz and McMahon 2005; Duprat and Sellier 2006; Nguyen et al. 2009).

Figures 3, 4, and 5 illustrate the process of the DWMLS algorithm. Figure 3 shows the initial $7(= 3 * 2 + 1)$ LHD samples along with the true limit state surface. Figure 4 shows the initially estimated MPFP1 along with two additional sampling points. Figure 5 shows two more iterations with MPFP2 and MPFP3 in a partially enlarged drawing. It is clear that the estimated MPFP converges to the true MPFP. The iteration history of interim MPFP search using the proposed method is presented in Table 3. The performance function is evaluated twelve times during the three iteration cycles before arriving at the final convergence.

Table 4 presents the results of failure probability P_f, reliability index β, MPFP and involved computational effort Ns using MCS (sample size $Ns = 10^6$), FORM, several recent response surface methods and proposed MLS and DWMLS method. For crude MCS, the coefficient of variation (COV) of the estimated failure probability P_f with sample size of Ns is:

$$\mathrm{COV} = \sqrt{\frac{1 - P_f}{N_s P_f}} \tag{29}$$

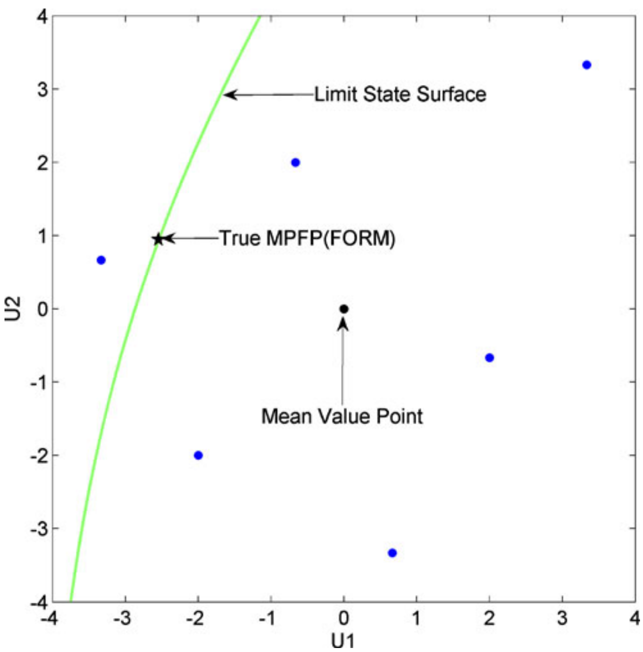

Fig. 3 The initial LHD and the mean value point for example 1(Step 1 of the proposed method)

While for all examples in this paper, the COV of P_f is presented in parenthesis beside the failure probability results. However, it should be noted that the exact results of P_f is estimated by the MCS with 10^6 samples and the exact β and MPFP are set as results get by classical FORM algorithm. Comparison of P_f aims at accuracy of the proposed

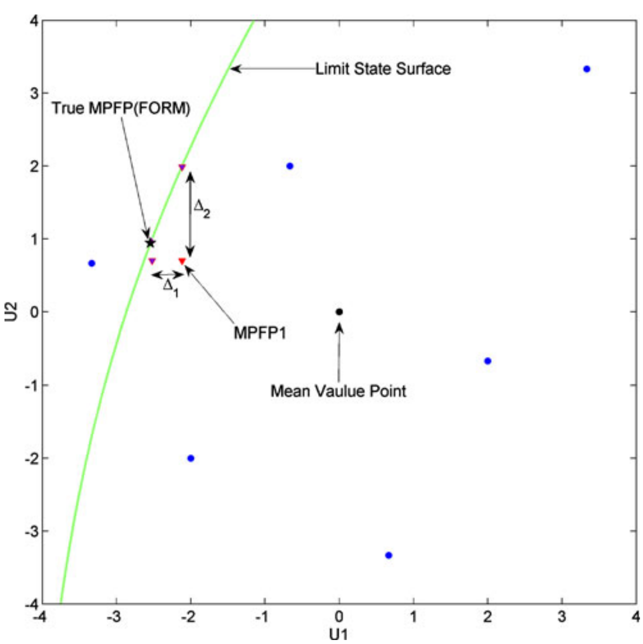

Fig. 4 Add n new experimental points based on previous MPFP for example 1(Step 6 of the proposed method)

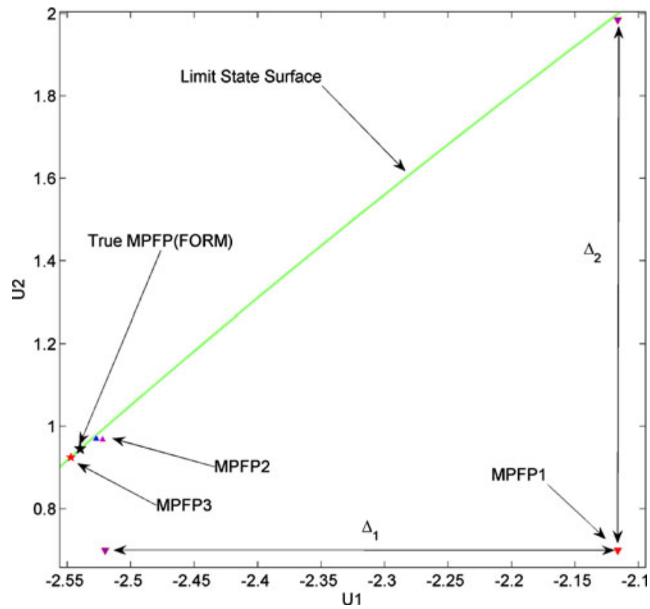

Fig. 5 Add one new experimental point based on previous MPFP for example 1(Step 4 of the proposed method)

method, while comparison of β and MPFP indicate the performance of proposed method in positioning and converging to the true MPFP.

As illustrated in Figs. 3, 4 and 5, the nonlinearity of the first example in the vicinity of the design point is relatively low, so the results of all methods are close each other. From Table 4, the proposed reliability method using MLS and DWMLS can estimate both P_f, β and MPFP well. However, with the same number of function evaluations ($Ns = 12$), the DWMLS method can obtain slightly more accurate estimation of reliability index β as well as failure probability P_f than MLS method. While compared with FORM and other recent methods, the proposed DWMLS method shows more accurate reliability estimation. Therefore, the results in Table 4 indicate that proposed DWMLS method improves not only the accuracy in positioning the MPFP and estimating reliability index β, but also the efficiency as only 12 function evaluations are needed to converge.

6.2 Example 2: dynamic response of a nonlinear oscillator

In order to investigate the performance of the proposed method in complex problems with more random variables

Table 3 Iteration history of DWMLS method—Example #1

Iteration	β	MPFP	N_S
1	2.23	[−2.12, 0.70]	7
2	2.70	[−2.52, 0.97]	3
3	2.71	[−2.55, 0.92]	2

Table 4 Summary of results—Example #1 (the digit with bold font are regarded as exact results for comparison and the accuracy of other results to compare with exact one are given in the parentheses below each digit)

Method	MPFP	β	P_f	N_S
Monte Carlo simulation	[−2.54,0.95]	2.71	3.68E-3 (COV=1.7%)	1.0E6
FORM (H–L algorithm)	[−2.54,0.95]	2.71	3.37E-3 (8.4%)	27
RSM in Kang (2010)	[−2.54,0.94]	2.71 (0.0%)	3.36E-3 (8.7%)	12
RSM in Nguyen (2009)	[−2.57,0.86]	2.71 (0.11%)	3.39E-3 (7.9%)	12
RSM in Kaymaz (2005)	[−2.56,0.82]	2.69 (0.89%)	3.62E-3 (1.6%)	8
RSM in Duprat (2006)	[−2.54,0.95]	2.71 (0.0%)	3.36E-3 (8.7%)	21
MLS + MCS	[−2.55,0.91]	2.71 (0.07%)	3.56E-3 (3.3%)	12
DWMLS + MCS	[−2.55,0.92]	2.71 (0.0%)	3.61E-3 (1.9%)	12

and greater non-linearity, the second example deals with a nonlinear undamped single degree of freedom system as presented in Fig. 6. This example is also used in several other studies (see, e.g. Echard et al. 2011; Rajashkhar and Ellingwood 1993; Schueremans and Gemert 2005). The nonlinear oscillator with random system parameters subjected to a rectangular pulse load with random duration and amplitude is presented, and the performance function is defined by:

$$g(c_1, c_2, m, r, t_1, F_1) = 3r - |z_{\max}| = 3r - \left| \frac{2F_1}{mw_0^2} \sin\left(\frac{w_0 t_1}{2}\right) \right| \quad (30)$$

where $w_0 = \sqrt{(c_1 + c_2)/m}$. The random parameters of six basic variables are listed in Table 5.

Table 6 presents the reliability analysis results by different methods in the recent literature along with proposed hybrid reliability method with MLS and DWMLS method. The reported results come from directional sampling (DS) and importance sampling (IS) methods combined with typical surrogate models (e.g., polynomials, splines, and neural network), and an active learning method combining Kriging and Monte Carlo simulation (AK-MCS). Table 6 shows that the results from DWMLS are accurate compared with FORM and other variants methods in estimating the failure probability with the least number of function evaluations. It is noted that the results reported by Schueremans and Gemert (2005) and by Echard et al. (2011) are based on interpreting the last column of Table 5 as a standard deviation, not as a COV.

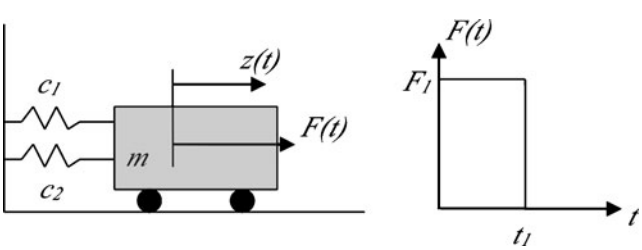

Fig. 6 Non-linear oscillator—system definition and applied load

6.3 Example 3: a cantilever beam

The above two examples demonstrated the performance of the proposed method in estimating the failure probability and locating the MPFP. The proposed method can also be applied to multiple performance functions; i.e., calculating the system probability of failure. In such a case, adaptive experimental design is applied to each performance function.

The third example considers system reliability with two limit state surfaces. A cantilever beam, as shown in Fig. 7, is subjected to a tip load of 200.0 N. Two failure criteria are considered: (i) the displacement at the tip of the beam should be less than 0.005 m, as expressed in (30), and (ii) maximum stress in the beam should be less than 33 MPa, as expressed in (31).

$$\text{Displacement limit state}: \quad g_1(X) = 0.005 - \frac{4PL^3}{Ebh^3} \leq 0 \quad (31)$$

$$\text{Stress limit state}: \quad g_2(X) = 33.0 \times 10^6 - \frac{12PL}{bh^2} \leq 0 \quad (32)$$

Table 5 Probabilistic distribution of random variables—Example 2

Random variable	Distribution type	Mean value	Standard deviation
c_1	Normal	1.0	0.10
c_2	Normal	0.1	0.01
m	Normal	1.0	0.05
r	Normal	0.5	0.05
t_1	Normal	1.0	0.20
F_1	Normal	1.0	0.20

Table 6 Summary of reliability results—Example #2 (the digit with bold font are regarded as exact results)

Method	N_S	P_f	β
Monte Carlo simulation	1E6	3.89E-2 (COV=1.9%)	1.72
FORM	84	4.29E-2	1.72
Directional Sampling(DS)*	1281	3.5E-2	1.81
DS+Polynomial*	62	3.4E-2	1.83
DS+Spline*	76	3.4E-2	1.83
DS+Neural Network*	86	2.8E-2	1.91
Importance Sampling(IS)*	6144	2.7E-2	1.93
IS+Polynomial*	109	2.5E-2	1.96
IS+Spline*	67	2.7E-2	1.93
IS+Neural Network*	68	3.1E-2	1.87
AK-MCS+U**	58	2.83E-2	1.91
AK-MCS+EFF**	45	2.85E-2	1.90
MLS+MCS	52	3.95E-2	1.73
DWMLS+MCS	43	3.94E-2	1.73

*the results come from reference Schueremans and Gemert (2005)
** the results come from reference Echard et al. (2011)

In the above equations, L, b, h indicate the length, width and height of the beam, whose probability distribution properties are shown in Table 7. The modulus of elasticity of the beam was taken to be 70.0 GPa. The system probability of failure is defined by

$$p_f = 1 - P[g_1(\mathbf{X}) \geq 0 \cup g_2(\mathbf{X}) \geq 0] \tag{33}$$

As the closed form expressions of performance functions are available, it is possible to estimate system reliability with Monte Carlo simulations. Table 8 listed the results of MCS with a million samples and the result estimated by the proposed DWMLS method. It should be noted that 98 samples in the DWMLS method include all the points used to locate two limit state surfaces and corresponding two MPFPs until convergence. In order to inspect the accuracy and efficiency of the DWMLS method on finding each MPFP, the FORM algorithm is used for each limit state function to estimate an individual reliability index β for comparison in Table 9. From the results in Table 8,

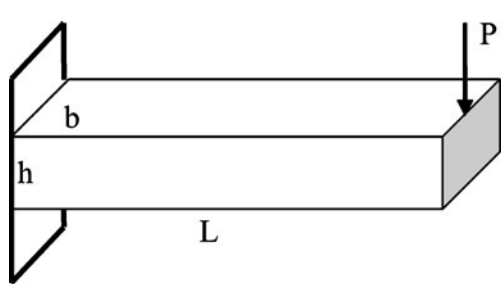

Fig. 7 Simply supported beam

Table 7 Probabilistic characteristics of the basic random variables of the cantilever beam

Random variable	Mean (m)	Standard deviation	Distribution type
L	0.90	0.090	Normal
b	0.08	0.008	Normal
h	0.04	0.004	Normal

the failure probability of MLS method is more accurate than proposed DWMLS, this is because 34(=132–98) more experimental points are used in MLS. For highly nonlinear performance function g1(x), FORM has difficulty in convergence (Ns = 803), while DWMLS can position MPFP efficiently without a large number of function evaluations ($Ns = 98$).

6.4 Example 4: a nonlinear limit state function with ten random variables

In order to compare with other methods, we chose an analytical example, which had been tested by several methods. In the fourth example, the proposed method was extended to a nonlinear limit state function with ten random variables, which is considered as a high-dimensional example. The limit state function is defined as:

$$g(\mathbf{x}) = \sum_{i=1}^{10} x_i + 10x_1^2x_2^2 + x_2^2x_3^2 + x_3^2x_4^2 + x_4^2x_5^2 + x_5^2x_6^2 + x_7^2x_8^2 + x_8^2x_9^2 + x_9^2x_{10}^2 - 16 \tag{34}$$

Ten input random variables $\mathbf{x} = \{x_1,\ x_2,\ldots,x_{10},\}^{\mathrm{T}}$ follow the same normal distribution with a mean value $\mu_{\mathrm{x}} = 1.0$ and standard deviation $\sigma_{\mathrm{x}} = 0.2$.

The predicted failure probability based on the proposed hybrid method is presented in Table 10 along with the results from other methods. In the table, MPP-UDR-1, −2, and −3 stand for UDR methods, expanded at MPFP, with different sample sizes. Likewise, SGI-1 and −2 represent sparse grid sampling methods with different sample sizes of expansion points around the MPFP. As can be observed in the table, the performance of proposed hybrid analysis framework is undoubtedly comparable to both MPP-UDR

Table 8 Comparison of failure probability for the cantilever beam system

Method	P_f	N_S
Monte Carlo simulation	8.33E-3(COV=1.1%)	1E6
MLS + MCS	8.86E-3	132
DWMLS + MCS	8.98E-3	98

Table 9 Comparison of reliability index for the cantilever beam system

Method	g_1		g_2	
	N_S	β	N_S	β
FORM	803	2.50	24	2.52
DWMLS + MCS	–	2.55	–	2.54

and SGI methods. Generally, MPP-UDR yielded relatively large errors even with large function evaluation numbers. The SGI method proposed by Xiong et al. (2010) showed to be more accurate in predicting reliability, but it demanded far more numbers of function evaluation. In the case of this high-dimensional example, the proposed hybrid method performed better in terms of both accuracy and the number of function evaluations. At the same time, DWMLS showed to be more efficient than MLS in locating the MPFP even both obtained nearly the same reliability result. The reason for this performance improvement over other local integration or interpolation methods is mainly because the hybrid method can find the MPFP efficiently in the first stage and capture the global profile of limit state function in the second stage. Consequently, the proposed method turns out to be good at estimating reliability for a high-dimensional limit state function.

6.5 Example 5: ten-bar truss structure

As a practical example using finite element analysis, a ten-bar truss structure (as shown in Fig. 8) is considered. The ten-bar truss structure is a classical structural analysis problem and widely studied (Wei and Rahman 2007; Kang et al. 2010). The structure is simply supported at nodes 1 and 4, and is subjected to two concentrated loads P = 10^5 lb at nodes 2 and 3. The truss members, which have random cross-sectional areas A_i, $i = 1, 2, \ldots, 10$, are made of an aluminum alloy with Young's modulus E = 10^7 psi. The

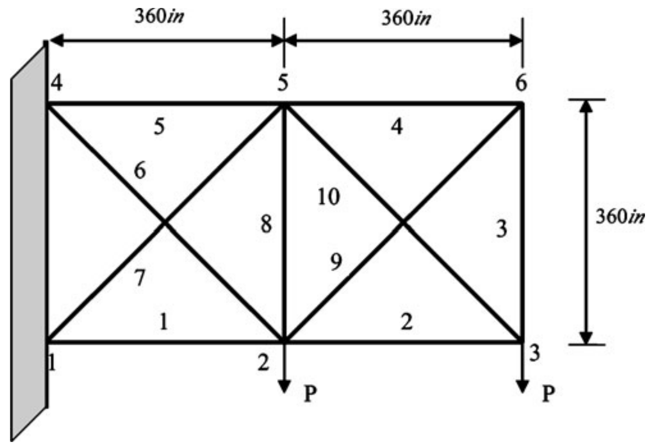

Fig. 8 A ten-bar truss structure

input random variables $\mathbf{X} = \{A_1, A_2, \ldots, A_{10}\}^T$ follow normal distribution and have a mean $\mu = 2.5\ in^2$ and standard deviation $\sigma = 0.5\ in^2$. The maximum vertical displacement $v(\mathbf{X})$, which occurs at node 3, is limited to $v_0 = 18\ in$. Therefore, the performance function is defined as:

$$g = v_0 - v_{\max}(\mathbf{X}) = 18.0 - v_{\max}(\mathbf{X}) \quad (35)$$

The structural analysis was done in MSC/NASTRAN, a commercial finite element analysis program. The estimation results are reported in Table 11.

Table 11 lists the predicted failure probability of ten-bar truss and the associated computational effort using FORM, several SORM, three variants of MPP-UDR, crude MCS (10^6 samples), and proposed hybrid reliability method

Table 10 Failure probability of example #4

Method	N_S	P_f	*Error (%)*
Monte Carlo simulation	10E6	0.0083 (COV=0.3%)	0
MPP-UDR-1*	131	0.0057	31.3
MPP-UDR-2*	251	0.0061	26.5
MPP-UDR-3*	971	0.0058	30.1
SGI-1*	509	0.0076	8.4
SGI-2*	709	0.0079	4.8
MLS + MCS	195	0.0076	8.4
DWMLS + MCS	155	0.0078	6.0

* the results come from reference Xiong et al. (2010)

Table 11 Failure probability of ten bar truss

Method	N_S	P_f	β
Monte Carlo simulation	10E6	0.139 (COV = 0.25%)	1.08
FORM(HL algorithm)	127	0.086	1.36
SORM(Breitung)	506	0.129	1.13
SORM(Hohenbichler)	506	0.152	1.03
SORM(Cai and Elishakoff)	506	0.147	1.05
MPP-UDR with linear integration*	187	0.146	1.05
MPP-UDR with quadratic integration*	187	0.140	1.08
MPP-UDR with simulation**	187	0.147	1.05
MLS + MCS	113	0.146	1.37
DWMLS + MCS	98	0.146	1.37

*the results come from reference Wei and Rahman (2007)
**the results come from reference Rahman and Wei (2006)

combined with ordinary MLS and DWMLS. From Table 10, both versions of MLS predict the failure probability more accurately than FORM and all three variants of SORM. On the other hand, there is no improvement on accuracy for DWMLS compared with ordinary MLS. This is because 15(=113–98) more experiments points are evaluated in the convergence of MPFP for ordinary MLS. The MPP-UDR method with quadratic integration shows the best accuracy. However, it can be noted that the UDR methods require about two times more function evaluations than the hybrid DWMLS method.

7 Conclusions

In this paper, a doubly weighted moving least squares and a two-stage hybrid reliability analysis scheme are proposed to improve the surrogate model for reliability analysis. The proposed method provides a larger weight to the point near the MPFP and turns to sample more experimental points closer to the limit state surface. From the benchmark examples #1 and #2, it was shown that, in comparison to classical MCS and FORM with various surrogate models, DWMLS improves the convergence speed, and locates the limit state function more accurately with a less number of sampling points. In the example #3, the proposed two-stage reliability analysis scheme was able to calculate system reliability by identifying multiple MPFPs. The last two examples have revealed the steady performance improvement of proposed hybrid method in high-dimensional nonlinear problems and it turns to be more favorable or at least comparable than several recent developed reliability methods.

However, it should be noted that the proposed method is not intended as a replacement of existing surrogate models and reliability methods, but as a possible complement and improvement to these methods. Furthermore, more studies are needed to extend the proposed method to reliability based design optimization of complex systems.

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