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Deciding Degree of Conservativeness in Initial Design Considering Risk of Future Redesign

Early in the design process, there is often mixed epistemic model uncertainty and aleatory parameter uncertainty. Later in the design process, the results of high-fidelity simulations or experiments will reduce epistemic model uncertainty and may trigger a redesign process. Redesign is undesirable because it is associated with costs and delays; however, it is also an opportunity to correct a dangerous design or possibly improve design performance. In this study, we propose a margin-based design/redesign method where the design is optimized deterministically, but the margins are selected probabilistically. The final design is an epistemic random variable (i.e., it is unknown at the initial design stage) and the margins are optimized to control the epistemic uncertainty in the final design, design performance, and probability of failure. The method allows for the tradeoff between expected final design performance and probability of redesign while ensuring reliability with respect to mixed uncertainties. The method is demonstrated on a simple bar problem and then on an engine design problem. The examples are used to investigate the dilemma of whether to start with a higher margin and redesign if the test later in the design process reveals the design to be too conservative, or to start with a lower margin and redesign if the test reveals the design to be unsafe. In the examples in this study, it is found that this decision is related to the variance of the uncertainty in the high-fidelity model relative to the variance of the uncertainty in the low-fidelity model.
[DOI: 10.1115/1.4034347]

1 Introduction

Engineering design is an iterative process. Early in the design process, such as at the preliminary design phase, engineers often utilize low-fidelity models which may be associated with high uncertainty. Model uncertainty is classified as *epistemic uncertainty* when it arises due to lack of knowledge, it is reducible by gaining more information, and it has only a single true (but unknown) value [1–3]. In addition, almost all engineering designs are subject to *aleatory uncertainty* (e.g., loading, material properties, etc.). The input parameter uncertainty is classified as aleatory if it is due to natural or inherent variability, it is irreducible, and it is a distributed quantity.[1] Later in the design process, when prototypes are tested or high-fidelity simulations are performed, new knowledge will become available that reduces epistemic uncertainty and may result in a decision to change the initial design. Changing the initial design, referred to as *redesign* or *engineering change* (EC), is an important issue for industry and engineering management [6,7]. Redesign is often viewed negatively because it is associated with costs and delays; however, it is also an opportunity for design improvement [6].

Research related to redesign, or engineering change, has mostly been performed at the system level requiring a high level of abstraction. These methods include the change prediction method (CPM) [8], the RedesignIT computer program [9], a pattern-based redesign methodology [10], a combination of a function-behavior-structure (FBS) linkage model with the CPM method [11], and a Monte Carlo simulation (MCS)-based method of estimating redesign risk [12].

At a lower level of abstraction, redesign is typically triggered when an initial design is later revealed to not meet specifications or constraints due to model uncertainty. Redesigning one component may trigger the propagation of changes throughout the system; however, this subsequent change propagation is not directly

[1]The distinction between aleatory and epistemic uncertainties is somewhat controversial and not always clear. Faber argues that the classification of uncertainty has a dependence on modeling scale as well as time [4] and O'Hagan and Oakley raised the question of whether there is any true randomness or if all uncertainty might be considered epistemic [5].

Contributed by the Design Automation Committee of ASME for publication in the Journal of Mechanical Design. Manuscript received March 3, 2016; final manuscript received July 17, 2016; published online September 12, 2016. Assoc. Editor: Shapour Azarm.

addressed in this study. Villanueva et al. simulated the effects of future tests and redesign on an integrated thermal protection system (ITPS) considering the effect of redesign on the uncertainty in the probability of failure [13]. Matsumura et al. compared reliability-based design optimization (RBDO) considering future redesign to traditional RBDO [14]. Villanueva et al. demonstrated the tradeoff between expected design performance and probability of redesign for the ITPS example [15]. Price et al. compared designer versus company perspectives on starting with a higher margin and possibly redesigning to improve performance to starting with a lower margin and possibly redesigning to improve safety [16]. This study develops a generalized formulation of the previously application-specific formulations [13,15,16] and explores how the degree of conservativeness in the initial design relates to the expected design performance after possible redesign. In related work, Price et al. introduced a Kriging surrogate to represent epistemic model uncertainty in order to consider spatial variations in model uncertainty in the context of simulating the effects of future tests and redesign [17].

Research regarding redesign is similar to research regarding design flexibility. Design flexibility might be summarized as the potential for a design to be easily modified in response to a change in requirements [18]. Similar to redesign, design flexibility is often understood to be a means of coping with uncertainty and in particular a means of handling time variant design requirements [18]. As demonstrated by De Neufville and Scholtes, the expected value of a project can be significantly increased if the project can be easily adapted to new circumstances in order to avoid downside risks or exploit opportunities [19]. This is similar to the idea presented in this study of improving expected design performance through redesign for safety and redesign for performance. Roser and Kazmer proposed the flexible design method which allows a designer to minimize total expected costs while considering possible design changes occurring later in the design process [20,21]. Roser et al. demonstrated an economic method for deciding between design changes with different levels of uncertainty and different associated costs [22].

Redesign is often caused by epistemic model uncertainty. If engineers had access to models that were capable of perfectly predicting design performance and the necessary resources to exercise them in the design process, then the initial design would definitely satisfy design constraints and redesign could largely be avoided. Assuming a known true model, reliability-based design optimization (RBDO) has mostly focused on ensuring a prescribed level of reliability given known aleatory parameter uncertainty [23–25]. Therefore, most RBDO formulations are implicitly conditional on the model of the system exactly matching the true physics of the system. Some studies have sought to specifically address the incorporation of model uncertainty into reliability-based design [26–28]. However, to compensate for all the lack of knowledge (i.e., epistemic model uncertainty) that is present at the initial design stage, the initial design may need to be very conservative. In reality, engineering design is an iterative process where over time designs are tested, experiments are performed, models are improved, and new knowledge is gained that reduces epistemic uncertainty. If there will be a future opportunity to reduce epistemic uncertainty and possibly change the initial design (i.e., redesign), then this may affect the selection of the initial design.

Typically, an initial design will have some *margin* relative to design constraints in order not only to improve safety, but also to provide some insurance against future redesign [29]. When selecting a margin for the initial design, designers face a dilemma in whether to start with a larger initial margin (i.e., more conservative initial design) and possibly performing redesign to improve performance versus starting with a smaller margin (i.e., less conservative initial design) and possibly performing redesign to restore safety. This decision to be more or less conservative in the design process is similar to the question of optimistic versus pessimistic design practices as explored by Thornton [30]. This paper proposes a general method for optimizing the margins governing a two-stage deterministic design process in order to control the epistemic uncertainty in the final design, design performance, and probability of failure. The method considers the probability of future redesign while selecting the initial design. This allows for the tradeoff between expected final design performance and redesign risk while still ensuring reliability. The method is demonstrated on a simple bar problem and then on an engine design problem.

The methods are described in Sec. 2. In Sec. 3, the method is applied to the design of a minimum weight uniaxial tension bar and then to the engine design of a supersonic business jet. The discussions and conclusions are presented in Sec. 4. Limitations of the proposed method and perspectives for future work are presented in Sec. 5.

2 Methods

The deterministic design process consists of selecting an initial design, testing the initial design, and possibly performing calibration and redesign. The process is controlled by an initial margin n_{ini}, lower and upper bounds on acceptable margins n_{lb} and n_{ub}, and a redesign margin n_{re}. These margins $\boldsymbol{n} = \{n_{\mathrm{ini}}, n_{\mathrm{lb}}, n_{\mathrm{ub}}, n_{\mathrm{re}}\}$ are optimized as described in Sec. 2.1. The optimizer calls a function to perform a crude Monte Carlo simulation (MCS) of epistemic error realizations as described in Sec. 2.2. The complete design, test, and possible calibration and redesign process is carried out for each realization of epistemic error as described in Sec. 2.3. Probability of redesign, expected probability of failure, and expected design cost are calculated from the MCS as described in Sec. 2.4.

2.1 Optimization of Margins.

The margins $\boldsymbol{n}$ are optimized to minimize the expected value of the design cost function subject to constraints on expected probability of failure and probability of redesign. The formulation of the optimization problem is

$$\begin{aligned} \min \quad & \mathbb{E}_E[\mathbb{E}_U[f(\boldsymbol{X}_{\mathrm{final}}, \boldsymbol{U})]] \\ \text{w.r.t} \quad & \boldsymbol{n} = \{n_{\mathrm{ini}}, n_{\mathrm{lb}}, n_{\mathrm{ub}}, n_{\mathrm{re}}\} \\ \text{s.t.} \quad & \mathbb{E}_E[P_{f,\mathrm{final}}] \le p_f^\star \\ & p_{\mathrm{re}} \le p_{\mathrm{re}}^\star \\ & n_{\mathrm{lb}} \le n_{\mathrm{ub}} \\ & \boldsymbol{n}_{\min} \le \boldsymbol{n} \le \boldsymbol{n}_{\max} \end{aligned} \tag{1}$$

where $\mathbb{E}_E[\cdot]$ is the expectation with respect to epistemic uncertainty, $\mathbb{E}_U[\cdot]$ is the expectation with respect to aleatory uncertainty, $f(\cdot,\cdot)$ is an objective function, $\boldsymbol{X}_{\mathrm{final}}$ is the vector of final design variables, $\boldsymbol{U}$ is a vector of aleatory random variables, $P_{f,\mathrm{final}}$ is the final probability of failure, and p_{re} is the probability of redesign. The final design and final probability of failure are epistemic random variables. In the objective function, the mean is first calculated with respect to aleatory uncertainty for each design realization and then the expectation is calculated over the means with respect to epistemic uncertainty. The optimization is based on an MCS as seen in Fig. 1. Solving the optimization problem for different values of $p_{\mathrm{re}}^\star$ results in a tradeoff between expected cost and probability of redesign. Covariance matrix adaptation evolution strategy (CMA-ES) with a penalization strategy to handle the constraints is used to solve the optimization problem [31].

2.2 Monte Carlo Simulation of Epistemic Model Error.

The epistemic model uncertainty and aleatory parameter uncertainty are treated separately. To represent epistemic model uncertainty, we introduce the epistemic random variables E_L and E_H to represent the error in the low- and high-fidelity models, respectively. In this work, it is assumed that design models improve over the course of the design process with low-fidelity models being available early and high-fidelity models becoming

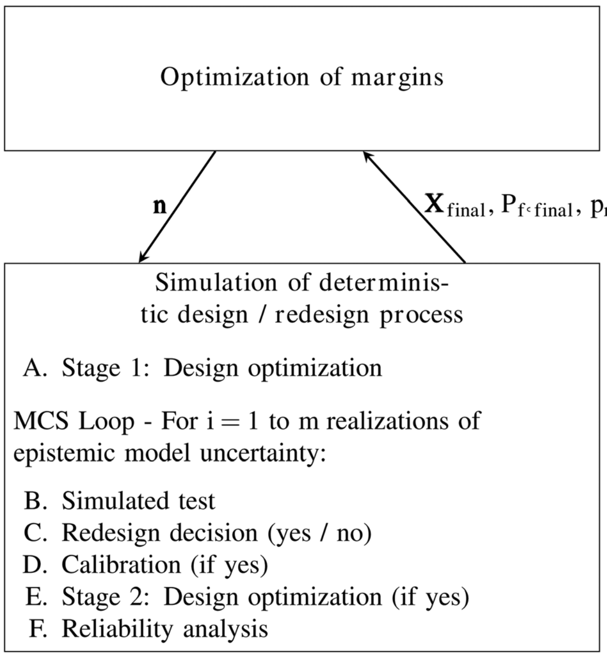

Fig. 1 The overall design process consists of optimization of the margins based on an MCS of the deterministic design/redesign process

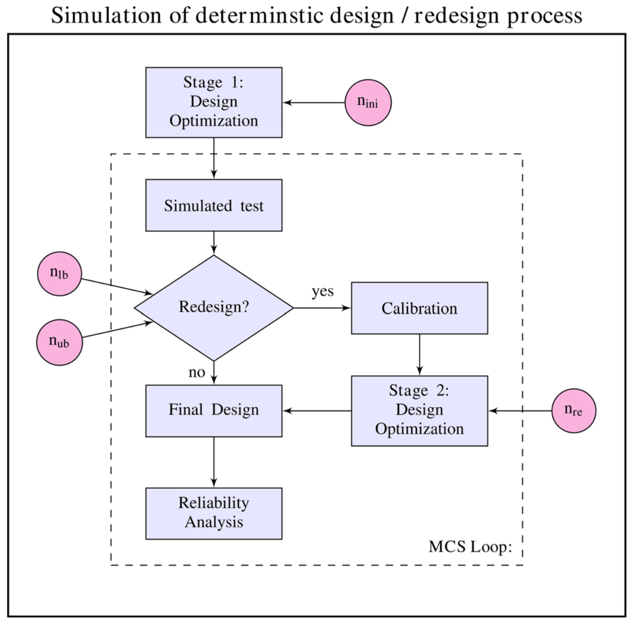

Fig. 2 Flowchart showing the steps in the two-stage deterministic design/redesign process (lower box from Fig. 1). Margins $n = \{n_{\text{ini}}, n_{\text{lb}}, n_{\text{ub}}, n_{\text{re}}\}$ are shown as inputs at relevant steps.

available later. For example, the high-fidelity model may be an actual experiment using a design prototype that was designed based on a low-fidelity computer simulation. In the proposed method, we assume that only the low-fidelity model is available and therefore predictions of the high-fidelity model and the true model are defined with respect to this known model. In the proposed design process, only the low-fidelity model is used during design optimization and all high-fidelity model evaluations are simulated. To simplify the propagation of mixed epistemic model uncertainty and aleatory parameter uncertainty, it is assumed that the discrepancy between the low- and high-fidelity models and between the high-fidelity and true model is constant with respect to design variables $\boldsymbol{x}$ and aleatory variables $\boldsymbol{U}$. The assumed relationship between the different fidelity models is

$$g_T(\boldsymbol{x},\boldsymbol{u}) = g_H(\boldsymbol{x},\boldsymbol{u}) + e_H = g_L(\boldsymbol{x},\boldsymbol{u}) + e_L \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^d$ is a vector of design variables, $\boldsymbol{U}$ is a vector of aleatory random variables with a realization $\boldsymbol{u} \in \mathbb{R}^p$, $g_T(\cdot,\cdot)$ is the true model, $g_H(\cdot,\cdot)$ is the high-fidelity model, $g_L(\cdot,\cdot)$ is the low-fidelity model, $e_H \in \mathbb{R}$ is the true error in the high-fidelity model, and $e_L \in \mathbb{R}$ is the true error in the low-fidelity model. It is assumed that the possible errors are known based on expert opinion or previous experience. Representing an expert's knowledge and beliefs about an unknown parameter as a probability distribution is referred to as elicitation. For a discussion of elicitation and how it relates to epistemic uncertainty, the reader is referred to the work of O'Hagan and Oakley [5]. For a more general discussion of elicitation methods, the reader is referred to the work of Kadane and Wolfson [32]. In this study, the errors E_L and E_H are modeled as two independent uniformly distributed epistemic random variables with $\mathrm{Var}(E_H) < \mathrm{Var}(E_L)$. Uniform distributions are used because we assume we do not have much information except for the lower and upper bounds of the error. In practice, other distributions can be used based on the available information.

The true model is predicted based on the distribution of error E_L as

$$G_T(\boldsymbol{x},\boldsymbol{u}) = g_L(\boldsymbol{x},\boldsymbol{u}) + E_L \tag{3}$$

Similarly, the high-fidelity model is predicted as

$$G_H(\boldsymbol{x},\boldsymbol{u}) = g_L(\boldsymbol{x},\boldsymbol{u}) + E_L - E_H \tag{4}$$

Let $g_T^{(i)}(\cdot,\cdot)$ denote a realization of $G_T(\cdot,\cdot)$ and $\boldsymbol{\Omega}_E$ denote the epistemic sampling space. It is assumed that there exists an epistemic realization, $\exists e_L^{(i)} \in \boldsymbol{\Omega}_E$, such that the realization corresponds to the true process, $g_T^{(i)}(\cdot,\cdot) = g_T(\cdot,\cdot)$. This follows from the assumption that the true relationship can be written as shown in Eq. (2) and the assumption that the epistemic random variable E_L includes the true model error. The mean of the possible errors is defined as $\bar{e}_L$ and $\bar{e}_H$. The mean prediction with respect to epistemic uncertainty of the high-fidelity model and true model are defined as $\bar{g}_H(\cdot,\cdot)$ and $\bar{g}_T(\cdot,\cdot)$, respectively.

A crude Monte Carlo simulation of $i = 1,\dots,m$ error realizations is performed. In Sec. 2.3, design/redesign process is described conditional on one pair of error samples. The deterministic design/redesign process is repeated for many different error realizations. Based on the MCS, the risk of redesign is estimated. Furthermore, the MCS explores how failing a future test is related to the final design performance and safety.

2.3 Deterministic Design/Redesign Process. A flowchart of the design/redesign process is shown in Fig. 2. The design process consists of selecting an initial design, a simulated evaluation of the initial design with a high-fidelity model, possible redesign, and a reliability assessment. In Sec. 2.3.1–2.3.3, the process is described conditional on the error realizations $E_L = e_L^{(i)}$ and $E_H = e_H^{(i)}$.

2.3.1 Initial Design. The selection of the initial design is based on a deterministic margin-based optimization problem

$$\begin{aligned} \min \quad & f(\boldsymbol{x},\boldsymbol{u}_{\text{det}}) \\ \text{w.r.t.} \quad & \boldsymbol{x} \\ \text{s.t.} \quad & \bar{g}_T(\boldsymbol{x},\boldsymbol{u}_{\text{det}}) - n_{\text{ini}} \geq 0 \\ & \boldsymbol{x}_{\min} \leq \boldsymbol{x} \leq \boldsymbol{x}_{\max} \end{aligned} \tag{5}$$

where $\boldsymbol{u}_{\text{det}}$ is a vector of deterministic values that are substituted for aleatory random variables. The bar accent in Eq. (5) indicates the average taken over the distribution of possible model errors. Note that if the low-fidelity model is believed to be unbiased, $\bar{e}_L = 0$, then the mean prediction of the true model is simply the low-fidelity model $g_L(\cdot,\cdot)$. The failure domain is defined with respect to the true (but unknown) model $g_T(\cdot,\cdot)$ as

$$\Omega_f(\boldsymbol{x}) = \{\boldsymbol{u} \in \boldsymbol{\Omega}_U | g_T(\boldsymbol{x},\boldsymbol{u}) < 0\} \quad (6)$$

where $\boldsymbol{\Omega}_U$ is the aleatory sampling space. Let $\boldsymbol{x}_{\text{ini}}$ denote the optimum design found from Eq. (5) using initial margin n_{ini}. It is assumed that the conservative values $\boldsymbol{u}_{\text{det}}$ are based on regulations (e.g., FAR §25.613 [33], FAR§25.303 [34]) and/or previous experience.

2.3.2 Testing Initial Design and Redesign Decision. Later in the design process, the initial design $\boldsymbol{x}_{\text{ini}}$ will be evaluated with the high-fidelity model to measure the margin. In the Monte Carlo simulation, the test is based on a simulated high-fidelity evaluation $g_H^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}})$. If $n_{\text{lb}} \le g_H^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}}) \le n_{\text{ub}}$, then the initial design will pass the test and be accepted as the final design. However, if $g_H^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}}) < n_{\text{lb}}$, then the design is unsafe and redesign will be performed to improve safety. If $g_H^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}}) > n_{\text{ub}}$, then redesign is performed to improve performance because the initial design is too conservative. An indicator function for the redesign decision is denoted $q^{(i)}$ which is one for redesign and zero otherwise. Redesign initiated due to a low margin $(g_H^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}}) < n_{\text{lb}})$ is referred to as redesign for safety and redesign initiated due to a high margin $(g_H^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}}) > n_{\text{ub}})$ is referred to as redesign for performance.

2.3.3 Model Calibration. Before redesign, the mean prediction of the true model $\bar{g}_T(\cdot,\cdot)$ is calibrated based on the test result. The model is calibrated deterministically based on the difference between the prediction and the high-fidelity evaluation of the initial design. The calibrated model is

$$g_{\text{calib}}^{(i)}(\boldsymbol{x},\boldsymbol{u}) = \bar{g}_T(\boldsymbol{x},\boldsymbol{u}) + e_{\text{calib}}^{(i)} \quad (7)$$

where $e_{\text{calib}}^{(i)} = g_H^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}}) - \bar{g}_T(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}})$. The calibrated model $G_{\text{calib}}(\cdot,\cdot)$ accounts for changes in the model that might occur during the future calibration. The calibration improves the model when the high-fidelity model is more accurate than the low-fidelity model, $|e_H^{(i)}| < |e_L^{(i)}|$. This simple method of calibration works well because of the underlying assumption that the model bias is constant as described in Eq. (2). Due to the assumption of constant model bias, the error in the low-fidelity model is canceled out during calibration and the calibrated model is simply equal to the high-fidelity model, $g_{\text{calib}}^{(i)}(\cdot,\cdot) = g_H^{(i)}(\cdot,\cdot)$.

2.3.4 Redesign. If the test is not passed, redesign will be performed to find a new design using the calibrated model $g_{\text{calib}}^{(i)}(\cdot,\cdot)$ and a new margin n_{re}. The deterministic design problem for selecting a new design after calibration is

$$\begin{aligned} \text{min} \quad & f(\boldsymbol{x},\boldsymbol{u}_{\text{det}}) \\ \text{w.r.t.} \quad & \boldsymbol{x} \\ \text{s.t.} \quad & g_{\text{calib}}^{(i)}(\boldsymbol{x},\boldsymbol{u}_{\text{det}}) - n_{\text{re}} \ge 0 \\ & \boldsymbol{x}_{\min} \le \boldsymbol{x} \le \boldsymbol{x}_{\max} \end{aligned} \quad (8)$$

Let $\boldsymbol{x}_{\text{re}}^{(i)}$ denote the solution to Eq. (8). The new design $\boldsymbol{X}_{\text{re}}$ is an epistemic random variable because it is conditional on the unknown outcome of the future high-fidelity evaluation. However, there is no inherent variability (i.e., aleatory uncertainty) in the design choice. The new design is a random variable only because it is unknown at the initial design stage. Note that the feasible design space of the redesign problem Eq. (8) is different than the feasible design space in the initial design problem Eq. (5) due to the calibration and the use of a margin n_{re} that may be different than n_{ini}. Conditional on the outcome of the future test, some designs with improved performance may become accessible during redesign that were previously considered infeasible or some designs that were previously considered reasonable may be revealed to be unsafe.

2.4 Probabilistic Evaluation. Each set of margins $\boldsymbol{n}$ results in a probability of redesign p_{re}, a final probability of failure after possible redesign $P_{f,\text{final}}$ (epistemic random variable), and a final cost $\mathbb{E}_U[f(\boldsymbol{X}_{\text{final}},\boldsymbol{U})]$ (epistemic random variable). Histograms of random variables are obtained based on a crude MCS as described in Sec. 2.2. The expected values with respect to epistemic model uncertainty that are used in Eq. (1) are obtained using numerical integration.

The probability of redesign is $p_{\text{re}} = \mathbb{E}_E[Q]$. After possible redesign, the final design is

$$\boldsymbol{x}_{\text{final}}^{(i)} = (1 - q^{(i)})\boldsymbol{x}_{\text{ini}} + q^{(i)}\boldsymbol{x}_{\text{re}}^{(i)} \quad (9)$$

The expected mean design cost after possible redesign is $\mathbb{E}_E[\mathbb{E}_U[f(\boldsymbol{X}_{\text{final}},\boldsymbol{U})]]$. The expected mean design cost can be written in terms of conditional probabilities as

$$\begin{aligned} \mathbb{E}_E[\mathbb{E}_U[f(\boldsymbol{X}_{\text{final}},\boldsymbol{U})]] = {} & (1 - p_{\text{re}})\mathbb{E}_U[f(\boldsymbol{x}_{\text{ini}},\boldsymbol{U})] \\ & + p_{\text{re}}\mathbb{E}_E[\mathbb{E}_U[f(\boldsymbol{X}_{\text{re}},\boldsymbol{U})]|Q = 1] \end{aligned} \quad (10)$$

where $\mathbb{E}_U[f(\boldsymbol{x}_{\text{ini}},\boldsymbol{U})]$ is the expected mean design cost conditional on passing the test and $\mathbb{E}_E[\mathbb{E}_U[f(\boldsymbol{X}_{\text{re}},\boldsymbol{U})]|Q = 1]$ is the expected mean design cost conditional on failing the test.

The final margin with respect to the high-fidelity model after possible redesign is

$$n_{H,\text{final}}^{(i)} = (1 - q^{(i)})g_H^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}}) + q^{(i)}g_H^{(i)}(\boldsymbol{x}_{\text{re}}^{(i)},\boldsymbol{u}_{\text{det}}) \quad (11)$$

where the high-fidelity model is equal to the calibrated model due to the calibration process as described in Sec. 2.3.3. The final margin with respect to the true model after possible redesign is

$$n_{T,\text{final}}^{(i)} = (1 - q^{(i)})g_T^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{u}_{\text{det}}) + q^{(i)}g_T^{(i)}(\boldsymbol{x}_{\text{re}}^{(i)},\boldsymbol{u}_{\text{det}}) \quad (12)$$

Due to epistemic model uncertainty, the true probability of failure is unknown. A realization of the probability of failure for the initial design is

$$p_{f,\text{ini}}^{(i)} = \mathbb{P}_U[g_T^{(i)}(\boldsymbol{x}_{\text{ini}},\boldsymbol{U}) < 0] \quad (13)$$

where $\mathbb{P}_U[\cdot]$ denotes the probability with respect to aleatory uncertainty. In the probability of failure calculation, epistemic model uncertainty is treated separately from the aleatory uncertainty. There is epistemic uncertainty in the true probability of failure with respect to aleatory uncertainty due to epistemic model uncertainty. In other words, there is a lack of knowledge regarding the probability of failure because it is calculated using a low-fidelity model which may have some error or bias. In reality, the true probability of failure of the final design does not depend on model fidelity. However, our knowledge of the true probability of failure depends on the uncertainty in our models. To account for model uncertainty, the probability of failure calculation is repeated conditional on different realizations of the true model $g_T^{(i)}(\cdot,\cdot)$ as shown in Eq. (13). After redesign, the probability of failure is

$$p_{f,\text{re}}^{(i)} = \mathbb{P}_U[g_T^{(i)}(\boldsymbol{x}_{\text{re}}^{(i)},\boldsymbol{U}) < 0] \quad (14)$$

The design variable $\boldsymbol{x}_{\text{re}}^{(i)}$ is an epistemic random variable because it is conditional on the outcome of the future test. The final probability of failure after possible redesign is

Table 1 Problem definition for uniaxial tension test example

	Description	Notation
Design variable	Cross-sectional area (mm^2)	$x = a$
Aleatory variables	Applied load, material strength	$\boldsymbol{U} = \{P, S\}$
Conservative values	Limit load, allowable strength	$\boldsymbol{u}_{\text{det}} = \{1600\,\text{N}, 15.35\,\text{MPa}\}$
Objective function	Cross-sectional area (mm^2)	$f(x) = a$
Limit-state function	Yielding	$g_L(x, \boldsymbol{U}) = S - P/a$
Target mean reliability		$p_f^\star = 1 \times 10^{-5}$

Table 2 Uncertain parameters for uniaxial tension test example

Parameter	Classification	Symbol	Mean, μ	C.O.V	Range	Distribution
Applied load	Aleatory	P (N)	1000	0.20	$(-\infty, \infty)$	Normal
Material strength	Aleatory	S (MPa)	20	0.12	$(-\infty, \infty)$	Normal
Error in low-fidelity model	Epistemic	E_L (MPa)	0	—	$[-4.35, 4.35]$	Uniform
Error in high-fidelity model	Epistemic	E_H (MPa)	0	—	$[-2.18, 2.18]$	Uniform

$$p_{f,\text{final}}^{(i)} = (1 - q^{(i)}) p_{f,\text{ini}}^{(i)} + q^{(i)} p_{f,\text{re}}^{(i)} \tag{15}$$

The expected probability of failure after possible redesign is $\mathbb{E}_E[P_{f,\text{final}}]$. The expected probability of failure can be written in terms of conditional probabilities as

$$\mathbb{E}_E[P_{f,\text{final}}] = (1 - p_{\text{re}})\mathbb{E}_E[P_{f,\text{ini}}|Q = 0] + p_{\text{re}}\mathbb{E}_E[P_{f,\text{re}}|Q = 1] \tag{16}$$

where $\mathbb{E}_E[P_{f,\text{ini}}|Q = 0]$ is the expected probability of failure conditional on passing the test and $\mathbb{E}_E[P_{f,\text{re}}|Q = 1]$ is the expected probability of failure conditional on failing the test. We can see from Eq. (16) that the expected final probability of failure is a weighted average of the expected probability of failure of the initial design and the expected probability of failure of the possible redesigns.

3 Test Cases

3.1 Uniaxial Tension Test

3.1.1 Problem Description. In this example, we consider the design of a minimum weight bar subject to uniaxial loading. The problem definition is shown in Table 1. Note that following from the formulation in Eq. (4), the high-fidelity model is assumed to have the same functional form as the low-fidelity model except for a constant discrepancy. The design is subject to aleatory uncertainty in loading and material properties. In addition, there is epistemic model uncertainty in the limit-state function describing the yielding of the bar. The uncertain parameters are defined as shown in Table 2. The bar is designed to minimize the mass, or equivalently cross-sectional area, subject to a stress constraint. The bar is designed using conservative values in place of random loads and material properties. Later in the design process, the bar will be tested (e.g., high-fidelity simulation or prototype test) and it will be redesigned if the margin with respect to the stress constraint is too high or too low.

The problem follows the general method described in Sec. 2. The limit-state function is a linear function of the aleatory parameters and all aleatory parameters are assumed to be normally distributed. Therefore, the computational cost is reduced by calculating the reliability index analytically for each realization of epistemic model error. The reliability index is the minimum distance from the origin to the limit-state function in standard normal space (for background, see [35]). Due to the simplicity of the design problem, the optimum deterministic design can be obtained directly by solving for the value of the design variable that satisfies the deterministic constraint.

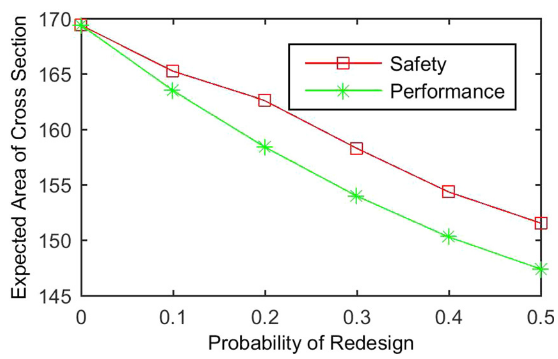

Fig. 3 Uniaxial tension test—comparison of expected cross-sectional area after possible redesign as a function of probability of redesign for redesign for performance (conservative initial design) versus redesign for safety (ambitious initial design)

3.1.2 Expected Performance Versus Probability of Redesign. Tradeoff curves for expected cost, $\mathbb{E}_E[f(\boldsymbol{X}_{\text{final}})]$, versus probability of redesign, p_{re}, are shown in Fig. 3. The tradeoff curves were obtained by solving Eq. (1) for several values of the constraint on probability of redesign, $p_{\text{re}}^\star$. The two curves correspond to the special cases of performing redesign only for performance and performing redesign only for safety. It was observed that redesign for performance was the global optimum solution and the optimum margins would converge to this solution when allowing for both redesign for safety and performance.

The expected mass of the bar decreases with increasing risk of redesign. When there is zero probability of redesign, the initial design must be conservative enough that the expected probability of failure is less than or equal to the target value of 1×10^{-5}. To meet the target on expected probability of failure, the initial design must be heavier. This is the design we would obtain if we optimized only n_{ini} to minimize the weight of the initial design with a constraint on expected probability of failure. Both curves start at this design because the probability of redesign is zero and therefore there is no difference between the redesign strategies. As the probability of redesign increases, redesign can be used to correct the initial design if the high-fidelity model reveals the

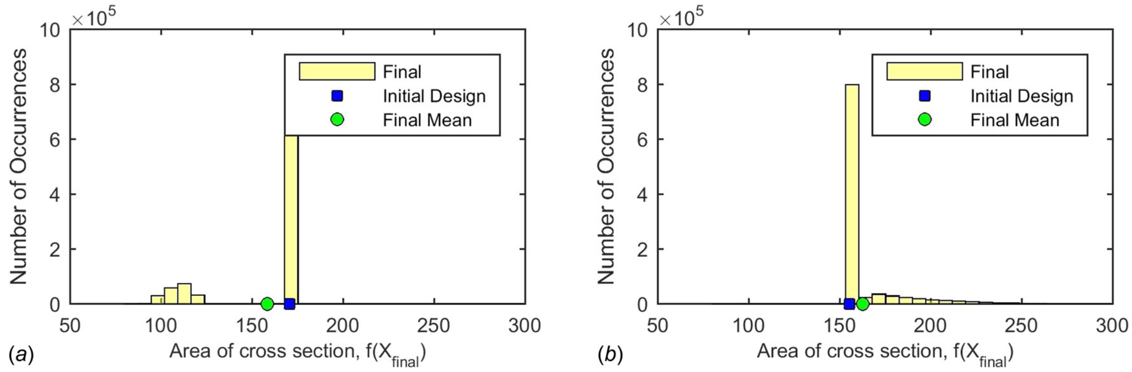

Fig. 4 Uniaxial tension test—epistemic uncertainty in cross-sectional area for 20% probability of redesign

Table 3 Results for uniaxial tension example for 20% probability of redesign

Description	Notation	Redesign for safety	Redesign for performance
Probability of redesign	p_{re}	0.20	0.20
Cost of initial design	$f(x_{\mathrm{ini}})$	155.5	170.7
Expected cost conditional on performing redesign	$\mathbb{E}_E[f(X_{\mathrm{re}})\|Q=1]$	191.2	109.4
Expected cost after possibly performing redesign	$\mathbb{E}_E[f(X_{\mathrm{final}})]$	162.7	158.4
Expected probability of failure of initial design	$\mathbb{E}_E[P_{f,\mathrm{ini}}]$	2.9×10^{-5}	0.9×10^{-5}
Expected probability of failure of initial design conditional on passing test	$\mathbb{E}_E[P_{f,\mathrm{ini}}\|Q=0]$	0.9×10^{-5}	1.2×10^{-5}
Expected probability of failure of new designs conditional on failing test	$\mathbb{E}_E[P_{f,\mathrm{re}}\|Q=1]$	1.3×10^{-5}	0.4×10^{-5}
Expected probability of failure after possibly performing redesign	$\mathbb{E}_E[P_{f,\mathrm{final}}]$	1.0×10^{-5}	1.0×10^{-5}

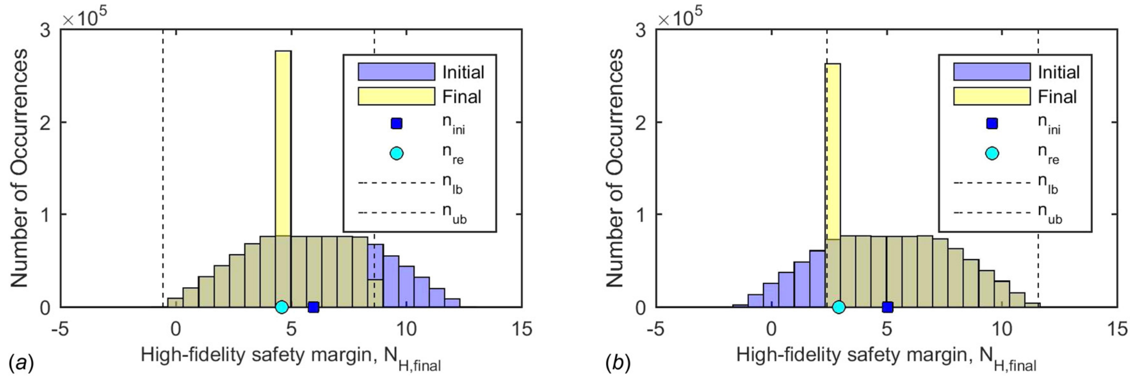

Fig. 5 Uniaxial tension test—epistemic uncertainty in margin with respect to high-fidelity model for 20% probability of redesign. Plots show overlapping transparent histograms.

margin is too high or too low. To explore the simulation in more detail, the points on the tradeoff curve corresponding to 20% probability of redesign were selected.

Histograms of the area of the cross section of the bar are shown in Fig. 4. Redesign for performance starts with a heavier design and redesign is used to reduce the weight if the initial design is revealed to be overly conservative. Redesign for safety starts with a lighter initial design and redesign adds weight if the initial design is revealed to be unsafe. It is observed that if redesign for performance is required, then the change in area is much larger than the change associated with redesign for safety. Based on the statistics in Table 3, the expected change in the cross-sectional area conditional on redesign for performance is -33%, whereas the change conditional on redesign for safety is about 23%. If we assume the effort and cost associated with redesign is proportional to the relative change in the design, then redesign for performance is more difficult and expensive than redesign for safety. Therefore, redesign for safety may be preferred over redesign for performance even if the expected performance is similar. Furthermore, the assumption of constant model bias is more reasonable when the change in the design is small, which could lead to less accurate calibration and probability of failure estimates when considering redesign for performance.

Histograms of the margin with respect to the high-fidelity model are shown in Fig. 5. Redesign for performance starts with a higher initial margin and redesign is performed if the margin is revealed to be above n_{ub}. Redesign for safety starts with a lower initial margin and redesign is performed if the margin is revealed to be below n_{lb}. If redesign is performed, then the design is adjusted during redesign optimization to have a margin of n_{re} as indicated by the peak at this location. Histograms of the margin with respect to the true model are shown in Fig. 6. In contrast to the margin with respect to the high-fidelity model in Fig. 5, the true margin does not depend on the error in the

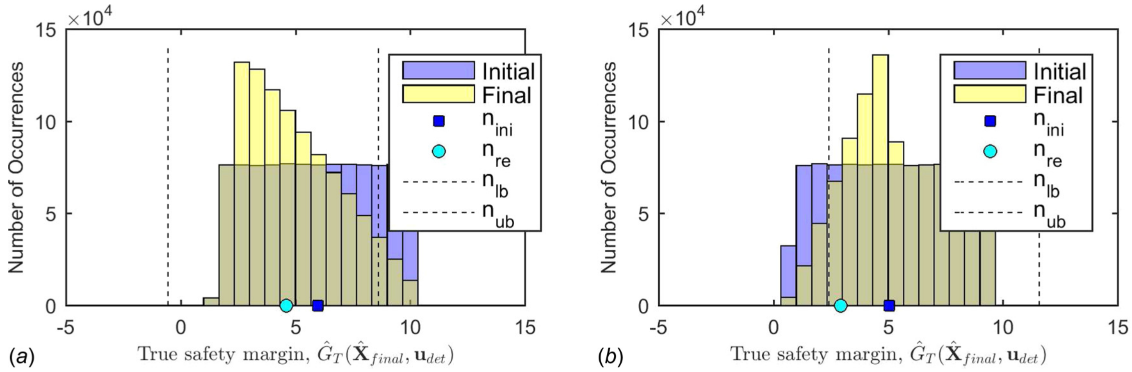

Fig. 6 Uniaxial tension test—epistemic uncertainty in margin with respect to true model for 20% probability of redesign. Plots show overlapping transparent histograms.

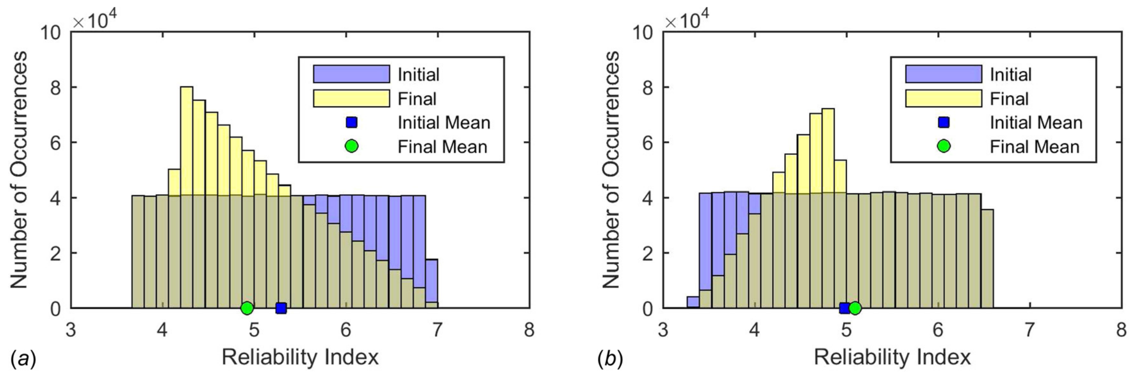

Fig. 7 Uniaxial tension test—epistemic uncertainty in reliability index for 20% probability of redesign. Plots show overlapping transparent histograms.

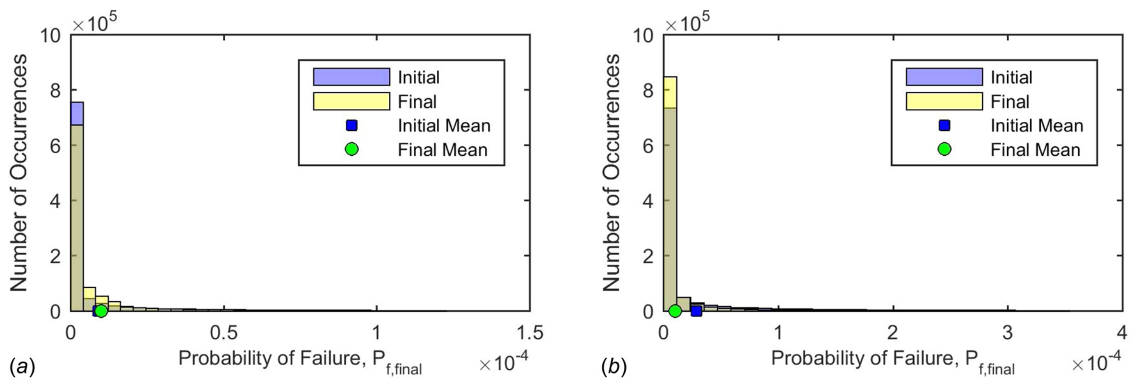

Fig. 8 Uniaxial tension test—epistemic uncertainty in failure for 20% probability of redesign. The figures are plotted with different scales to show the change in the tail of the distribution. Plots show overlapping transparent histograms.

high-fidelity model. The truncation of the true margin distribution is imperfect and the true margin after redesign does not exactly correspond to n_{re}. It should be noted that we have not included any requirement that the margin should be strictly positive. The margin is calculated with respect to conservative aleatory values, $\boldsymbol{u}_{det}$, and therefore it may be reasonable to employ a negative margin in order to saturate the expected probability of failure constraint.

Histograms of the reliability index are shown in Fig. 7. By redesigning based on the observed safety margin, the designer is correcting a dangerous or overly conservative design. Histograms of the probability of failure are shown in Fig. 8. Both strategies are optimized to obtain the same expected probability of failure after redesign. Redesign for safety reduces the expected probability of failure by correcting the design if it is revealed to be unsafe and redesign for performance starts with a more conservative design with lower expected probability of failure. Based on the statistics in Table 3, if planning to redesign for safety, the expected probability of failure of the initial design before the test is more than three times greater than when planning to redesign for performance. Interestingly, if the initial design passes the test without redesign, then the expected probability of failure of the initial design is actually about 25% lower when considering redesign for safety compared to redesign for performance.

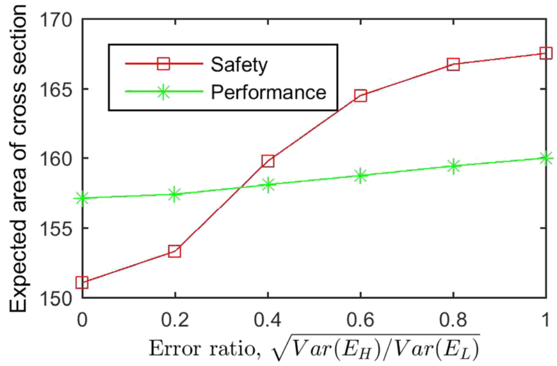

Fig. 9 Uniaxial tension test—redesign for safety is preferred when high-fidelity model error is low, but redesign for performance is preferred when high-fidelity model error is high. Plot is for fixed probability of redesign of 20%.

3.1.3 Expected Performance Versus Level of High-Fidelity Model Error. In some situations, the designer may be able to control the accuracy of the high-fidelity model. For example, it may be possible to develop a model with less uncertainty by investing more time and resources. In this case, the designer may be interested in how the choice of margins depends on the amount of uncertainty in the high-fidelity model. Even if the designer is unable to influence the accuracy of the high-fidelity model, it is interesting to explore the effect of varying this uncertainty in order to illustrate how it influences the choice of whether to redesign for safety or redesign for performance. To explore the effect of the error in the high-fidelity model, the ratio of the standard deviation of the error in the high-fidelity model relative to the standard deviation of the error in the low-fidelity model, $\sqrt{\mathrm{Var}(E_H)/\mathrm{Var}(E_L)}$, was varied from zero to one. The standard deviation of the error of the low-fidelity model was held fixed and both distributions had means of zero. An error ratio of zero corresponds to no error in the high-fidelity model and a ratio of one corresponds to having the same error distributions for both models. For each point on the curves, the margins were optimized by solving Eq. (1) for a fixed probability of redesign of 20%. As shown in Fig. 9, redesign for safety is preferred when the error in the high-fidelity model is low but redesign for performance is preferred when the error in the high-fidelity model is high. Note that for the tradeoff curve shown in Fig. 3, the ratio of the errors in the models was $\sqrt{\mathrm{Var}(E_H)/\mathrm{Var}(E_L)} = 0.5$.

3.2 Supersonic Business Jet Engine Design

3.2.1 Problem Description. This example is based on the propulsion discipline design problem from the Sobieski supersonic business jet (SSBJ) problem [36]. The design problem is to minimize engine weight subject to a constraint on the maximum normalized throttle setting. The problem is based on the scaling of a baseline engine to meet a thrust requirement. If the engine is designed to provide the required thrust when operating near idle throttle, then the resulting engine design is unreasonably large and heavy. If the engine is designed to provide the required thrust when operating at full throttle, then the engine design can be smaller and lighter. However, there is epistemic uncertainty in the low-fidelity prediction of the thrust output and therefore it is desirable to have some margin to increase the probability that the as-built engine can provide sufficient thrust. In addition, the thrust output of the engine varies with Mach number and altitude. In this example, we consider that the engine is designed to operate for a distribution of altitudes (aleatory uncertainty).

The throttle setting is defined as the ratio of the engine output thrust relative to the maximum available thrust at a given altitude and Mach number. A throttle setting of one indicates maximum power at a given altitude and Mach number and a throttle setting of 0.01 is idle thrust. The net available thrust of the engine increases with Mach number and decreases with altitude. A nondimensional throttle setting variable, x, is created by normalizing the throttle with respect to the point of maximum thrust of the baseline engine. The nondimensional throttle setting is defined as

$$x = S_{\mathrm{out}}/S_0 \tag{17}$$

where S_{out} is the output thrust and $S_0 = 16168$ lbf is the maximum thrust of the baseline engine. If the required thrust S_{req} is different than the thrust provided by the baseline engine, the baseline engine design is scaled to match the new requirement. In this example, we assume a fixed thrust requirement $S_{\mathrm{req}} = 40{,}000$ lbf. The engine scale factor (ESF) is defined as

$$\mathrm{ESF} = \frac{S_{\mathrm{req}}}{2S_{\mathrm{out}}} = \frac{S_{\mathrm{req}}}{2xS_0} \tag{18}$$

where the value of two in the denominator reflects the fact that two engines are used on the jet. The weight of the engine W_E is approximated as following a power law relationship with engine scale factor

$$W_E = 2W_{\mathrm{BE}}(\mathrm{ESF})^{1.05} \tag{19}$$

where $W_{\mathrm{BE}} = 4360$ lb is the weight of the baseline engine.

A response surface of the engine performance map for the baseline engine calculates maximum available thrust S_{avail} at a given Mach number M and altitude h. The response surface sets an upper bound on throttle, x_{ub}, when normalized by S_0

$$x_{\mathrm{ub}}(M,h) = \frac{S_{\mathrm{avail}}(M,h)}{S_0} = \frac{1}{S_0}\left(\alpha_0 + \alpha_1 M + \alpha_2 h + \alpha_3 M^2 + 2\alpha_4 Mh + 2\alpha_5 h^2\right) \tag{20}$$

where the coefficients are listed in Table 4. The plot of the engine performance map response surface in Fig. 10 shows that the thrust decreases as the altitude increases.

In this example, we are interested in minimizing the weight of the engine subject to a constraint on maximum throttle. The problem definition is shown in Table 5. Note that following from the formulation in Eq. (4), the high-fidelity model is assumed to have the same functional form as the low-fidelity model except for a constant discrepancy. We consider aleatory uncertainty in the altitude and epistemic model uncertainty in the maximum throttle, x_{ub}, as defined in Table 6. The problem follows the general method described in Sec. 2. The engine is designed using a conservative value in place of random altitude. Later in the design process, the engine will be tested (e.g., high-fidelity simulation or prototype test) and it will be redesigned if the margin with respect to the throttle constraint is too high or too low. That is, the engine will be redesigned if it provides insufficient thrust or the thrust is so large that it is worth redesigning to use a smaller, lighter engine.

Table 4 Coefficients for calculating throttle upper bound (Eq. (20))

Coefficient	Value
α_0	1.1484×10^4
α_1	1.0856×10^4
α_2	-5.0802×10^{-1}
α_3	3.2002×10^3
α_4	-1.4663×10^{-1}
α_5	6.8572×10^{-6}

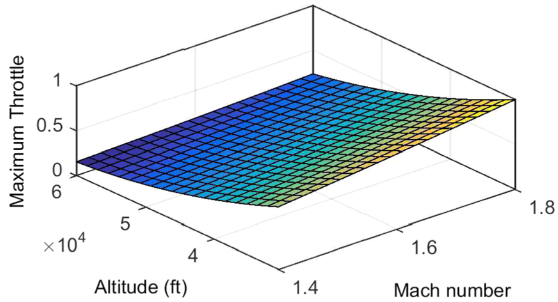

Fig. 10 A response surface of the engine performance map calculates maximum available thrust at a given Mach number, *M*, and altitude, *h*. The throttle setting is normalized to one at an altitude of approximately 32,000 ft and Mach 1.9.

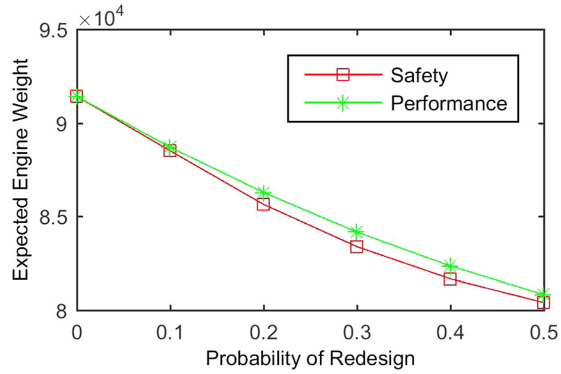

Fig. 11 SSBJ Engine—comparison of expected engine weight after possible redesign as a function of probability of redesign for redesign for performance (conservative initial design) versus redesign for safety (ambitious initial design)

Table 5 Problem definition for SSBJ Example

	Description	Notation
Design variable	Throttle	x
Aleatory variable	Altitude (ft)	$U = H$
Conservative value	Max altitude	$u_{\text{det}} = 56,770\,\text{ft}$
Objective function	Engine weight (lbs)	$f(x) = W_E(x)$
Limit-state function	Maximum throttle	$g_L(x, U) = x_{\text{ub}}(H) - x$
Target mean reliability		$p_f^\star = 1 \times 10^{-3}$

The probability of failure is estimated based on a Monte Carlo simulation. The throttle should be set to the upper bound to minimize the engine weight. Therefore, deterministic design optimization was avoided by setting the throttle to the upper bound minus the margin.

3.2.2 Expected Performance Versus Probability of Redesign. Tradeoff curves for expected cost, $\mathbb{E}_E[f(\boldsymbol{X}_{\text{final}})]$, versus probability of redesign, p_{re}, are shown in Fig. 11. The tradeoff curves were obtained by solving Eq. (1) for several values of the constraint on probability of redesign, $p_{\text{re}}^\star$. The two curves correspond to the special cases of performing redesign only for performance and performing redesign only for safety. It was observed that redesign for safety was the global optimum solution and the optimum margins would converge to this solution when allowing for both redesign for safety and performance. This result is different from the example in Sec. 3.1 where redesign for performance was preferred. To explore the simulation in more detail, the points on the tradeoff curve corresponding to 20% probability of redesign were selected.

Histograms of the throttle are shown in Fig. 12 and histograms of engine weight are shown in Fig. 13. Redesign for performance starts with a heavier design (lower throttle setting) and redesign increases the throttle setting to reduce the weight if the initial design is revealed to be overly conservative. Redesign for safety starts with a lighter initial design (higher throttle setting) and redesign reduces the throttle which increases the weight if the initial design is revealed to be unsafe. It is observed that if redesign for performance is required then the change in weight is much larger than the change associated with redesign for safety. Based on the statistics in Table 7, the expected change in the weight conditional on redesign for performance is −29%, whereas the change conditional on redesign for safety is about 16%. If we assume the effort and cost associated with redesign is proportional to the relative change in the design, then redesign for performance is again more difficult and expensive than redesign for safety as was observed in the uniaxial tension test example. This supports the initial finding for the uniaxial tension test example that redesign for safety may be preferred over redesign for performance even if the expected performance is similar. As also noted on the uniaxial tension test example, the large design change associated with redesign for performance may lead to less accurate calibration and probability of failure estimates due to the assumption of constant model bias.

Histograms of the margin with respect to the high-fidelity model are shown in Fig. 14. As was observed in the bar example, redesign for performance starts with a higher initial margin and redesign for safety starts with a lower initial margin.

Histograms of the probability of failure are shown in Fig. 15. Redesign for safety reduces the expected probability of failure substantially by correcting the design if it is revealed to be unsafe. In other words, redesign for safety truncates the tail of the distribution corresponding to an unsafe initial design. The change in the expected probability of failure is much less significant with redesign for performance because the expectation is not very sensitive to the very low probability of failure realizations that are changed when redesigning for performance. Based on the statistics in Table 7, redesign for safety reduce the expected probability of failure by a factor of seven, whereas redesign for performance increases the expected probability of failure by only about 4%.

3.2.3 Expected Performance Versus Level of High-Fidelity Model Error. The ratio of the standard deviation of the error in the high-fidelity model relative to the standard deviation of the error in the low-fidelity model, $\sqrt{\text{Var}(E_H)/\text{Var}(E_L)}$, was varied from zero to one. For each point on the curves, the margins were optimized by solving Eq. (1) for a fixed probability of redesign of 20%. As shown in Fig. 16, redesign for safety is preferred when the error in the high-fidelity model is low but redesign for performance is preferred when the error in the high-fidelity model is high. The overall trends are similar to those observed for the example in Sec. 3.1. Note that for the tradeoff curve

Table 6 Uncertain Parameters for SSBJ Example

Parameter	Classification	Symbol	Mean, μ	C.O.V	Range	Distribution
Altitude	Aleatory	H (ft)	52,500	0.05	[45,000, 60,000]	Truncated normal
Error in low-fidelity model	Epistemic	E_L	0	—	[−0.0375, 0.0375]	Uniform
Error in high-fidelity model	Epistemic	E_H	0	—	[−0.0075, 0.0075]	Uniform

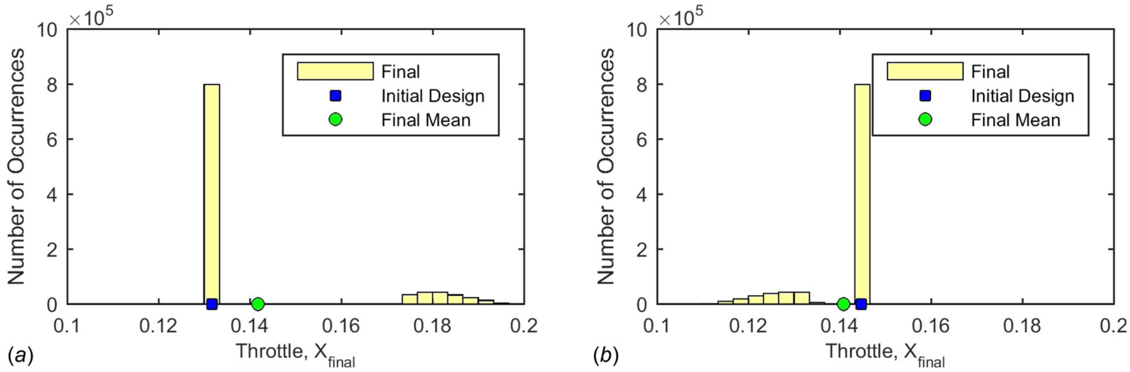

Fig. 12 SSBJ Engine—epistemic uncertainty in throttle setting for 20% probability of redesign

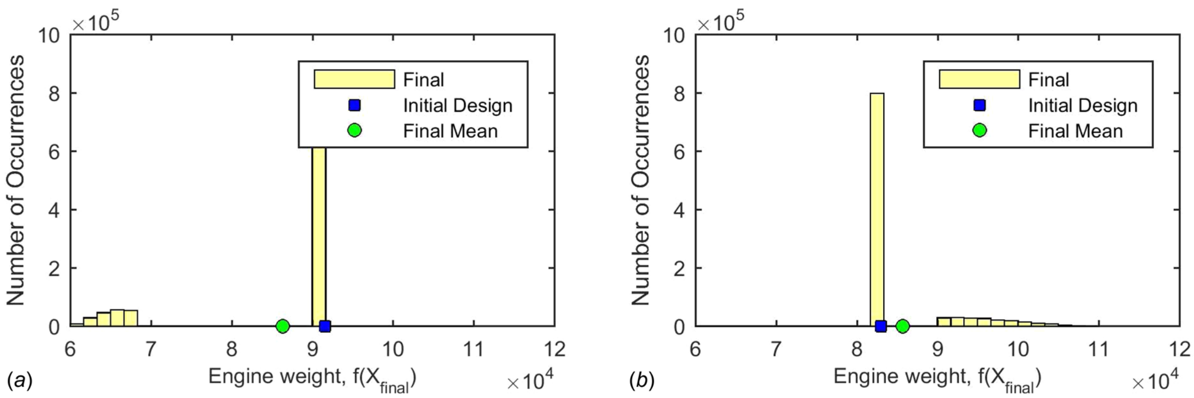

Fig. 13 SSBJ Engine—epistemic uncertainty in engine weight for 20% probability of redesign

Table 7 Results for SSBJ example for 20% probability of redesign

Description	Notation	Redesign for safety	Redesign for performance
Probability of redesign	p_{re}	0.20	0.20
Cost of initial design	$f(x_{\mathrm{ini}})$	8.30×10^4	9.16×10^4
Expected cost conditional on performing redesign	$\mathbb{E}_E[f(X_{\mathrm{re}})\|Q=1]$	9.64×10^4	6.52×10^4
Expected cost after possibly performing redesign	$\mathbb{E}_E[f(X_{\mathrm{final}})]$	8.57×10^4	8.63×10^4
Expected probability of failure of initial design	$\mathbb{E}_E[P_{f,\mathrm{ini}}]$	6.94×10^{-3}	0.96×10^{-3}
Expected probability of failure of initial design conditional on passing test	$\mathbb{E}_E[P_{f,\mathrm{ini}}\|Q=0]$	1.05×10^{-3}	1.20×10^{-3}
Expected probability of failure of new designs conditional on failing test	$\mathbb{E}_E[P_{f,\mathrm{re}}\|Q=1]$	0.80×10^{-3}	0.18×10^{-3}
Expected probability of failure after possibly performing redesign	$\mathbb{E}_E[P_{f,\mathrm{final}}]$	1.00×10^{-3}	1.00×10^{-3}

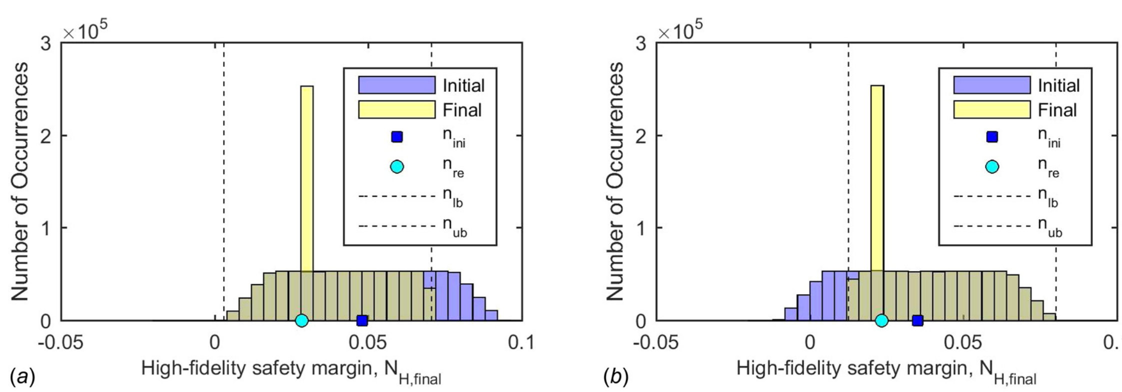

Fig. 14 SSBJ Engine—epistemic uncertainty in margin with respect to high-fidelity model for 20% probability of redesign. Plots show overlapping transparent histograms.

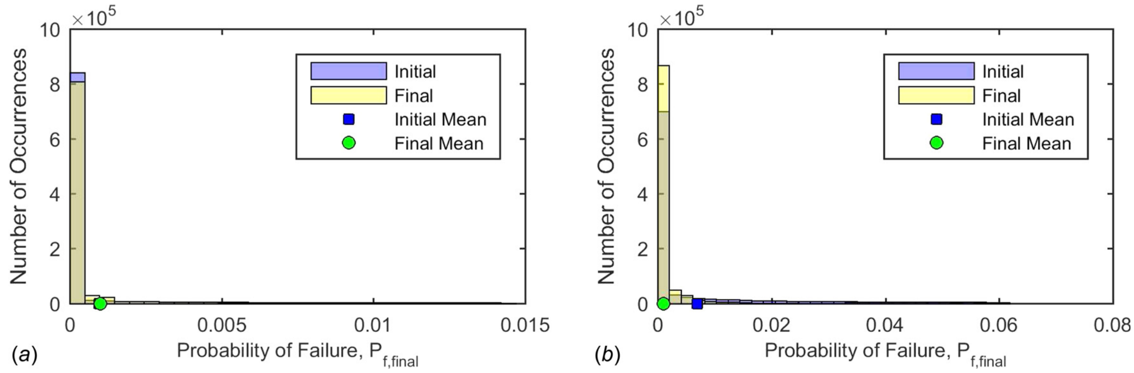

Fig. 15 SSBJ Engine—epistemic uncertainty in probability of failure for 20% probability of redesign. The figures are plotted with different scales to show the change in the tail of the distribution. Plots show overlapping transparent histograms.

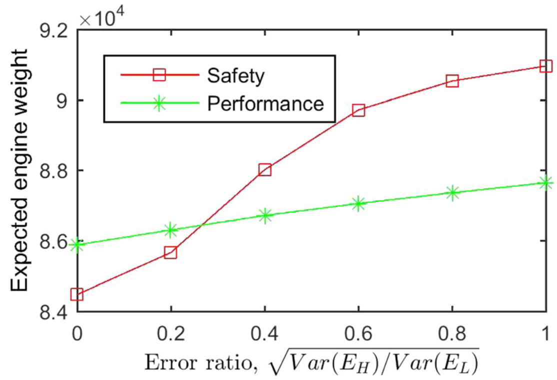

Fig. 16 SSBJ Engine—redesign for safety is preferred when high-fidelity model error is low, but redesign for performance is preferred when high-fidelity model error is high. Plot is for fixed probability of redesign of 20%.

shown in Fig. 11 the ratio of the errors in the models was $\sqrt{\mathrm{Var}(E_H)/\mathrm{Var}(E_L)} = 0.2$.

4 Discussion and Conclusion

This study presented a generalized formulation of a two-stage margin-based design/redesign process considering the effects of a future test and possible redesign. The margins that control the deterministic design/redesign process are optimized to minimize the expected value of the design cost function (i.e., maximize expected performance) while satisfying constraints on probability of redesign and expected probability of failure. The future test result (i.e., high-fidelity evaluation of initial design or prototype test) is an epistemic random variable that is predicted based on the distributions of possible errors in the low- and high-fidelity models. Future test results are simulated in order to calculate the probability of redesign, the possible designs after calibration and redesign, and the final distribution of probabilities of failure. By considering that the design may change later in the design process conditional on the outcome of the future test, it is possible to trade off between the risk of having to redesign later in the design process and the associated performance and/or reliability benefits.

When considering epistemic model uncertainty in a design constraint, the designer faces a dilemma in whether to start with a larger initial margin (i.e., more conservative initial design) and possibly redesign to improve performance versus starting with a smaller margin (i.e., less conservative initial design) and possibly redesigning to restore safety. This study analyzes this decision when there is a fixed but unknown constant bias between the low-fidelity model, high-fidelity model, and true model. In the examples in this study, it is found that the decision of whether to start with a higher initial margin and possibly redesign for performance, or to start with a lower initial margin and possibly redesign for safety, depends on the ratio of the standard deviation of the uncertainty in the high-fidelity model relative to the standard deviation of uncertainty in the low-fidelity model.

It was observed that the redesign for safety strategy was strongly influenced by the amount of error in the high-fidelity model. It is hypothesized that the amount of error in the high-fidelity model has a stronger influence on the redesign for safety strategy because the error interferes with the process of truncating dangerous designs. The benefit of redesign for safety is that it prevents a dangerous initial design from successfully passing the test. This substantially reduces the expected probability of failure which in turn allows the initial design to be less conservative. However, if there is a large amount of error in the high-fidelity model, then a dangerous initial design may pass the test unnoticed. Even if this is unlikely, the possibility of a high probability of failure has a significant influence on the mean probability of failure. To compensate, the initial design must be more conservative. On the other hand, when considering redesign for performance, it is not a problem if a very safe (i.e., overly conservative) initial design passes the test.

It is observed that redesign for safety and redesign for performance result in different distributions of performance (e.g., weight). Redesign for performance capitalizes on the fact that it may be possible to obtain a substantial improvement in performance if the initial design is revealed to be much too conservative. The performance improvement is large but the probability of obtaining this benefit is small when the probability of redesign is small. The initial design must be more conservative since redesign is only used to improve performance and not to restore safety. Redesign for safety attempts to obtain better initial design performance by allowing for the possibility that redesign may be necessary to restore safety. If the initial design is revealed to be unsafe, then it is found that a small design change is usually sufficient to restore safety. When the probability of redesign is small, the initial design is likely to pass the test and be accepted as the final design. Redesign for safety allows for a better initial design than redesign for performance. However, redesign for performance has the advantage that it may be possible to skip the redesign process when time constraints outweigh the possible performance benefits of redesign. Another interesting interpretation of the results is that the degree of design change may be considered proportional to the cost or effort associated with redesign. Therefore, redesign for performance may be associated with higher expected costs or effort due to the large design changes even if the expected performance and probability of redesign are similar to that of redesign for safety. This finding seems to support the industry practice of primarily redesigning to correct safety issues rather than to improve performance.

5 Limitations and Future work

This study is based on the assumption that there is a fixed but unknown constant bias between the low-fidelity model, high-fidelity model, and true model. If the model error is constant across the joint design/aleatory space, then the reduction in epistemic model uncertainty does not depend on the location where the high-fidelity model is evaluated. If the model error is not constant, then it may incentivize starting with a lower margin in order to have a high-fidelity evaluation close to the limit surface $g(\boldsymbol{x}, \boldsymbol{u}) = 0$. In related work, a Kriging surrogate is introduced to model epistemic uncertainty in order to account for spatial correlations in model uncertainty [17].

The proposed method may be computationally expensive because it involves a Monte Carlo simulation (MCS) of a design/redesign process nested inside a global optimization problem. To reduce the computational cost, surrogate models can be fit to the mean probability of failure and mean design cost as a function of the margins as described in Appendix [17]. Surrogate models were not used in the examples in this study because the design models were not computationally expensive.

Federal Aviation Administration regulations mandate the use of conservative material properties such as A-basis or B-basis and conservative loads based on a factor of 1.5 times limit loads. These requirements can be incorporated into the specification of conservative values $\boldsymbol{u}_{\text{det}}$ used in the proposed method. However, these values may not be optimal. Even if the choice of conservative values is constrained by regulations, a designer may be interested in quantifying the performance penalty associated with the values specified in regulations relative to other choices. For more general problems, regulations governing the choice of conservative values may not exist and a designer may be interested in finding the best possible design values. Future work will investigate optimizing the values $\boldsymbol{u}_{\text{det}}$ in addition to the margins.

In this study, a constraint was placed on the expected probability of failure during the optimization of margins. By constraining the expected probability of failure, it is possible to arrive at an optimum set of margins that not only have some very safe designs but also have some unsafe designs. To avoid this situation, additional constraints should be included that consider the spread of the probability of failure distribution (e.g., superquantile [37]).

In this study, the proposed method was illustrated on some simple example problems. The method could easily be applied to examples with additional design variable and/or aleatory variables. Additional design constraints with mixed uncertainty could theoretically be handled by including another set of margins specific to each constraint. Models with higher computational cost could be handled by fitting surrogates to the expected performance and expected probability of failure with respect to the margins in order to remove the design models from the global optimization problem (see Appendix).

Acknowledgment

This research was supported by Air Force Office of Scientific Research (Contract No. 84796) and ONERA—The French Aerospace Lab. This support is gratefully acknowledged.

Nomenclature

e = epistemic model error
$\mathbb{E}[\cdot]$ = expected value operator
$f(\cdot,\cdot)$ = objective function
$g(\cdot,\cdot)$ = limit-state function
n = margin
p_{re} = probability of redesign
p_f = probability of failure
$\mathbb{P}[\cdot]$ = probability operator
q = redesign indicator function
$\boldsymbol{U}$ = aleatory random variable vector
$\text{Var}(\cdot)$ = variance operator
$\boldsymbol{x}$ = design variable vector

Subscripts

det = deterministic value
E = epistemic uncertainty
f = failure
final = final design after possible redesign
H = high-fidelity model
ini = initial design
L = low-fidelity model
lb = lower bound
re = design after redesign
T = true model
ub = upper bound
U = aleatory uncertainty

Superscripts

(i) = epistemic realization
$\star$ = target value in optimization

Accents

$\bar{}$ = mean value

Appendix: Reducing the Computational Cost Through the Use of Surrogate Models

The proposed method may be computationally expensive because it involves a Monte Carlo simulation (MCS) of a design/redesign process nested inside a global optimization problem. To reduce the computational cost, surrogate models can be fit to the expected cost and expected probability of failure as a function of the margins. A design of experiment (DoE) is performed over the margin design space $\boldsymbol{n}_{\min} \leq \boldsymbol{n} \leq \boldsymbol{n}_{\max}$ where $\boldsymbol{n} = \{n_{\text{ini}}, n_{\text{lb}}, n_{\text{ub}}, n_{\text{re}}\}$. The MCS of the design/redesign process is performed for each point in the DoE to calculate the expected cost, $\mathbb{E}_E[\mathbb{E}_U[f(\boldsymbol{X}_{\text{final}}, \boldsymbol{U})]]$, and the expected probability of failure, $\mathbb{E}_E[P_{f,\text{final}}]$. It is recommended that a noninterpolating surrogate model, such as Kriging with nugget, be used in order to filter some of the noise introduced by the MCS. After creating the surrogate models, the optimization problem in Eq. (1) can be solved with respect to the surrogate models. After finding the optimum margins, it is recommended that another MCS be performed using the optimum margins to obtain the detailed simulation results and assess the accuracy of the surrogates at the optimum.

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