Struct Multidisc Optim (2017) 55:1091–1119
DOI 10.1007/s00158-016-1579-y

REVIEW ARTICLE

On design optimization for structural crashworthiness and its state of the art

Jianguang Fang[1] · Guangyong Sun[1] · Na Qiu[2] · Nam H. Kim[2] · Qing Li[1]

Received: 30 July 2015 / Revised: 21 August 2016 / Accepted: 23 August 2016 / Published online: 16 September 2016

Abstract Optimization for structural crashworthiness and energy absorption has become an important topic of research attributable to its proven benefits to public safety and social economy. This paper provides a comprehensive review of the important studies on design optimization for structural crashworthiness and energy absorption. First, the design criteria used in crashworthiness and energy absorption are reviewed and the surrogate modeling to evaluate these criteria is discussed. Second, multiobjective optimization, optimization under uncertainties and topology optimization are reviewed from concepts, algorithms to applications in relation to crashworthiness. Third, the crashworthy structures are summarized, from generically novel structural configurations to industrial applications. Finally, some conclusions and recommendations are provided to enable academia and industry to become more aware of the available capabilities and recent developments in design optimization for structural crashworthiness and energy absorption.

Keywords Literature review · Crashworthiness optimization · Surrogate model · Uncertainty · Multiobjective optimization · Topology optimization · Energy absorption

✉ Guangyong Sun
sgy800@126.com

✉ Qing Li
qing.li@sydney.edu.au

Jianguang Fang
fangjg87@gmail.com

1 School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, Sydney, NSW 2006, Australia

2 Department of Mechanical & Aerospace Engineering, University of Florida, Gainesville, FL 32611, USA

1 Introduction

1.1 Motivation of crashworthiness optimization

Motorization brings two significant challenges to the modern society. Firstly, road and vehicle safety becomes increasingly important, which has notably heightened legislative requirements by introducing more effective protective systems to the vehicle. Secondly, there is an ever-growing concern in environment and sustainability, which largely push up the lightweighting standards to reduce fuel consumption. For these reasons, the automotive industry has devoted a substantial effort to deliver more crashworthy vehicles for addressing these two competing issues concurrently.

Vehicle crash brings increasing concerns from socioeconomic aspects. Each year vehicle crash leads to about 1.2 million deaths and many more injuries worldwide, becoming one of the biggest public health issues facing modern society. In USA, the most motorized country, for example, the crash-induced fatality has remained at a considerably high level, though certain reduction, over the past decade (Fig. 1), leading to the direct annual cost of US$277 billion, equivalent to nearly US$900 per head or 1.9 % of real Gross Domestic Product (GDP) of the country (http://www.nhtsa.gov/). Fig. 1 also provides the fatality data in China, a rapidly developing motorization country. A total of 58,539 people were killed in accidents in 2013 (http://www.stats.gov.cn/.) Such high socioeconomic burden places increasing attention in road and vehicle safety. As an important means, crashworthiness design of vehicle structures has proven highly effective, through which at least 43 % of potential fatalities could have been prevented and much more injuries avoided as per the study by O'Neill (2009), making crashworthiness research and development draw growing interest over recent years. For example, front rails of automobiles and trains (Fig. 2) have

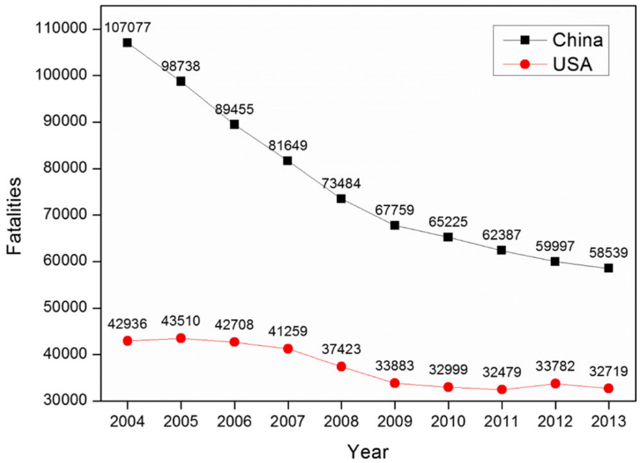


Fig. 1 Fatalities caused by motor vehicle crashes in USA and China

been extensively studied to protect passengers from fatal or severe injuries during the collision. To rate crashworthiness of vehicle, the new car assessment program (NCAP) was established to provide consumers with comparative ratings. In many ways, NCAP has largely lifted the standard of consumer's expectation and vehicle crashworthiness, virtually making all the new vehicles become five-star (highest) in order to enter the market (O'Neill (2009)).

On the other hand, lightweighting of the vehicle has been driven by emergent concerns in fuel consumption (over 1 billion Liters per day for light vehicles such as passenger cars) and environment due to rapid motorization (Davis et al. (2013)). The statistics show that the fuel consumption is directly proportional to vehicle weight; specifically, 10 % reduction of weight could lead to 6–8 % saving in fuel consumption (Zhang et al. (2007b)). Furthermore, lightweighting increases the range of the vehicle, which is a determinant for the use of the electric vehicle (EV). Lured by such a heightening requirement, a series of recent articles in *Nature* and *Science* clearly indicated a replenished research interest and significant endeavor in pursuing lightweight materials and structures (Kim et al. (2015)). Yet the question remaining to be addressed is whether their crashworthiness meets the requirements or there is a solution in balancing crashworthiness and lightweighting most effectively. Over the past two decades, design optimization has been developed as a powerful tool to seek a highest possible crashworthiness and lightest possible structure for various vehicles.

1.2 Theme of this review

In general, a problem of crashworthiness optimization can be formulated mathematically as:

$$\begin{cases} \min\ \mathbf{f}(\mathbf{x}) \\ s.t.\quad \mathbf{g}(\mathbf{x}) \le \mathbf{0} \\ \qquad \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \end{cases} \tag{1}$$

where $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are the objective vector and constraint vector, respectively. $\mathbf{x}$ denotes the vector of design variables.

This review of literature will be conducted based on (1) as follows. (1) classification of crashworthiness criteria adopted in various optimization objectives and constraints; (2) formulations of crashworthiness criteria (objectives and constraints) for optimization; (3) optimization strategies for size, shape and topology under single-/multiobjective with/without uncertainties; and (4) applications of crashworthiness optimization, ranging from novel structural configurations, such as sectional profile of energy absorbers, to entire vehicular and other engineering structures.

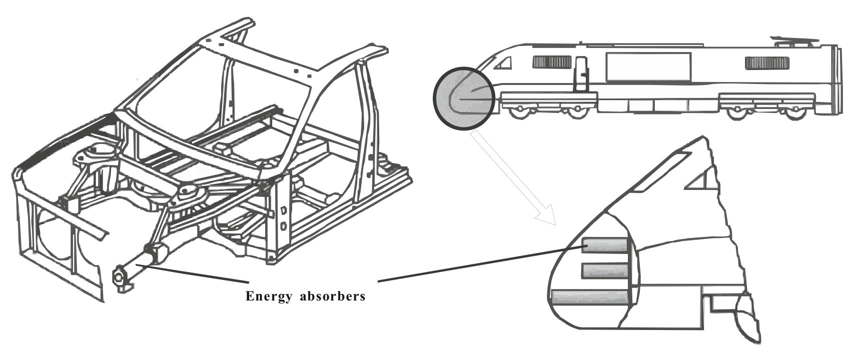
2 Design criteria for crashworthiness and energy absorption

2.1 Classification of crashworthiness criteria

2.1.1 Injury-based metrics

From a biomechanics point of view, occupants' responses to a crash can be measured by such indices as head injury criteria (HIC), chest acceleration, chest deflection and femur loads under impact (Du Bois et al. (2004)). These indices are affected by the vehicle crash pulse (*CP*), the magnitude of intrusion (*Intr*) into the occupant compartment, restraint system, and vehicle interiors. Among them, *CP* and *Intr* are the direct consequences of structural crashworthiness. As a variant of *Intr*, intrusion velocity (*IntrV*) is also utilized as a design

Fig. 2 Energy absorbers in automobile/train structures (Marsolek and Reimerdes (2004))



criterion in the literature. In general, a high acceleration implies a large impact force exerted on occupants and can result in a high risk of injury to occupants. For this reason, peak acceleration ($a_{\max}$) and peak crash force ($F_{\max}$) during impact are extensively employed as design criteria for optimization. Table 1 in Appendix summarizes commonly used injury-based metrics in literature.

2.1.2 Energy-based metrics

The crashworthy structures are expected to absorb as much energy as possible so as to reduce the kinetic energy transmitting to the occupants. Hence, the amount of energy absorption (*EA*) has been drawn exhaustive attention by researchers.

$$EA(d) = \int_0^d F(s)\mathrm{d}s \tag{2}$$

where *F(s)* is the instantaneous impact force at the crash distance *s* and *d* the total crash displacement concerned for measuring the energy absorption.

To take into account the mass efficiency, the specific energy absorption (*SEA*) defined as the *EA* per unit mass has also been widely used.

$$SEA(d) = \frac{EA(d)}{M} \tag{3}$$

where *M* is the mass of the structure.

Other criteria in relation to energy absorption capacity include the crash load efficiency (*CFE*), defined as the ratio of the mean crash force (F_{avg}) to peak force $F_{\max}$; and the load uniformity (*LU*), which is a reciprocal of CFE.

$$CFE(d) = \frac{F_{avg}(d)}{F_{\max}(d)} \times 100\% \tag{4}$$

$$LU(d) = \frac{F_{\max}(d)}{F_{avg}(d)} \times 100\% \tag{5}$$

$$F_{avg}(d) = \frac{EA(d)}{d} \tag{6}$$

The usage ratio (*UR*) of the energy absorber can also be employed to assess the crashworthiness,

$$UR(d) = \frac{d}{l} \times 100\% \tag{7}$$

where *l* is the total length of the structure. The effective crash distance d_{eff} is calculated as the deformation at which $CFE(d) \times UR(d)$ has the maximum value (Hanssen et al. (2000)). Then, the other criteria can be calculated during the displacement from 0 to d_{eff}.

In order to maximize the material usage, topology optimization often seeks internal energy density (*IED*) of each element to be as uniform as possible over the whole design domain. Table 2 in Appendix summarizes the energy-based metrics in literature.

2.2 Remarks on design criteria

Note that the selection of design criteria prior to optimization has been considered critically important in order to obtain a most 'beneficial' optimal design. However, the researchers have not reached a consensus yet to date. For example, Marzbanrad and Ebrahimi (2011) and Shakeri et al. (2007) preferred a long effective distance to make full use of the energy-absorbing capacity, whereas the others (e.g. Fang et al. (2014a), Shi et al. (2013a), Shi et al. (2013b), Wang and Shi (2014)) preferred a short effective distance with the expectation of a lowest possible intrusion. Taking the front side rails in frontal impact as an example, if they deform too much, the passenger compartment might be intruded severely by the engine booth. On the other hand, if the rails crash a short distance, only a little kinetic energy can be dissipated through progressive deformation and thus high deceleration could be yielded. Also some researchers (e.g., Zhang et al. (2008), Bi et al. (2010)) constrained F_{avg} above a certain level to maintain high energy absorption, while others (e.g., Zarei and Kroger (2006), Yang and Qi (2013), Zarei and Kroger (2008a), Zarei and Kroger (2008b), Toksoy and Güden (2011)) suggested to keep F_{avg} at a low level to reduce the risk of occupant injury. Horstemeyer et al. (2009) compared the two different criteria under side impact, i.e. the energy absorption of collapsed components and an injury-based metric (in terms of accelerations) of the dummy. They revealed that the injury-based design, which differs a lot from the energy-based design, could achieve a much safer structure. The uniform internal energy density (IED) criterion in topology optimization was also challenged by Witowski et al. (2012) because uniform IED only represents the whole structure absorbs energy uniformly, rather than the maximum amount of *EA* in the whole structure.

For industrial applications, the selection of design criteria is closely related to the loading scenario considered (please see Section 5.2 for further details). As the ultimate goal of crashworthiness optimization is to ensure the passenger's safety, it can be more judicious to consider the damage of the dummy during crashes although its modeling is fairly complicated (Horstemeyer et al. (2009)). When it comes to the safety components used in real world, more realistic loading conditions and design criteria should be considered altogether to realize their required functionality in a context of the entire vehicle system. For example, design criteria under more realistic loads, such as lateral bending (Kim et al. (2002), Xiang et al. (2006), Zarei and Kroger (2008b), Zhang et al. (2009), Fang et al. (2014b)) and oblique loads (Reid and Reddy (1986), Zarei and Kroger (2008a), Qi et al. (2012), Tarlochan et al.

(2013), Zhang et al. (2014b)) should also be taken into account for crashworthiness design of crashing components. In addition, from the practical point of view, it is recommended that the roles of each safety component/system should be analyzed based on the whole vehicle safety (Zhu et al. (2016)). As such, the design criteria can be integrated to achieve the best performance of the vehicular structure as a whole.

3 Formulations of crashworthiness criteria in optimization

3.1 Analytical functions

It is indispensable to formulate the crashworthiness performance quantitatively for optimization. The first approach is perhaps to develop proper analytical models of these performance criteria with respect to design variables (Chen (2001), Kim (2002), Hanssen et al. (2001), Kim et al. (2002)). For example, Chen (2001) derived the closed-form theoretical expressions of *EA* during a bending collapse and combined compression/bending deformation for a thin-walled beam with a ultralight filler. Kim (2002) derived the analytical function of SEA and then incorporated it into the optimization of a multi-cell tube. Hanssen et al. (2001) used several formulas in terms of foam density, wall thickness, column width, wall material strength and total component length to optimize the square column with foam filler. Note that analytical functions can only be applied to such tubes with simple geometries (e.g., square and circular tubes) subject to strong mechanical assumptions, and was more extensively used at earlier days when the computational resources were not affordable to accomplish the crashing simulation for design optimization.

3.2 Direct coupling with finite element analysis

With the increasing computational capacity, numerical methods, represented by nonlinear finite element analysis (FEA), have proven effective to predict the crashworthiness. The question is if it is possible to directly couple FEA with optimization algorithms for crashworthiness problems. Or, whether is it realistic to iteratively call FEA evaluations in the optimization process. Mathematical programming-based structural optimization often requires gradient information of the objectives and constraints to determine a searching direction towards an optimal solution. In a very early work, Yang et al. (1994) showed that it is feasible but fairly costly to conduct the optimization for crash simulations, where the design sensitivities were calculated by using the forward finite difference method with 1 % step size for each design variable. For highly nonlinear problems, the simulated responses often contain numerical noise, making it difficult to calculate gradients accurately (Zabaras et al. (2003)).

To overcome this difficulty, gradient-free or namely zeroth-order methods, which do not need gradient information, seem to be more suitable. Of them, population-based algorithms such as Monte Carlo simulation (MCS) methods, simulated annealing (SA) and genetic algorithms (GA) could be a useful choice. The advantage of gradient-free methods is that they may converge to the global optimum. However, computational cost could be prohibitive as they often require a large number of function evaluations before convergence, and there is no universally good criterion to determine the convergence. A practical compromise is to limit the number of nonlinear FEA runs by the predefined population size and number of generations, which may however make it difficult to yield a global optimum.

Nevertheless, in an industrial context, the mathematical optimum may be of less practical interest while an efficient improvement is intended. In this regard, Rzesnitzek et al. (2002) proposed a two-stage optimization method for crashworthiness problems, in which the first stage conducts a stochastic optimization using MCS for a large number of design variables to acquire an optimal solution away from the initial design and identify a small number of significant design variables. Redhe et al. (2004) pointed out that the stochastic optimization should not be used for problems with less than 10-15 design variables; and the more design variables the problem has, the more efficient the stochastic optimization is. In the multidisciplinary optimization of car bodies, Duddeck (2008) recommended to couple population-based algorithms with FEA for the design of the frontal impact problem, which has a highly non-regular crash responses due to high nonlinearity and bifurcations. More recently, Xu et al. (2014) used the benchmark problems to conduct direct coupling based optimization. From their study, when sufficient computational resources are available, direct coupling method could be promising in terms of the performance and feasibility of optimization. Xu et al. (2015) developed a data mining-based strategy to improve the efficiency of population-based algorithms. The historic information was utilized to identify and eliminate low-quality and repetitive designs based on clustering analysis. The algorithm considers both the explorative search at the earlier stage and exploitative search at the later stage. The authors also recommended using larger population and fewer generations to take advantages of parallel computing.

Besides, the equivalent static load (ESL) method (Choi and Park (1999), (Kang et al. (2001), Shin et al. (2007), Park (2011)) offers a simplified and approximate approach to the evaluation of crashworthiness performance. The applications can be found in Jeong et al. (2008), Jeong et al. (2010), Yi et al. (2012) and Lee et al. (2015). The major advantage of ESL is that a nonlinear crashworthiness problem is converted to a linear static problem, which is much cheaper and more stable, computationally. However, its limitations are also obvious due to the intrinsic nature of simplifications and assumptions (Witowski et al. (2012)).

3.3 Surrogate modeling

In crashworthiness optimization, direct coupling method may be inefficient (if not impossible) since iterative nonlinear FEA during optimization usually require enormous computational efforts and take the high risk of premature simulation failure prior to a proper convergence. As a result, surrogate models (or metamodels) are more often used as an alternative for formulating the design criteria in terms of an explicit function of design variables in advance of optimization, which has proven an effective and sometimes a unique approach (Wang and Shan (2007), Forrester et al. (2008)). The idea of surrogate modeling is to construct an approximate function based upon a series of sampling evaluations, in which design space is typically sampled using the design of experiment (DoE) methods (Fig. 3). Then, the FEA is performed at these sample points to establish surrogate models with a certain confidence of approximation for crashworthiness optimization.

3.3.1 Polynomial response surface (PRS) model

In many cases, the polynomial basis functions are found simple yet effective to establish a surrogate model. For example, a quadratic PRS model can be expressed as (Montgomery (1996)),

$$\hat{y}(\mathbf{x}) = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_i^2 + \sum_{i=1}^{n-1} \sum_{j>i}^{n} b_{ij} x_i x_j \tag{8}$$

where b_0, b_i, b_{ii} and b_{ij} are the unknown coefficients, x_i is the i-th design variable, and n is the total number of design variables. $\hat{y}(\mathbf{x})$ is the approximation to the actual value $y(\mathbf{x})$ from FEA.

Considering the fact that the number of unknown coefficients is $n_s = (n+1) \times (n+2)/2$, it is recommended to generate more than twice more samples than n_s to prevent overfitting of coefficients, which implies the number of design variables in PRS model can be critical for determining the computational cost. To address this issue, stepwise regression (Draper and Smith (1981), Yang et al. (2000), Gu et al. (2001)) can be implemented to screen the terms in PRS that have relatively little contribution to the design criteria.

3.3.2 Radial basis function (RBF) model

Radial basis function model was developed for scattered multivariate data interpolation by using a series of basis functions that are symmetric and centered at each sampling point. Radial basis functions are typically formulated as (Hardy (1971)):

$$\hat{y}(\mathbf{x}) = \sum_{j=1}^{m} c_j p_j(\mathbf{x}) + \sum_{i=1}^{n_s} \lambda_i \varphi(r(\mathbf{x}, \mathbf{x}_i)) \tag{9}$$

where m is the number of the polynomial terms, c_j is the coefficient for polynomial basis function $p_j(\mathbf{x})$, and n_s is the number of sample points. λ_i is the weighted coefficient for the term for the i-th variable, $r(\mathbf{x}, \mathbf{x}_i)$ is the Euclidean distance expressed in terms of $\|\mathrm{x} - \mathrm{x}_i\|$. $\varphi(r)$ is the radial basis function.

3.3.3 Kriging (KRG) model

The Kriging model (Sacks et al. (1989)) was originally developed for mining and geostatistical applications involving spatially and temporally correlated data. The KRG model is composed of a global model $f(\mathbf{x})$ and a local departure $Z(\mathbf{x})$:

$$y(\mathbf{x}) = f(\mathbf{x}) + Z(\mathbf{x}) \tag{10}$$

where $y(\mathbf{x})$ is the unknown function of interest, $f(\mathbf{x})$ models the global trend of the function of interest, and $Z(\mathbf{x})$ models the correlation between the points by a stochastic process whose mean is zero and variance is σ^2.

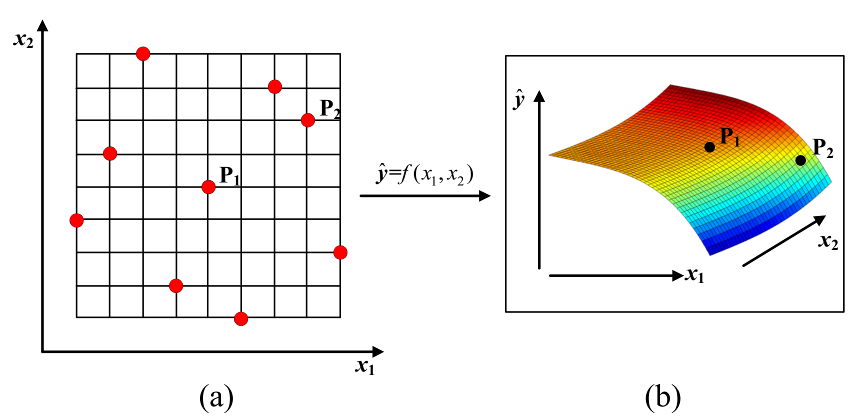


Fig. 3 Surrogate modeling: (a) Design of experiment, (b) Construction of surrogate models

$Z(\mathbf{x})$ provides local deviations, and the covariance between different points is modeled as:

$$\mathrm{Cov}\left(Z(\mathbf{x}_i), Z(\mathbf{x}_j)\right) = \sigma^2 \mathbf{R}\left[R\left(\mathbf{x}_i, \mathbf{x}_j\right)\right] \tag{11}$$

where $\mathbf{R}$ is a correlation matrix defined by correlation function $R(\mathbf{x}_i, \mathbf{x}_j)$ as follows:

$$R\left(\mathbf{x}_i, \mathbf{x}_j\right) = \exp\left[-\sum_{k=1}^{n} \theta_k \left|x_i^k - x_j^k\right|^2\right] \tag{12}$$

where θ_k is the unknown correlation parameter used to fit the model, and x_i^k and x_j^k are the k-th components of sample points $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively.

3.3.4 Artificial neural network (ANN) model

Artificial neural network is a powerful method to formulate the relationship between a set of input variables and output results in complex systems (Zurada (1992)). It is composed of some parallel numerical computing units, namely neural elements. The neural elements are linked according to specific topological network functions. Thus, a neural model will be defined by using connection weights and biases parameters. These parameters are trained from the training sample data set by using specific optimization algorithm, which adjusts the values of the weights between elements. The training data consists of pairs of design variables and output responses.

Other types of surrogate models used in crashworthiness optimization include support vector regression (SVR) (Smola and Schölkopf (2004)) and multivariate adaptive regression splines (MARS) (Friedman (1991)) etc.

3.3.5 Comparison of different surrogate models

Regarding the selection of surrogate models, researchers have provided some general guidelines, which may also be useful in crashworthiness optimization. Simpson et al. (2001) pointed out that PRS works well in the problems with <10 variables and the problems with random errors. They also claimed that ANN should be good for very large design problems (~10,000 variables) while KRG is able to handle the problems with <50 variables. Jin et al. (2001) compared surrogate models under multiple modeling criteria. RBF was found most insensitive to DoE sample size in most situations in terms of accuracy and robustness, while KRG is very sensitive to the noise because it interpolates the sample data.

There have been some comparative studies of surrogate models in relation to crashworthiness problem in literature (e.g., Fang et al. (2005), Forsberg and Nilsson (2006), Zhu et al. (2009) , Shi et al. (2012)). For instance, Fang et al. (2005) compared quadratic PRS and RBF for fitting nonlinear responses in a frontal collision and found that PRS was able to produce a satisfactory approximation to *EA*, while the RBF models performed better to approximate $a_{\max}$. Also, the RBF models can yield more accurate optimization results. Forsberg and Nilsson (2006) compared the linear PRS and KRG with the same updating scheme in the region of interest. KRG was found to enable to improve the sequential behavior of the optimization algorithm at earlier iterations of the optimization process. However, KRG could be problematic if a constraint was violated after several iterations and linear PRS seemed more easily to find a feasible solution. Shi et al. (2012) proposed to select the best surrogate model using a Bayesian metric under data uncertainty, thereby determining a proper sample size for large scale real-life problems. It can be concluded that the selection of a surrogate model is largely case dependent. In other words, no unique surrogate model is able to produce the most accurate result for all cases (Yang et al. (2005)). Furthermore, the most accurate surrogate model may not necessarily provide the most promising optimum (Song et al. (2013)). Therefore, the concurrent use of multiple surrogate models are recommended by Song et al. (2013) to seek for a better optimum since the time of constructing surrogate models is negligible compared to that of acquiring DoE data. Another practice of using multiple surrogates is to construct ensembles of surrogate models as follows.

3.3.6 Ensemble of surrogates

Typically, obtaining data required for developing surrogate is computationally expensive, and the use of an ensemble was first introduced by Bishop (1995) to take full advantage of all the individual surrogates to extract as much information as possible with a relatively low computational cost. Using the weighted-sum formulation, the ensemble of surrogates can be expressed as:

$$\hat{y}_{Ens}(\mathbf{x}) = \sum_{i=1}^{N} \omega_i(\mathbf{x}) \hat{y}_i(\mathbf{x}) \tag{13}$$

where $\hat{y}_{Ens}(\mathbf{x})$ denotes the predicted response by the ensemble of surrogates $\hat{y}_i(\mathbf{x})$, N is the number of the individual surrogate in the ensemble, $\hat{y}_i(\mathbf{x})$ and $\omega_i(\mathbf{x})$ are the surrogate response and the corresponding weight factor of the i-th surrogate models, respectively.

To determine the weight factors for surrogate models, different strategies have been developed. Zerpa et al. (2005) set the values of the weight factors for each surrogate model to be inversely proportional to the estimate of the prediction variance. Goel et al. (2007) proposed a heuristic weight scheme based on the generalized mean square cross-validation error (GMSE). Viana et al. (2009b) proposed to select the weight factors following an approach based on minimizing the mean square error. Acar and Rais-Rohani (2009) proposed an optimal weighted surrogate approach to determining the weight factors via optimization. The difference between the last two is that Acar and Rais-Rohani's approach obtains the weights through an optimization process, while Viana's approach obtains the weights through an analysis expression (Zhou et al. (2011)). Note that while the determination of proper weight factors associated with individual surrogates can be based upon global and/or local measures (Acar (2010b)), the above-mentioned strategies used the global method (i.e., $\omega_i(\mathbf{x})=\omega_i$). The more sophisticated local methods, in which $\omega_i(\mathbf{x})$ varies over the design space, were also proposed by Acar (2010b).

When coming to crashworthiness optimization, ensembles of surrogates have also shown their advantages over individual surrogates (Acar (2010b); Acar and Solanki (2009a); Hamza and Saitou (2012); Pan and Zhu (2011a); Yin et al. (2014a)). For example, Acar and Solanki (2009a) examined ensembles of surrogates in two cases of offset frontal and side impacts and found that for all crash responses of interest the ensemble of surrogates outperformed all individual surrogates. Pan and Zhu (2011a) demonstrated that forming an ensemble of surrogates could help avoid a misleading optimum in a design optimization of vehicle roof structures.

The ideal scenario for using ensembles of surrogates would be that individual surrogates have different prediction values but similar overall prediction accuracies on the entire design domain, so that prediction errors cancel out when aggregation of the prediction is performed. That is to say, one should ensure the accuracy and diversity of individual surrogates to better make advantage of surrogate ensembles. Unfortunately, surrogates with a comparable accuracy were found often highly correlated (Viana et al. (2009a)). One should also keep in mind that when applying ensembles of surrogates to crashworthiness optimization, the global accuracy is less interesting than the ability to lead to the global optimum (Viana et al. (2014)).

3.3.7 Efficient global optimization

When performing a surrogate-based optimization, a basic assumption is that the surrogate model is sufficiently accurate and all we need to do is to find the optimum design using the established surrogate model (Forrester and Keane (2009)). However, the surrogate model constructed using initial samples will probably not be accurate in the local region of the final optimum. It is common to *exploit* this local region by sequentially positioning additional samples inside. These infill points are then used to update the surrogate model until the optimum converges to the final location properly, which seems to be attractive to more accurately locate a local optimum rather than the true global optimum (Forrester and Keane (2009)).

On the other hand, *exploring* design space is a strategy to increase the global accuracy of a surrogate model. It is straightforward to add sequential samples to the sparse regions of design space. If error estimates are available for the surrogate model, those points with large errors can be a candidate for increasing the accuracy of the surrogate. For example, Chen et al. (2014) and Sun et al. (2014a) used the maximum mean squared error of KRG model to determine new sampling points in the framework of sequential optimization.

Considering both *exploitation* and *exploration*, Efficient Global Optimization (EGO) (Schonlau (1998)) has been proposed to add new sampling points iteratively which contribute toward global optimization. The EGO algorithm uses KRG models because they provide not only the surrogate prediction but also error estimates. The expected improvement (*EI*) is maximized to find the sequential sampling points at each iteration as (Schonlau (1998)),

$$EI(\mathbf{x}) = \begin{cases} \left(f_{\min}-\hat{y}(\mathbf{x})\right)\Phi\left(\frac{f_{\min}-\hat{y}(\mathbf{x})}{\hat{s}(\mathbf{x})}\right) + \hat{s}(\mathbf{x})\phi\left(\frac{f_{\min}-\hat{y}(\mathbf{x})}{\hat{s}(\mathbf{x})}\right) & \text{if } \hat{s}(\mathbf{x}) > 0 \\ 0 \quad \text{if } \hat{s}(\mathbf{x}) = 0 \end{cases} \tag{14}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density and the cumulative distribution functions of the standard normal distribution. $\hat{s}(\mathbf{x})$ is the KRG prediction error, which is also called as the mean squared error. If $\hat{s}(\mathbf{x})>0$, the first term in . (14) is the difference between the current minimum and the predicted value multiplied by the probability that $y(\mathbf{x})$ is smaller than the current best $f_{\min}$. Therefore, the first term becomes large when $\hat{y}(\mathbf{x})$ is likely smaller than $f_{\min}$. The second term is the standard deviation of $y(\mathbf{x})$ multiplied by the probability density at $y(\mathbf{x})=f_{\min}$. This term becomes great when there is a high uncertainty of the prediction (which probably appears far away from the existing samples as the KRG model goes exactly through samples). To take into account the constraints, . (14) can be multiplied by the probabilities $P(\mathbf{g}(\mathbf{x})\leq\mathbf{0})$ to obtain the constrained version of *EI* (EI_c) so that each constraint is met.

$$EI_c(\mathbf{x}) = EI(\mathbf{x})P(\mathbf{g}(\mathbf{x}) \leq \mathbf{0}) \tag{15}$$

Schonlau (1998) proposed to generate sequential sampling scheme by maximizing *EI* to yield the sequential points. This criterion balances the exploration of design space and exploitation of the local region around an optimum, which has been applied to the crashworthiness design of a cylindrical tube by Lee et al. (2002). The *EI* criterion assumes that KRG model parameters were estimated accurately based on the existing sample data. Otherwise, the iteration process could converge very slowly or even not at all (Forrester and Keane (2009)). Besides, since a KRG model often underestimates the uncertainty, extra care should be taken to avoid the premature termination by setting an overly stringent threshold.

Another extension of EGO is the multiobjective versions of *EI*, which were established recently (e.g., Keane (2006), Shimoyama et al. (2013), Couckuyt et al. (2014)). Unlike the definition of single objective *EI* in (14) and (15), multiobjective *EI* could become cumbersome as the dimension of the objective function increases.

Traditionally, EGO adds one single point to the sample set at a time, which may not be thought efficient when computational resources are available for parallel computing and the main concern is the wall-clock time (rather than the number of simulations). To take advantage of parallel computing capabilities, Schonlau (1998) sought the global maximum of the *EI* criterion and then temporarily added the KRG predicted value at this point to sample data (assuming that the model is correct at this location). The KRG model is then constructed with the sample data and the maximization of *EI* criterion is continued to seek other new samples until a predefined number of new samples has been obtained. Sobester et al. (2004) proposed to locate a number of maxima of *EI* (which is usually extremely multimodal) using either a gradient-based optimization algorithm with multiple restarts or a genetic algorithm with clustering and sharing. Then, those locations of the maxima can be evaluated in parallel and the process is repeated until convergence. Viana et al. (2013) proposed a multiple surrogate efficient global optimization algorithm to add multiple samples per optimization cycle, in which uncertainty estimates of other models was imported from the KRG model. However, these parallel versions of EGO have not been substantially validated for their effectiveness in crashworthiness optimization. Hamza and Shalaby (2014) used three infill criteria to generate new multiple samples, and their algorithm was successfully applied to a crashworthiness problem after being tested on four benchmark mathematical functions. Most recently, Haftka et al. (2016) conducted a comprehensive survey on parallel surrogate-assisted global optimization and the interested readers are strongly recommended to acquire more insightful information from this article.

3.3.8 Successive surrogate modeling (SSM)

The general idea of the EGO is to add sequential sampling points iteratively at the regions of interest so that the accuracy of surrogate models is improved locally and globally. The other option for sequential sampling is to use successive response surface method (SRSM or successive surrogate modeling, SSM) (Kurtaran et al. (2002)), in which the region of interest (RoI) is gradually shrunk to a smaller area around the optimum by panning and zooming within the design space (original RoI) during the iterations (Fig. 4).

In the successive surrogate modeling method, the center of RoI at the $(k+1)$-th iteration is the optimum $\mathbf{x}^{(k)*}$ of the k-th iteration, and its size of RoI is a fraction of the size of the k-th

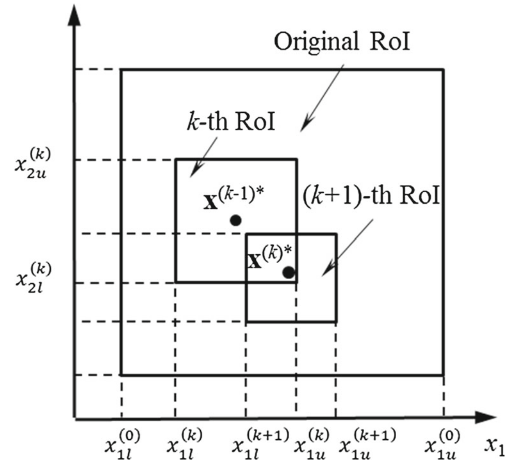


Fig. 4 Updating process of RoI

iteration. The fraction parameter λ_i for the i-th design variable is calculated based upon the distance between the optimum and the center of the current RoI:

$$\lambda_i^{(k+1)} = \eta + (\gamma - \eta)\frac{\left|x_i^{(k)*} - \left(x_{il}^{(k)} + x_{iu}^{(k)}\right)/2\right|}{\left(x_{iu}^{(k)} - x_{il}^{(k)}\right)/2} \tag{16}$$

The maximum value of $\lambda_i^{(k+1)}$ ($\lambda^{(k+1)} = \max \lambda_i^{(k+1)}$ $(i = 1,..,n)$) is applied to all design variables during the iterations. Then, the lower and upper bounds of the i-th design variable of $(k+1)$-th subregion can be determined by:

$$\begin{cases} x_{il}^{(k+1)} = \max\left\{x_i^{(k)*} - \lambda^{(k+1)}\left(x_{iu}^{(k)} - x_{il}^{(k)}\right)/2, x_{il}^{(0)}\right\} \\ x_{iu}^{(k+1)} = \min\left\{x_i^{(k)*} + \lambda^{(k+1)}\left(x_{iu}^{(k)} - x_{il}^{(k)}\right)/2, x_{iu}^{(0)}\right\} \end{cases} \tag{17}$$

where $x_{il}^{(0)}$ and $x_{iu}^{(0)}$ are the lower and upper bounds of the entire design space.

While SSM has been demonstrated to be able to identify the optimum region for various crashworthiness problems (Kurtaran et al. (2002), Craig et al. (2005), Liang and Le (2009), Liu et al. (2014)), iterative resampling in SSM might be prohibited in practice as crashworthiness simulations are rather expensive computationally. Implementation of inherited Latin hypercube design (Wang (2003)), which is a technique to inherit previous sample points, might help reduce the required number of sample points in subsequent iterations. The other limitation might be that the continuity between subsequent approximations is not well guaranteed and the information obtained at the previous iterations is difficult to be taken into account (Naceur et al. (2006)). With recent developments in high-performance computing (HPC), parallel computing has become a trend in optimization. Thanks to its parallel nature within each iteration, the speed and efficiencies of SSM can be realized by using multiple resources (Sheldon et al. (2011)). Recently, Stander (Stander (2012), Stander (2013)) extended SSM to solve multiobjective optimization problems by introducing a Pareto Domain Reduction (PDR) technique. Irregular sub-regions of the Pareto optimal front were used as sampling domains, which shrunk iteratively to explore the neighborhood of the Pareto optimal front. This technique was demonstrated to have a comparable accuracy to the direct genetic algorithm while using a much smaller number of simulations.

For readers' reference, Table 3 in Appendix summarizes the studies on surrogate modeling techniques used in the literature for crashworthiness optimization.

4 Optimization strategies

4.1 Multiobjective optimization (MOO)

Like most real-life engineering applications, crashworthiness optimization can be often characterized by a number of design criteria. In the literature, many crashworthiness optimization articles have dealt with multiple design objectives. Due to conflicts between these objectives, often a rational approach to such a problem is to generate a set of solutions (namely Pareto solutions) that provide acceptable overall performance in terms of all these objectives rather than a single one. These solutions are compared using the non-dominated approach which does not introduce preference on any of objective functions in prior. In this approach, solution $\mathbf{x}^{(1)}$ dominates solution $\mathbf{x}^{(2)}$ if: (1) $\mathbf{x}^{(1)}$ is feasible and $\mathbf{x}^{(2)}$ is not, or both of them are infeasible but $\mathbf{x}^{(1)}$ is closer to the feasible boundary; or (2) feasible solution $\mathbf{x}^{(1)}$ is not worse than feasible solution $\mathbf{x}^{(2)}$ in all the objectives and $\mathbf{x}^{(1)}$ is strictly better than $\mathbf{x}^{(2)}$ in at least one objective (Coello et al. (2004)). Otherwise, none of the solutions dominates the other and they are both non-dominated.

Currently, there appear to be two popular ways of dealing with multiobjective optimization in crashworthiness problems. Firstly, one can formulate a combined cost function $F(\mathbf{x})$ to indirectly represent the contributions of multiple objectives through a single function; and then performs a single objective optimization (Forrester and Keane (2009)). In this approach, the linearly weighted sum technique is the most commonly-used formulation in crashworthiness optimization, as:

$$F(\mathbf{x}) = \sum\nolimits_{i=1}^{k} w_i f_i(\mathbf{x}), \quad 0 \le w_i \le 1, \quad \sum\nolimits_{i=1}^{k} w_i = 1 \tag{18}$$

where w_i is the weighting factor (to emphasize the relative importance) of the i-th objective function $f_i(\mathbf{x})$.

Secondly, one can conduct the multiobjective optimization using population-based algorithms directly without formulating a combined cost function, of which multiobjective particle swarm optimization (MOPSO) (Raquel and Naval (2005)) and non-dominated sorting genetic algorithm II (NSGA-II) (Deb et al. (2002)) are two popular algorithms frequently used in crashworthiness problems. Table 4 in Appendix summarizes the previous works on crashworthiness optimization with multiple objectives and Fig. 5 displays the percentage of the different optimization methods used in the literature, where MOGA stands for general multi-objective genetic algorithm and NSGA-II is a special version of MOGA. It is noted that more than 60 % publications have adopted the direct evolutionary algorithms to seek non-dominated solutions.

Although it is of considerable limitation to generation of preferable Pareto solutions, combined cost function methods still contributed to around 40 % of multiobjective crashworthiness optimization in the literature for its simplicity. Note that a linear weighted sum method is impossible to obtain a proper solution in the non-convex portions on the Pareto frontier. Theoretical reasons for this deficiency have been given by Das and Dennis (1997) and Messac et al. (2000b). If nonlinear weighted sum method is used, this limitation might be avoided but the form of the function to be used is difficult to decide consistently. In addition, in the linear weighted sum method, varying the weighting factor from 0 to 1 homogeneously cannot guarantee an even distribution of Pareto points. Das and Dennis (1997) illustrated the necessary conditions for a series of weighted sum interactions to create an even spread of points on the Pareto curve in the objective space.

In crashworthiness optimization, lightweighting of structure and its crashing performance are frequently conflictive, and design criteria (e.g. SEA and F_{max},) could also strongly compete with each other during optimization (e.g., Khakhali et al. (2010), Fang et al. (2014b)). Under this circumstance, generating a complete representation of non-dominated Pareto solutions in objective space is meaningful and could provide insightful information for decision-making.

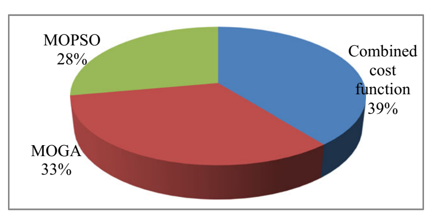
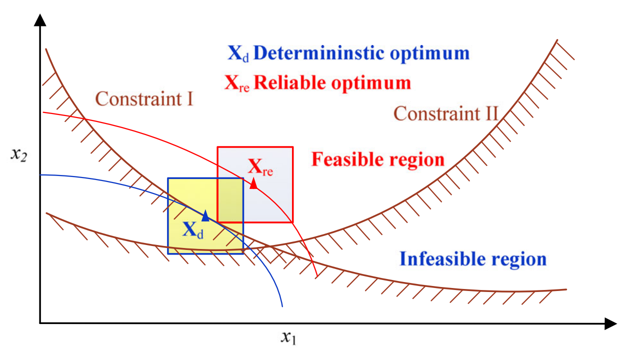


Fig. 5 Percentage of MOO methods used for crashworthiness optimization in literature

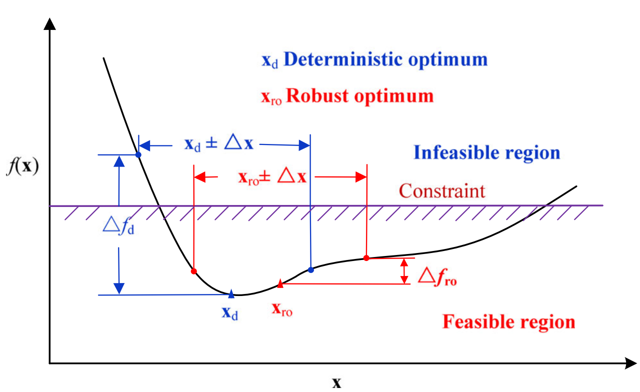
4.2 Optimization under uncertainties

4.2.1 Definition of optimization under uncertainties

Most (if not all) real-life engineering problems involve some degree of uncertainties in loading conditions, material properties, geometries, manufacturing tolerances and actual usage, etc. It must be pointed out that usually a deterministic optimization tends to push a design toward one or more constraints until the constraints become active, thereby leaving no room for accommodating various uncertainties. Therefore, reliability-based optimization (RBO), which aims to seek a reliable optimum by converting the deterministic constraints into probabilistic counterparts representing that probability of design infeasibility is restricted to a pre-specified level, has been widely applied to engineering problems. As shown in Fig. 6a, let $\mathbf{x}_d$ represent the deterministic optimum and $\mathbf{x}_{re}$ represent the reliable optimum in the design space (x_1-x_2 space), which is divided into infeasible and feasible regions by the constraints. Since the deterministic optimum $\mathbf{x}_d$ is located on the boundary of the constraint, it may fall to the infeasible region when uncertainties are present. On the other hand, the reliable optimum $\mathbf{x}_{re}$ moves away to create a gap from the boundary of the constraint so that it can still be within the feasible region when uncertainties are present.



(a)



(b)

Fig. 6 Illustrations of design optimization with uncertainties (a) Reliability-based optimization (RBO) and (b) Robust design optimization (RDO)

Moreover, conventional design likely leads to a large scatter of optimal performance due to uncertainties, which may not only cause significant fluctuations from the desired performance, but also increase life-cycle costs, including inspection, repair and other maintenance expenses (Fang et al. (2015a)). Thus, the concept of robust design optimization (RDO) is to reduce the scatter of the structural performance without eliminating the source of uncertain variability. This approach has drawn increasing attention for solving real-world problems recently (e.g. Park et al. (2006), Beyer and Sendhoff (2007), Yao et al. (2011)). As shown in Fig. 6b, let the *x*-axis represent the uncertain parameter, e.g., random design variable or noise factor, and the vertical axis represents the value of an objective function $f(\mathbf{x})$ to be minimized. Of these two optimal solutions $\mathbf{x}_d$ and $\mathbf{x}_{ro}$ as pointed, x_2 is considered more robust as a variation of $\pm\Delta\mathbf{x}$ in the design variable and/or noise factor does not alter the objective function too much ($\Delta f_{ro} << \Delta f_d$). On the contrary, $\mathbf{x}_d$ appears highly sensitive to the parametric perturbation and often cannot be recommended as a design in practice, even though it has a better nominal value than $\mathbf{x}_{ro}$. It is noted that a robust-based optimization places more emphasis on the stability of the objective, while a reliability-based optimization pays more attention to the feasibility of the constraint.

To accommodate uncertainties, reliability-based optimization (RBO) has been adopted in crashworthiness problems. A general RBO problem can be expressed mathematically as:

$$\begin{cases} \min\ \mathbf{f}(\mathbf{x}) \\ s.t.\quad P(\mathbf{g}(\mathbf{x}) \le \mathbf{0}) \ge R_\mathbf{t} \\ \qquad \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \end{cases} \tag{19}$$

where R_t denotes the reliability level and $P(\cdot)$ stands for the probability function of satisfying the constraints ($\mathbf{g}(\mathbf{x}) \le \mathbf{0}$).

It is commonly acknowledged that a robust design was firstly proposed by Japanese engineer Genichi Taguchi, named as the Taguchi method to improve the quality of manufactured goods and makes the product performance less sensitive to variations of variables beyond the control of designers. A general robust design optimization (RDO) problem can be formulated mathematically as:

$$\begin{cases} \min\quad \mathbf{F}\left(\boldsymbol{\mu}_f(\mathbf{x}), \boldsymbol{\sigma}_f(\mathbf{x})\right) \\ s.t.\quad \mathbf{g}(\mathbf{x}) \le \mathbf{0} \\ \qquad \mathbf{x}^L \le \mathbf{x} \le \mathbf{x}^U \end{cases} \tag{20}$$

where $\boldsymbol{\mu}_f(\mathbf{x})$ and $\boldsymbol{\sigma}_f(\mathbf{x})$ are the vectors of mean and standard deviation of the objectives, respectively.

To enhance a design in terms of both reliability and robustness, RBO and RDO can be combined and referred to as reliability-based robust design optimization (RBRDO), which can be formulated as:

$$\begin{cases} \min & \mathbf{F}\left(\boldsymbol{\mu}_f(\mathbf{x}), \boldsymbol{\sigma}_f(\mathbf{x})\right) \\ s.t. & P(\mathbf{g}(\mathbf{x}) \leq \mathbf{0}\ ) \geq R_t \\ & \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{cases} \tag{21}$$

In the literature, *design for six sigma* (DFSS) can be regarded as a special case of RBRDO. Six sigma is a quality philosophy involving the statistical tools within a structured methodology for gaining the knowledge needed to achieve better, faster and less expensive products than competitors (Breyfogle III (2003)). The term sigma refers to as standard deviation (σ), measuring the dispersion of a set of data around the mean value (μ) of the data.

While the six sigma approach aims to reduce the number of defects, DFSS offers a powerful tool to optimize the products in a cost-effective and simple fashion to meet the customer's requirements (Antony (2002)). The two goals in DFSS are: (1) striving to maintain performance within acceptable limits consistently (reliability); and (2) striving to reduce performance variation and thus increase robustness. With this concept, (21) can be revised as

$$\begin{cases} \min & \boldsymbol{\mu}_f(\mathbf{x}) + 6\boldsymbol{\sigma}_f(\mathbf{x}) \\ s.t. & \boldsymbol{\mu}_g(\mathbf{x}) + 6\boldsymbol{\sigma}_g(\mathbf{x}) \leq \mathbf{0} \\ & \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{cases} \tag{22}$$

4.2.2 Methods of uncertainty analysis in optimization

Monte Carlo simulation The problems defined in (19)–(22) involve a procedure to obtain the values of probabilistic objectives and constraints. One of the effective yet simple approaches could be Monte Carlo simulation (MCS). Using a large number of samples, Monte Carlo simulation allows the estimation to the probability of feasibility as follows,

$$P(g(\mathbf{x}) \leq 0\ ) = \frac{1}{Q}\sum_{i=1}^{Q} I(\mathbf{x}) \tag{23}$$

where Q is the total number of samples and $I(\mathbf{x})$ is an indicator function defined as

$$I(\mathbf{x}) = \begin{cases} 1 & \text{if } g(\mathbf{x}) \leq 0 \\ 0 & \text{if } g(\mathbf{x}) > 0 \end{cases} \tag{24}$$

Note that in (23), Q independent sets of design variables are obtained from sampling techniques on the basis of the probability distribution of input random variables. Thus, MCS is also referred to as sampling-based method (Helton et al. (2006)). MCS is also a conventional method of quantifying robustness (Yao et al. (2011)), allowing determining the means and standard deviations of objectives in . (19)- (21).

$$\begin{cases} \mu_f(\mathbf{x}) \cong \dfrac{\sum_{i=1}^{Q} f(\mathbf{x}_i)}{Q} \\ \sigma_f^2(\mathbf{x}) \cong \dfrac{1}{Q-1}\displaystyle\sum_{i=1}^{Q}\left(f(\mathbf{x}_i) - \mu_f(\mathbf{x})\right)^2 \end{cases} \tag{25}$$

If $\mathbf{x}_i$ is independent, the laws of large numbers allow us to achieve any degree of accuracy by increasing Q. The accuracy of MCS estimation can be quantified with the standard error defined as:

$$err = \frac{\sigma_f(\mathbf{x})}{\sqrt{Q}} \tag{26}$$

The error is, therefore, unrelated to the problem dimension (i.e., the number of design variables), which is very appealing for large-scale problems. And the error is proportional to $1/\sqrt{Q}$, implying that the improvement of accuracy by one order of magnitude will require 100 times more samples. Such computational cost can be prohibitive in the application for complex and highly nonlinear problems such as crashworthiness analysis.

On the other hand, the minimum sampling size required for the desired reliability level $P(g(\mathbf{x}) \leq 0\ )$, as suggested by Tu et al. (1999), is:

$$Q = \frac{10}{1\text{-}P[g(\mathbf{x}) \leq 0]} \tag{27}$$

which indicates that for a 10 % estimated probability of failure; about 100 function evaluations (e.g., nonlinear FEA runs in crashworthiness analysis) are required with some confidence on the first digit of failure prediction. To verify an event having a 1 % failure probability; about a 1000 structural analyses are required, which would be usually considered too expensive and some alternatives may be needed.

To apply MCS to crashworthiness optimization, the use of surrogate models has been advocated by many researchers (e.g., Acar and Solanki (2009b), Fang et al. (2014a), Gu et al. (2001), Gu et al. (2013), Khakhali et al. (2010), Koch et al. (2004), Lönn et al. (2011), Shi et al. (2013b)). After validation, the surrogate models can be used to evaluate the function values with a very large number (e.g., up to millions) of times around each design point at a relatively low computational cost.

In conventional design optimization, the accuracy of a surrogate model is a major concern because the purpose of a surrogate model is to find the optimum function value. In reliability analysis, however, the region where the limit state changes its sign, as in (23), is more important than the value of the function itself. Therefore, it is more important to identify the feasible region that satisfies the constraint conditions than accuracy in predicting function values. Ramu et al. (2007) explored the situation when the function is insensitive to input variables, the error in reliability tends to be amplified due to a relatively large error in identifying the failure (infeasible) region.

When surrogate models are used for reliability analysis, random variables are selected as input. For design optimization, on the other hand, design variables are selected as input to the surrogate models. When the mean of a random variable is used as a design variable, it is possible to use the surrogate model for both reliability analysis and design optimization. In many cases, however, the design variables are different from random variables. In such a case, it is necessary to extend the dimension of surrogate models to include both design and random variables (Qu and Haftka (2004)).

The main issue in surrogate models is the accuracy. Once an accurate surrogate model is available, any method can be applied to calculate reliability or to perform RBDO because the function evaluation using the surrogate model is extremely cheap. Therefore, the conventional MCS can be used to calculate the reliability. However, due to sampling uncertainty, using the conventional MCS can cause some difficulties in calculating sensitivity information during RBDO. Lee et al. (2011) proposed a method of calculating sensitivity using score functions especially when input variables are correlated.

Approximate moment approach (AMA) When a function is linear and input variables are normally distributed, the function is also normally distributed. In such a case, it is much easier to calculate the reliability. When a function is mildly nonlinear, it is possible to approximate the function as a linear or a quadratic function. Taylor series methods can be implemented to approximate statistical moments of system output. The statistical approximations of $f(\mathbf{x})$ using the first-order and second-order Taylor's expansions are expressed in (28) and (29), respectively:

$$\begin{cases} \mu_f(\mathbf{x}) \cong f(\mathbf{x})_{\overline{\mathbf{x}}} \\ \sigma_f^2(\mathbf{x}) \cong \sum_{i=1}^{n} \left(\frac{\partial f(\mathbf{x})}{\partial x_i} \right)_{\overline{\mathbf{x}}} \sigma_{x_i}^2 \end{cases} \tag{28}$$

$$\begin{cases} \mu_f(\mathbf{x}) \cong f(\mathbf{x})_{\overline{\mathbf{x}}} + \frac{1}{2} \sum_{i=1}^{n} \left(\frac{\partial^2 f}{\partial x_i^2} \right)_{\overline{\mathbf{x}}} \sigma_{x_i}^2 \\ \sigma_f^2(\mathbf{x}) \cong \sum_{i=1}^{n} \left(\frac{\partial f(\mathbf{x})}{\partial x_i} \right)_{\overline{\mathbf{x}}} \sigma_{x_i}^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\partial^2 f}{\partial x_i \partial x_j} \right)_{\overline{\mathbf{x}}}^2 \sigma_{x_i}^2 \sigma_{x_j}^2 \end{cases} \tag{29}$$

where $\sigma_{x_i}^2$ represents the variances of the i-th variable x_i and $\overline{\mathbf{x}}$ denotes the mean of the variable vector. To analytically evaluate the statistical information using (28) and (29), function $f(\mathbf{x})$ should be known. However, such a function $f(\mathbf{x})$ may not probably be easy to derive for various crashworthiness criteria. Again, surrogate models can serve as an alternative to approximating $f(\mathbf{x})$ (e.g. Sun et al. (2014a), Sinha (2007), Chen et al. (1996)). This could lead to significant improvement in computational efficiency. However, it might be problematic as the combination of these two kinds of approximation methods (i.e., Taylor's expansions and surrogate models) could cause inaccurate results. Alternatively, derivatives of responses with respect to random variables can be solved by numerical methods (i.e. finite difference methods). Note that $(n+1)$ and $(n+1)(n+2)/2$ analyses are needed for the first and second order Taylor's expansions, respectively. Therefore, Taylor's series approximations will become more expensive with increasing n, but can still be more efficient than MCS (Koch et al. (2004)). Other drawbacks of AMA can be found in Youn and Choi (2004b).

Dual surrogate model (DSM) Following the work by Vining and Myers (1990), dual surrogate models (DSM) have been used in crashworthiness (e.g., Sun et al. (2011), Lönn et al. (2010)) in which two surrogate models are created, one for the mean and the other for the variance or standard deviation of a response. Two types of variables are considered in such a system: namely design (controllable) variables and noise (uncontrollable) variables (Jin et al. (2003)). For constructing DSMs, a cross product array needs to be generated, where design variables are arranged in the inner array while noise variables in the outer array. In each set of design variables, the simulation is repeated several times to capture the mean and standard deviation of responses. Then, they are approximated as the functions of the design variables using surrogate modeling for the optimization. One issue of this approach is how to reasonably generate the outer array to capture accurate statistical information in an efficient manner. Besides, only noise variables can be uncertain and design variables are assumed to be deterministic, which could limit spectrum of applications whose uncertainties in design variables may not be neglected. For this purpose, Aspenberg et al. (2012) proposed a method to use global and local surrogate models for constructing DSMs, in which uncertainties in both design variables and noise variables can be considered.

Most probable point based reliability analysis In addition to the above-mentioned quantification methods for uncertainty, the most probable point (MPP) based reliability analysis has been employed to the crashworthiness optimization (Youn et al. (2004), Rais-Rohani et al. (2010), Sinha (2007)). First Order Reliability Method (FORM) (Hohenbichler and Rackwitz (1982)) is the most commonly used MPP based method in practical applications, which fits a tangential hyperplane to the limit state hypersurface at MPP. In RBO, the probabilistic constraint can be assessed to check whether or not the relibility index has been achieved (greater than the target value), which is called reliability index approach (RIA) (Sinha (2007)). The RIA is often found to be associated with high computational cost and/or with lack of robustness (sometimes simply fails to converge). That is, when the system is very safe, the reliability index approaches infinity. To overcome this difficulty, an alternative performance measure approach (PMA) was proposed (Tu et al. (1999); Youn et al. (2004)). In PMA, instead of finding MPP point that satisfies the limit state constraint, the value of limit state is minimized on the points that satisfies the reliability constraint. The advantage of PMA is that it can always find a solution, while its disadvantage is that the value of the reliability is not available.

It should be noted that even with the abovementioned methods for evaluating the probabilistic constraint, the implementation of RBO could be still fairly challenging for large-scale problems (Youn and Choi (2004a), Youn and Choi (2004b)). Moreover, for such application areas as crashworthiness design, lack of sensitivity information leads to considerable difficulties for performing RBO (Youn and Choi (2004a), Youn and Choi (2004b)). To tackle this problem, surrogate modeling has been integrated into the reliability analysis for complex problems (Youn and Choi (2004a), Youn and Choi (2004b), Youn et al. (2004)).

Different from the complexity of reliability analysis, some alternative simplification methods have also been proposed to translate the constraint with uncertainty into a quasi-deterministic constraint so as to balance the computational cost and accuracy. One of these methods, namely worst case analysis (Parkinson et al. (1993)), has been used in crashworthiness design by Zhang et al. (2007a), Zhu et al. (2009) and Baril et al. (2011).

4.2.3 Uncertainty based optimization for crashworthiness

Table 5 in Appendix summarizes the research works on crashworthiness optimization with uncertainties in the literature, from which we can classify them according to the sources of uncertainties as follows:

1) Manufacturing uncertainties. The uncertainties induced by manufacturing processes account for the discrepancy between the nominal design and corresponding real product. They may include parameters such as geometry (thickness, shape), material properties (Young's modulus, Poisson's ratio, density, yield stress, tangent modulus, etc) of crashing structures.
2) Operational uncertainties. Uncertainties present in different operational conditions upon crashing, such as occupant mass, impact speed, impact position, impact angle, and barrier, etc.
3) Modeling uncertainties. These are related to mathematical and numerical modeling techniques for extracting crashing performances. For example, numerical errors in FEA and uncertainties in surrogate modeling (Zhang et al. (2013a), Zhang et al. (2013b), Zhu et al. (2013)) should be considered in crashworthiness design.

Note that although various approaches of uncertainty optimization have been developed and adopted in various crashworthiness designs, the optimum results have seldom been validated experimentally in a statistical fashion. The validation of uncertainty optimization in crashworthiness should be an important topic in the future, which could lay a solid foundation to the widespread use in industries.

4.3 Topology optimization in crashworthiness

The topology optimization for structural crashworthiness began with Mayer et al's work (Mayer et al. (1996)). They used the homogenization technique and optimality criteria algorithm to distribute elemental material in a progressive fashion. In their work, internal energy was accounted as the objective function subject to a mass constraint. Their method was applied to the design of a three-dimensional automotive rear rail. Pedersen (2003a) proposed a topology optimization method for two-dimensional frame ground structure. The objective aimed to obtain a desired energy absorption history for a crushed structure, where the plastic beam elements could undergo large rotations and translation. Analytical sensitivity was derived to avoid expensive calculation of numerical gradients. However, the contact between elements was ignored because of the number of discontinuities and numerical instabilities associated with the highly nonlinear phenomena. The further work done by the same author can also be found in Pedersen (2003b) and Pedersen (2004).

Soto (2004) presented a heuristic non-gradient methodology to vary the density within the design domain for a prescribed distribution of plastic strains and stresses with a mass constraint. This methodology utilizes a density approach with two base materials (i.e., stiff and extremely soft ones), to represent a foam-like structure. Forsberg and Nilsson (2007) devised another non-gradient technique by using thickness as the design variable with one base material. However, through varying the thickness of each element, this methodology can only handle plate or shell structures. Huang et al. (2007) used the bi-directional evolutionary optimization (BESO) technique to design energy absorbing structures, where a discrete sensitivity of element was derived to address two principal design criteria, i.e. absorbed energy per unit volume and absorbed energy ratio.

Ortmann and Schumacher (2013) proposed a graph and heuristic based topology optimization technique for the design of profile cross-sections of crashing structures. They divided the optimization problem into two different loops. In the outer loop, the topology and shape of the structure are optimized based on expert knowledge; while in the inner loop the size and shape optimization takes place. This method is only capable of addressing the topological design of cross-sections (i.e., it can only solve the problems with a 2D design space though the structure investigated can be 3D).

Based on the hybrid cellular automaton (HCA) method (Tovar et al. (2007)), Patel et al. (2009) more recently proposed a heuristic (non-gradient) approach to addressing continuum-based topology optimization for structural crashworthiness. Similarly to a fully-stressed design, all elements in the structure were expected to contribute to the energy absorption through plastic deformation; and thus the optimum was achieved to obtain a uniform internal energy density in the whole structure. Based on their work, commercial software LS-TaSC was developed (Roux (2011)), which allows generating optimal crashworthy configurations. However, this method still needs to overcome the following limitations (Witowski et al. (2012)): (1) While the uniform IED likely helps produce a better topology with more even distribution of energy absorption in the material, it may unnecessarily ensure an overall maximum of energy absorption. (2) The inclusion of constraint (e.g. the maximum displacement) is realized indirectly by using the mass constraint. (3) Since HCA accumulates material distribution in the areas with high stresses and strains, the risk of rupture in these areas should be taken into account. (4) Although this method is in heuristic nature without sensitivity information, it takes a large number of iterations prior to convergence. To address the third problem, Guo et al. (2011) proposed a strain-based, dynamic multi-domain topology optimization technique for crashworthy structures, in which the optimization was reformulated for dynamically dividing two different subdomains in terms of the plastic strain limit. During the optimization, the material in low-strain subdomain was distributed by driving the IED of each material element to the prescribed target. The material in high-strain subdomain was distributed to reduce the effective plastic strain to the limiting value so as to ensure the integrity of the structure. Bandi et al. (2013) presented a new method in the HCA framework to optimize the crashworthy structures with controlled energy absorption. Again, the design domain was divided into two subdomains for different requirements. That is to say that the flexible subdomain close to the incident end was devised to provide cushioning effect (lower peak force), while the stiff subdomain close to the support (distal) end was to maintain the integrity of the entire structure.

Topological optimization is perhaps one of the most difficult problems being addressed in crashworthiness design to date. That is because of considerable complexity of obtaining topological sensitivity or optimality criteria for effectively addressing the crash dynamic process involving material and geometric nonlinearities, contact, strain rate etc. For this reason, alternative methods have been adopted to simplify such dynamic nonlinear problem through equivalent static and/or linear counterpart. Christensen et al. (2012) used the inertia relief method as a practical tool for crashworthiness topology optimization of a body-in-white. Chuang and Yang (2012) pointed out that the inertia relief method fails to fully support crashworthiness topology optimization and special attention is required for the definition of the loads in applications. Despite its limitations as mentioned in Section 3.2, the ESL method has also been employed for simplifying crashworthiness topology optimization problems recently (Kaushik and Ramani (2014)).

5 Optimization of crashworthy structures

5.1 Configurations for energy-absorbing structures

To achieve better crashworthiness performance, various novel configurations of structures have been proposed and further optimized as an energy absorber during crashes, as summarized in Table 6 of Appendix. The following sub-sections will briefly outline these categories of studies.

5.1.1 Thin-walled tubes

Thin-walled tubes have been exhaustively investigated in crashworthiness design by using analytical, numerical and experimental methods. Alexander (1960) was amongst one of the pioneers who derived a closed-form formula for calculating average crushing force. Wierzbicki and Abramowicz (1983), Abramowicz and Jones (1984) and Abramowicz and Jones (1986) also carried out experimental and theoretical studies on the axial crushing of tubes subjected to static and dynamic loads in the early stage. More recently, thin-walled tubes with various geometric sections have been studied for crashworthiness, such as circular, square/polygonal, conical/ tapered and hat etc, to seek for optimal designs (please refer to Table 6).

5.1.2 Multi-cell tubes

In general, the number of angular elements (corners) in a tubal cross-section largely determines the energy absorption and crashing behaviors (Wierzbicki and Abramowicz (1983) and Abramowicz and Wierzbicki (1989)). It is therefore expected to design thin-walled tubes with multiple cells and internal webs for achieving better energy-absorbing characteristics. Crashworthiness optimization has been also introduced to the design of various multi-cell tubes (Table 6). For example, Hou et al. (2008b) adopted surrogate modeling to optimize the single, double, triple and quadruple cell sectional columns, aiming to maximize the *SEA* and minimize the peak force $F_{\max}$. Zhang et al. (2008) found that for bitubal columns with internal ribs, an appropriate combination of the side length of the inner profile, inner and outer walls and strong ribs are preferred for best energy absorption. Liu et al. (2014) pointed out that the multi-cell section with double vertical internal stiffeners can absorb more energy and they further optimized this novel structure for the application to automotive front rails.

5.1.3 Foam-filled structures

Substantial research efforts on foam-filled structures have been devoted through various experimental (e.g., Seitzberger et al. (1997), Gupta and Velmurugan (1999), Santosa et al. (2000)), analytical (e.g., Gupta and Velmurugan (1999)) and numerical methods (e.g., Seitzberger et al. (1997), Santosa et al. (2000)). These studies demonstrated that foam-filled structures can undergo large deformation at nearly constant load. The presence of the foam-filler materials in thin-walled structures helps improve crashing stability and collapse modes, thereby enhancing the overall crashworthiness (Borvik et al. (2003), Seitzberger et al. (2000), Santosa et al. (2000)). However, the crashworthiness performance is highly dependent on the foam density and geometrical configurations (Seitzberger et al. (1997), Reyes et al. (2004)). To address this issue, optimization techniques were used to select best possible combination of tube geometry and foam density in both simple tubes (e.g., Yang and Qi (2013), Hou et al. (2009), Bi et al. (2010)) and complex structures (e.g., Kim (2001), Hanssen et al. (2006) and Villa et al. (2011).

5.1.4 Tailor-welded blank (TWB) and tailor-rolled blank (TRB) structures

To maximize the functionality of material in crashworthiness and energy absorption, substantial efforts have been devoted to the applications of proper tailor-welded blanks (TWB) structures (Ahmetoglu et al. (1995), Abdullah et al. (2001), Kinsey et al. (2000)). The TWB technology consists of laser-welded sheet metals with different thicknesses and different materials for a single workpiece. Crashworthiness optimization of TWB structures often aims to seek the best partition of different materials and thicknesses of each blank for both lightweighting and crash behaviors. For example, Pan et al. (2010) optimized a TWB based B-pillar structure to minimize the weight subject to the crashworthiness constraints of vehicular roof crush and side impact. Xu et al. (2013) demonstrated that the multi-component TWB structure can be optimized to further enhance the crashworthiness and reduce the weight.

Different from TWB, the TRB technology varies the blank thickness by a rolling process, which leads to a continuous thickness variation in the sheet. Chuang et al. (2008) demonstrated the feasibility of design optimization for TRB technology to achieve a better functional performance and reduce the mass of a vehicle structure.

5.1.5 Composite structures

One option to achieve lightweight design is to replace heavy metallic materials with light composites. Although most composite materials display little plastic characteristics, properly designed composite materials could absorb more energy per unit mass than the conventional metals (Ramakrishna (1997)). Lanzi et al. (2004b) optimized the shape of a composite cylindrical energy absorber and found that the moderate eccentricity and conicity led the structures to have higher energy absorption efficiency and less mass. Zarei et al. (2008) found that the optimized composite crash box could absorb around 17 % more energy with 26 % lower weight than the optimized aluminum counterpart. Belingardi et al. (2013) optimized the cross-sectional shape, wall thickness and transverse curvature of the E-Glass pultruded bumper and they achieved comparable energy absorption with steel and E-Glass fabric bumpers but better progressive failure mode with reduced peak load. In the work by Paz et al. (2014), the optimal GFRP honeycomb-filled tube improved the specific energy absorption by 40 % with a similar peak load or a lowered the peak load (by 37 %) with similar mass and energy absorption capacity. Duan et al. (2014) studied the crashworthiness optimization of a tapered sinusoidal specimen made of fiber reinforced polymers materials. Their optimal results showed that a smaller ratio of the thickness to the radius of the specimen was often beneficial to the enhancement of specific energy absorption and reduction of the peak force.

The energy absorption performance of composite structures can be tailored by controlling various material structures and parameters, such as fiber type, matrix configuration, fiber architecture, specimen geometry, process condition and fiber volume fraction (Jacob et al. (2002)). However, the existing optimization literature considered only geometrical parameters as design variables and it remains unknown whether or not crashworthiness optimization can be applied to the problems involving other material and process parameters; and from this perspective, composite structures still have considerable room to be further optimized for better crashworthiness.

5.1.6 Functionally graded structures

To tailor the crashworthiness performance, functionally-graded materials and structures are drawing increasing attention more recently. For example, Sun et al. (2010a) investigated the crashing characteristics of functionally graded foam (FGF)-filled columns, in which the foam has a gradient density along the axial direction, and they sought the best possible exponential gradient via a multiobjective optimization. Fang et al. (2014b) compared the bending behavior of different FGF-filled structures and found the FGF structures is able to generate more competent designs than the uniform counterpart.

In addition to FGF, the concept of functionally graded thickness (FGT) structures has also been introduced to crashworthiness optimization. Sun et al. (2014b) proposed a novel tube with a longitudinal variation of thickness and identified the best possible thickness gradient for achieving the highest *SEA* and lowest F_{max} using multiobjective optimization. Note that during crashing process severe deformation with combined bending and membrane deformation often takes place near the corners of tubes (Kim (2002)). Zhang et al. (2014a) extended the concept of transverse FGT to the square tube by placing more material to the corners and discovered 30–35 % increase in energy absorption efficiency. To take the advantages of FGT and multi-cell structures, Fang et al. (2015b) proposed the transverse FGT multi-cell tubes; and they found that the graded structures can generate more competent Pareto solutions in terms of F_{max} and *SEA* (Fig. 7a). Under almost the same value of F_{max}, the crashing force of the FGT tube maintains at a higher level overall than that of the uniform tube seen in Fig. 7b, where the shaded area between these two curves is the additional energy absorbed by the FGT structure. More recently, there has been also research work to investigate the optimization for double gradient structures (i.e., an FGF-filled tube with FGT) (Fang et al. (2016)). The study of these functionally graded structures should be extended to engineering applications.

5.2 Industrial applications

Over the past two decades, structural optimization has been widely used for crashworthiness design of a full-scale vehicle in the automotive industry. For example, Liao et al. (2008b) investigated a multiobjective optimization problem of frontal crash safety of a full-scale vehicle, where both full frontal and 40 % offset frontal crashes were considered based on polynomial surrogate modeling method with stepwise regression. A set of Pareto solutions was generated using NSGA-II, which provided the decision maker with insightful information. Acar and Solanki (2009b) performed a system reliability based optimization of an automotive structure for crashworthiness and analyzed the reliability allocation in different failure modes. They evaluated the effect of various uncertainty reduction measures and plotted the tradeoff of uncertainty reduction measures, system reliability, and structural weight. Liang

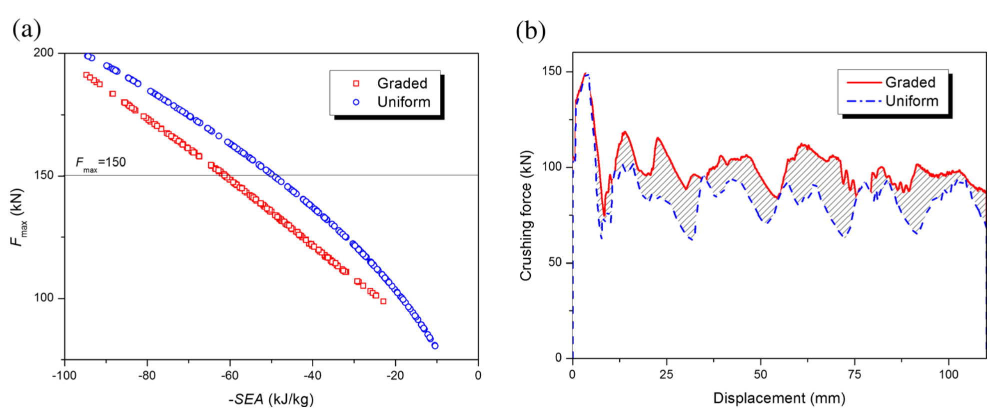


Fig. 7 Optimization results of FGT and uniform multi-cell tubes (Fang et al. (2015b)): a) Pareto front; b) crushing force

and Le (2009) investigated an SSM based crashworthiness optimization for bus structures to maintain survival space and reduced occupant injury when a rollover occurs. The results showed the side wall deformations were reduced by 49.2 and 39.4 % for upper and lower frames respectively, while the bus weight was only increased by 1.6 %. Goel et al. (2010) proposed an efficient resource allocation of NSGA-II with 1,000 simulations and a real-life vehicle model comprised of 58,000 elements was used for design testing. Use of a moderate population size would provide a reasonable trade-off among convergence to Pareto front, diversity of non-dominated solutions and computational cost in a parallel framework. The results also showed that for the problems with a small feasible region, the number of feasible solutions can be significantly increased in the first few generations involving about 200 simulations. The optimal design could lead to more than 50 % reduction in the peak acceleration and almost 6 % increase in the time to reach zero velocity while remaining the mass and maximum intrusive displacement unchanged. Kiani et al. (2013) investigated an RBF-based optimization of automotive structures considering multiple crashes and vibration scenarios. While the intrusion, acceleration, internal energy in full frontal, offset frontal and side impacts and three frequencies in vibration characteristics satisfied the constraints, their study achieved a weight saving of 3.6 % by optimizing 20 components of the vehicle.

Real-world loading conditions of crashworthiness optimization mainly include the front crash, side crash, and rollover in the literature (as summarized in Table 7). In front impact, deformable yet stiff front structures with crumple zones are required to absorb the kinetic energy so as to reduce the crash energy transmitting to occupants. Besides, intrusion into the occupant compartment should also be prevented, especially in the case of offset crashes. As a result, crash pulse (*CP*) (e.g., Craig et al. (2005), Goel et al. (2010), Liao et al. (2008a)) or its variants peak acceleration $a_{\max}$ (e.g., Hamza and Saitou (2012), Parrish et al. (2012), Zhu et al. (2012), Gu et al. (2013)) and peak force $F_{\max}$ (e.g., Zhu et al. (2009), Wang et al. (2011b)) and intrusion *Intr* (e.g., Redhe and Nilsson (2004), Yang et al. (2005), Craig et al. (2005)) are often used as design criteria in optimization for a front impact. In the side impact, side structures are expected to minimize intrusion and to prevent doors from opening. Due to little space for deformation, *Intr* is the emphasis in structural optimization under side impact (e.g., Bojanowski and Kulak (2011), Wang et al. (2011a), Aspenberg et al. (2012), Xu et al. (2013)). To ensure the rollover safety, a large resistance force and/or a small intrusion in the roofing structure, are commonly adopted to be the design criteria (e.g., Liang and Le (2009), Pan and Zhu (2011a), Bojanowski and Kulak (2011), Su et al. (2011)).

Because of the complexity of crashworthiness in the context of full-scale vehicles, the computational cost of FEA in optimization iterations can be rather high. Thus, surrogate models have been commonly recognized as an effective alternative in industry applications. However, the "*curse of dimensionality*" problem arises when the number of design variables increases. In other words, the computational cost for obtaining a large number of training points can make the surrogate modeling less attractive and even infeasible (Koch et al. (1999)). There seems to be two kinds of approaches to address this issue in literature. First, direct coupling population-based optimization algorithms with FEA could carry out the optimization more efficiently when the number of design variables becomes large (Duddeck (2008), Xu et al. (2014), Xu et al. (2015)). It was also recommended to use hybrid approaches to combine both surrogate modeling and direct coupling methods (Redhe et al. (2004)), in which the optimal result from the direct coupling method was used as a starting point for subsequent further surrogate-based optimization. Second, variable screening techniques could be used to reduce the dimensionality of the problem before performing a surrogate model based optimization (Simpson et al. (2004)). In this regard, Craig et al. (2005) identified some important variables through an analysis of variance (ANOVA) based on linear surrogate models. Liang and Le (2009) investigated the capability of energy absorption of the components to reduce the dimension of the optimization problem. Su et al. (2011) simply considered the components with high strain energy during the bus rollover as the influential structures. Hou et al. (2012) proposed a method of unreplicated saturated factorial design to screen out the less important variables for a vehicle crash. In industrial applications, the difficulty with variable screening may be due to the presence of multiple responses (Simpson et al. (2001)). In other words, complex industry problems should inevitably consider multiple performance responses and each response may unfortunately require different important design variables, which results in the breakdown of the variable screening process.

Automotive safety components/subsystems have been separated from the vehicle for crashworthiness optimization in literature. Longitudinal rails, bumpers, side pillar structures are some typical examples for optimizing the crashworthiness of components as listed in Table 8.

Real-world road systems were considered in crashworthiness optimization for vehicular structures. The guardrail is a common traffic facility on roads, and its design affects the functionality of absorbing the kinetic energy and redirecting the errant vehicle (Hou et al. (2014b), Kurtaran et al. (2002), Yi et al. (2012)).

As discussed above, there are three different levels of systems in crashworthiness optimization of automotive structures: component level, vehicle level, and vehicle-road facility level. A futuristic problem arising is how to integrate the designs of different system levels to ultimately improve the occupant safety.

As a broad topic, crashworthiness optimization also applies to other industries in real-life scenarios. In the aerospace engineering, crashworthiness optimization has been applied to subfloor structures of helicopters by several researchers (Bisagni et al. (2002), Hajela and Lee (1997), Lanzi et al. (2004a), Astori and Impari (2013)). In the maritime industry, crashworthiness optimization can enhance the energy-absorbing capacity of the fender structure (Jiang and Gu (2010)).

6 Conclusions

With the growing socioeconomic concerns on transport safety and fuel consumption, significance of designing crashworthy structures has been more and more recognized. Computational optimization provides a powerful tool to achieve best possible crashworthiness with lightest structures. This article provides a state-of-the-art review of a range of key issues from structural configurations, crashworthiness criteria, modeling strategies, to multiobjective, uncertainties, and industrial applications. From this comprehensive review, we can draw the following conclusions and recommendations for futuristic crashworthiness optimization.

- Due to the complexity of crashworthiness problem, surrogate modeling has been the most popular and feasible approach to formulating the design criteria for optimization, especially in small scale problems (e.g. < 20 design variables). Use of multiple surrogate models is recommended for addressing modeling accuracy and optimization effectiveness. To make full use of high-performance computing (HPC), efficient global optimization (EGO) and successive surrogate modeling (SSM) methods can be conducted with parallel computing to save the wall-clock time.
- For real-word problems with a large number of design variables, direct coupling population-based optimization algorithms with finite element analysis (FEA) may be more suitable; and data mining techniques are drawing increasingly attention for enhancing the optimization efficiency of population-based optimization algorithms.
- Population-based optimization algorithms are recommended for dealing with multiobjective problems for their capability of producing well-distributed Pareto solutions for decision making.
- Due to their limitations, equivalent methods (i.e., inertia relief and equivalent static loads) might not be a general strategy for crashworthiness topology optimization. Rigorous formulation still needs further studies for addressing nonlinearity and stability issues in topological sensitivity analysis.
- When uncertainties are presented, crashworthiness optimization could become challenging. Surrogate-based Monte Carlo simulation (MCS) is recommended to quantify the data of uncertainties during optimization iterations. Besides, the experimental validation of robust and/or reliable optimization results is recommended, whenever possible, as a focus of future studies prior to applying such uncertainty optimizations to real world more extensively.
- Various novel structures and materials have been proposed and optimized to enhance the crashworthiness. While rather promising, composite structures may still need to be further studied for tailoring better crashing performance. The concept of functionally graded structures has proven considerable effectiveness in the crashworthiness and their further applications in the industry could be a promising topic of study in the future.
- For designs of each component, their individual roles and design optimization should be addressed in an integrated form subject to proper design criteria so that they can perform the best as a whole.
- Some natural hierarchical materials have demonstrated remarkable energy absorption and impact resistance (McKittrick et al. (2010)). Bio-inspired design and microstructural optimization of composites could be a promising area of study, which could open up a new avenue pushing the lightweighting for crashworthiness to a new level.

Acknowledgments This work was supported by Australian Research Council (ARC) DECRA and future fellowship scheme.

Appendix

Table 1 Injury-based metrics

Metrics	Publications
HIC	Hong and Park (2003), Youn et al. (2004), Zhu et al. (2008), Oman and Nilsson (2010), Yi et al. (2012)
CP	Wu et al. (2002), Craig et al. (2005), Goel et al. (2010), Liao et al. (2008a), Liao et al. (2008b), Stander (2012), Stander (2013)
Intr	Rakheja et al. (1999), Kurtaran et al. (2002), Redhe et al. (2004), Redhe and Nilsson (2004), Craig et al. (2005), Forsberg and Nilsson (2006), Redhe and Nilsson (2006), Zhang et al. (2007a), Duddeck (2008), Liao et al. (2008a), Liao et al. (2008b), Shin et al. (2008), Zhu et al. (2008), Acar and Solanki (2009b), Acar and Solanki (2009a), Horstemeyer et al. (2009), Liang and Le (2009), Acar (2010b), Acar (2010a), Goel et al. (2010), Bojanowski and Kulak (2011), Pan and Zhu (2011b), Su et al. (2011), Wang et al. (2011a), Zhu et al. (2011), Hamza and Saitou (2012), Hou et al. (2012), Parrish et al. (2012), Yi et al. (2012), Yildiz and Solanki (2012), Zhu et al. (2012), Gu et al. (2013), Kiani et al. (2013), Shi et al. (2013a), Shi et al. (2013b), Xu et al. (2013), Zhang et al. (2013b), Hamza and Shalaby (2014), Hou et al. (2014a), Kiani et al. (2014), Wang and Shi (2014), Lönn et al. (2009), Aspenberg et al. (2012), Stander (2012), Stander (2013)
Intrusion Velocity (*IntrV*)	Yang et al. (2000), Blumhardt (2001), Gu et al. (2001), Fu and Sahin (2004), Koch et al. (2004), Youn et al. (2004), Sinha (2007), Sinha et al. (2007), Zhu et al. (2008), Baril et al. (2011), Wang et al. (2011a), Rangavajhala and Mahadevan (2013), Xu et al. (2013), Zhang et al. (2013a), Zhu et al. (2013), Hou et al. (2014a), Marklund and Nilsson (2001)
$a_{\max}$	Rakheja et al. (1999), Blumhardt (2001), Kurtaran et al. (2002), Pedersen (2003b), Redhe et al. (2004), Fang et al. (2005), Forsberg and Nilsson (2006), Redhe and Nilsson (2006), Cristello and Kim (2007), Shin et al. (2008), Horstemeyer et al. (2009), Goel et al. (2010), Jeong et al. (2010), Rais-Rohani et al. (2010), Sun et al. (2010b), Wang et al. (2010), Pan and Zhu (2011b), Sun et al. (2011), Zhu et al. (2011), Gu et al.), Hamza and Saitou (2012), Parrish et al. (2012), Zhu et al. (2012), Gu et al. (2013), Ingrassia et al. (2013), Kiani et al. (2013), Zhang et al. (2013b), Zhu et al. (2013), Abbasi et al. (2014), Hamza and Shalaby (2014), Hou et al. (2014a), Hou et al. (2014b), Kiani et al. (2014), Mohammadiha and Beheshti (2014), Aspenberg et al. (2012)
$F_{\max}$	Hanssen et al. (2001), Kurtaran et al. (2002), Lee et al. (2002), Pedersen (2003b), Redhe et al. (2004), Forsberg and Nilsson (2006), Hou et al. (2007), Huang et al. (2007), Zhang et al. (2007a), Hou et al. (2008a), Liu (2008a), Liu (2008c), Liu (2008b), Hou et al. (2009), Zhang et al. (2009), Zhu et al. (2009), Jiang and Gu (2010), Kaya and Oeztuerk (2010), Khakhali et al. (2010), Liu (2010b), Liu (2010a), Shariati et al. (2010), Sun et al. (2010a), Wang et al. (2010), Allahbakhsh et al. (2011), Hou et al. (2011), Pan and Zhu (2011b), Wang et al. (2011b), Wang et al. (2011a), Yin et al. (2011b), Zhu et al. (2011), Najafi and Rais-Rohani (2012), Qi et al. (2012), Zhang et al. (2012), Zhu et al. (2012), Belingardi et al. (2013), Esfahlani et al. (2013), Gedikli (2013), Song et al. (2013), Tang et al. (2013), Yang and Qi (2013), Yin et al. (2013), Duan et al. (2014), Fang et al. (2014b), Fang et al. (2014a), Hou et al. (2014c), Mohammadiha and Beheshti (2014), Najafi et al. (2014), Nguyen et al. (2014), Paz et al. (2014), Qi and Yang (2014), Sun et al. (2014a), Sun et al. (2014b), Tran et al. (2014), Yin et al. (2014a), Yin et al. (2014b), Zheng et al. (2014), Redhe et al. (2002), Jansson et al. (2003), Hunkeler et al. (2013), Bisagni et al. (2002), Lanzi et al. (2004a)

Table 2 Energy-based metrics

Metrics	Publications
EA	Mayer et al. (1996), Yamazaki and Han (1998), Rakheja et al. (1999), Shi and Hagiwara (2000), Hanssen et al. (2001), Kim (2001), Kim et al. (2002), Kurtaran et al. (2002), Lee et al. (2002), Lanzi et al. (2004b), Fang et al. (2005), Hanssen et al. (2006), Zarei and Kroger (2006), Yang and Qi (2013), Zhang et al. (2007a), Zhang et al. (2007c), Liu (2008a), Liu (2008c), Liu (2008b), Zarei and Kroger (2008a), Zarei and Kroger (2008b), Zarei and Kroger (2008c), Acar and Solanki (2009b), Acar and Solanki (2009a), Horstemeyer et al. (2009), Hou et al. (2009), Zhang et al. (2009), Zhu et al. (2009), Kaya and Oeztuerk (2010), Lönn et al. (2010), Wang et al. (2010), Marzbanrad and Ebrahimi (2011), Pan and Zhu (2011b), Sun et al. (2011), Zhu et al. (2011), Ghamarian and Zarei (2012), Gu et al.), Parrish et al. (2012), Zhu et al. (2012), Esfahlani et al. (2013), Gu et al. (2013), Tang et al. (2013), Zhang et al. (2013b), Paz et al. (2014), Redhe et al. (2002), Milho et al. (2004)
SEA	Zarei and Kroger (2006), Hou et al. (2007), Zarei and Kroger (2007), Hou et al. (2008a), Zarei and Kroger (2008a), Zarei and Kroger (2008c), Zhang et al. (2009), Bi et al. (2010), Jiang and Gu (2010), Khakhali et al. (2010), Liu (2010b), Liu (2010a), Shariati et al. (2010), Sun et al. (2010a), Sun et al. (2010b), Acar et al. (2011), Allahbakhsh et al. (2011), Hou et al. (2011), Guo et al. (2011), Marzbanrad and Ebrahimi (2011), Toksoy and Güden (2011), Yin et al. (2011a), Yin et al. (2011b), Ghamarian and Zarei (2012), Qi et al. (2012), Zhang et al. (2012), Belingardi et al. (2013), Gedikli (2013), Song et al. (2013), Yang and Qi (2013), Yin et al. (2013), Chen et al. (2014), Duan et al. (2014), Fang et al. (2014b), Fang et al. (2014a), Hou et al. (2014c), Nguyen et al. (2014), Qi and Yang (2014), Sun et al. (2014a), Sun et al. (2014b), Tran et al. (2014), Yin et al. (2014a), Yin et al. (2014b), Zhang et al. (2014a), Zhang et al. (2014b), Zheng et al. (2014), Hunkeler et al. (2013), Costas et al. (2014), Lanzi et al. (2004a), Zhu et al. (2016)
CFE/ LU	Avalle et al. (2002), Chiandussi and Avalle (2002), Avalle and Chiandussi (2007), Shakeri et al. (2007), Acar et al. (2011), Marzbanrad and Ebrahimi (2011), Gedikli (2013), Zhang et al. (2014b), Zhou et al. (2014), Costas et al. (2014)
F_{avg}	Xiang et al. (2006), Zarei and Kroger (2006), Yang and Qi (2013), Zarei and Kroger (2008a), Zarei and Kroger (2008c), Zhang et al. (2008), Bi et al. (2010), Toksoy and Güden (2011), Ghamarian and Zarei (2012), Najafi and Rais-Rohani (2012), Song et al. (2013), Fang et al. (2014a), Najafi et al. (2014), Sun et al. (2014a), Bisagni et al. (2002), Lanzi et al. (2004a)
d_{eff} / *UR*	Shakeri et al. (2007), Marzbanrad and Ebrahimi (2011), Zhang et al. (2012), Shi et al. (2013a), Shi et al. (2013b), Wang and Shi (2014)
IED	Forsberg and Nilsson (2007), Patel et al. (2009)

Table 3 Surrogate models in crashworthiness optimization

Surrogate models		Publications
Individual surrogate models	PRS	Blumhardt (2001), Kim (2001), Han and Yamazaki (2003), Hong and Park (2003), Xiang et al. (2006), Zarei and Kroger (2006), Zhang et al. (2007a), Zhang et al. (2007c), Liu (2008a), Liu (2008c), Liu (2008b), Shin et al. (2008), Zarei and Kroger (2008a), Zarei and Kroger (2008b), Zarei and Kroger (2008c), Hou et al. (2009), Bi et al. (2010), Kaya and Oeztuerk (2010), Liu (2010b), Liu (2010a),Shariati et al. (2010), Sun et al. (2010a), Zhang et al. (2010), Allahbakhsh and Saetni (2011), Baril et al. (2011), Hou et al. (2011), Guo et al. (2011), Sun et al. (2011),Toksoy and Güden (2011), Yin et al. (2011b), Bae and Huh (2012), Ghamarian and Zarei (2012), Hou et al. (2012), Qi et al. (2012), Yin et al. (2013), Zhu et al. (2013), Duan et al. (2014), Hou et al. (2014a), Hou et al. (2014c), Kiani et al. (2014), Mohammadiha and Beheshti (2014), Nguyen et al. (2014), Sun et al. (2014b), Tran et al. (2014), Zhang et al. (2014a), Lönn et al. (2009), Redhe et al. (2002), Avalle et al. (2002), Chiandussi and Avalle (2002), Avalle and Chiandussi (2007), Marklund and Nilsson (2001), Jansson et al. (2003), Esfahlani et al. (2013), Yang et al. (2000), Gu et al. (2001), Fu and Sahin (2004), Koch et al. (2004), Youn et al. (2004), Sinha (2007),Sinha et al. (2007), Liao et al. (2008b), Liao et al. (2008a), Aspenberg et al. (2012), Lönn et al. (2011), Sun et al. (2010b), Redhe and Nilsson (2004)
	KRG	Redhe and Nilsson (2006), Zhang et al. (2012),Yang and Qi (2013), Zhang et al. (2013a), Zhang et al. (2013b), Fang et al. (2014b), Qi and Yang (2014), Zhang et al. (2014b), Fang et al. (2014a)
	RBF	Hamza and Shalaby (2014), Lanzi et al. (2004b), Horstemeyer et al. (2009), Rais-Rohani et al. (2010), Bojanowski and Kulak (2011), Su et al. (2011), Farkas et al. (2012), Najafi and Rais-Rohani (2012), Yildiz and Solanki (2012), Gu et al. (2013), Kiani et al. (2013), Hou et al. (2014b), Acar (2010a), Aspenberg et al. (2012)
	ANN	Sun et al. (2010b), Hajela and Lee (1997),Shi and Hagiwara (2000), Khakhali et al. (2010), Jiang and Gu (2010), Zhu et al. (2008), Marzbanrad and Ebrahimi (2011), Bisagni et al. (2002), Lanzi et al. (2004a)
	SVR	Aspenberg et al. (2012), Pan et al. (2010), Wang et al. (2010), Paz et al. (2014)
Comparison of surrogate models		Wang et al. (2011a), Fang et al. (2005), Forsberg and Nilsson (2006), Costas et al. (2014), Acar and Solanki (2009b), Gedikli (2013), Xu et al. (2013),Yin et al. (2014b), Shi et al. (2013a), Shi et al. (2013b) , Yin et al. (2011a), Zhu et al. (2012)
Multiple surrogate models (including ensemble of surrogates)		Zhu et al. (2009), Pan and Zhu (2011b), Song et al. (2013), Zheng et al. (2014), Yin et al. (2014a), Pan and Zhu (2011a), Zhu et al. (2011), Hamza and Saitou (2012), Parrish et al. (2012)
Sequential sampling		Najafi et al. (2014), Chen et al. (2014), Lee et al. (2002), Kurtaran et al. (2002), Craig et al. (2005), Hou et al. (2007), Liang and Le (2009), Sun et al. (2014a), Fang et al. (2014a), Sheldon et al. (2011), Stander (2012), Stander (2013)

Table 4 Algorithms used in crashworthiness Optimization

Methods	Publications
Combined cost function	Hamza and Shalaby (2014), Cristello and Kim (2007), Zhang et al. (2009), Parrish et al. (2012), Rais-Rohani et al. (2010), Mohammadiha and Beheshti (2014), Wang et al. (2010), Ghamarian and Zarei (2012), Kaya and Oeztuerk (2010), Zarei and Kroger (2006), Acar et al. (2011), Yildiz and Solanki (2012), Hou et al. (2012), Farkas et al. (2012), Hou et al. (2014b), Tran et al. (2014), Fang et al. (2005), Ingrassia et al. (2013), Marzbanrad and Ebrahimi (2011), Shakeri et al. (2007), Zhu et al. (2008), Hou et al. (2008a), Costas et al. (2014)
MOGA	Hou et al. (2011), Paz et al. (2014), Kiani et al. (2014), Lanzi et al. (2004b), Najafi et al. (2014), Xu et al. (2013), Hamza and Saitou (2012), Gu et al. (2013), Liao et al. (2008a), Sun et al. (2014b), Guo et al. (2011), Zheng et al. (2014), Hou et al. (2014a), Bojanowski and Kulak (2011), Liao et al. (2008b), Sinha et al. (2007), Zhang et al. (2012), Sinha (2007), Goel et al. (2010), Jiang and Gu (2010), Xu et al. (2015), Stander (2012), Stander (2013)
MOPSO	Hou et al. (2009), Qi and Yang (2014), Sun et al. (2010a), Yin et al. (2011a), Fang et al. (2014a), Sun et al. (2011), Yin et al. (2014b), Yin et al. (2011b), Qi et al. (2012), Duan et al. (2014), Yin et al. (2014a), Zhang et al. (2014b), Nguyen et al. (2014), Fang et al. (2014b), Yin et al. (2013), Yang and Qi (2013)

Table 5 Research works on crashworthiness optimization with uncertainties

Publications	Sources of uncertainties	Uncertainty						
		Optimization classification		Uncertainty analysis				
		Robust	Reliability	MCS	AMA	DSM	MPP	Others
Yang et al. (2000)	Barrier height, barrier hitting position		√	√				
Gu et al. (2001)	Barrier height, barrier hitting position		√	√				
Youn et al. (2004)	Sheet thickness, material property, barrier height, barrier hitting position		√				√	
Fu and Sahin (2004)	Sheet thickness, material property, barrier height, barrier hitting position	√	√					√
Koch et al. (2004)	Sheet thickness, material property, barrier height, barrier hitting position	√	√		√			
Sinha (2007)	Sheet thickness, material yield stress		√		√		√	
Sinha et al. (2007)	Sheet thickness, material yield stress	√	√					√
Zhang et al. (2007a)	Sheet thickness, material yield stress	√	√					√
Acar and Solanki (2009b)	Material parameter, occupant mass, impact speed, error in FEA, error in our ignorance		√	√				
Zhu et al. (2009)	Sheet thickness, yield limit	√	√	√				√
Rais-Rohani et al. (2010)	Stress-strain parameter, impact speed, offset distance, occupant mass		√				√	
Lönn et al. (2010)	Geometrical parameters	√				√		
Khakhali et al. (2010)	Material density, material yield stress, plastic modulus, sheet thickness	√	√	√				
Zhu et al. (2011)	Sheet thickness, material yield stress		√	√				
Baril et al. (2011)	Sheet thickness, material yield stress	√						√
Sun et al. (2011)	Material density, material yield stress, Young's modulus	√	√			√		
Farkas et al. (2012)	Geometrical parameters, sheet thickness		√					√
Aspenberg et al. (2012)	Sheet thickness, material scaling	√				√		
Shi et al. (2013b)	Sheet thickness		√	√				
Zhu et al. (2013)	Surrogate modeling uncertainty		√					√
Gu et al. (2013)	Sheet thickness	√	√	√				
Rangavajhala and Mahadevan (2013)	Sheet thickness, yield stress	√	√					
Zhang et al. (2013a)	Sheet thickness, surrogate modeling uncertainty	√	√					√
Zhang et al. (2013b)	Sheet thickness, surrogate modeling uncertainty	√	√					√
Chen et al. (2014)	Sheet thickness, yield stress		√	√				
Najafi et al. (2014)	Young's modulus, yield stress, tangent modulus	√		√		√		
Fang et al. (2014a)	Sheet thickness, foam density	√	√	√				
Sun et al. (2014a)	Sheet thickness, foam density	√	√		√			

Table 6 Crashworthiness optimization of energy absorbers with different configurations

Structures		Works
Thin-walled tubes	Cylindrical	Sun et al. (2014a), Zarei and Kroger (2006), Kurtaran et al. (2002), Lee et al. (2002), Wang et al. (2010), Marzbanrad and Ebrahimi (2011), Lanzi et al. (2004b), Ghamarian and Zarei (2012)
	Square/ polygonal	Yamazaki and Han (1998), Lönn et al. (2010), Allahbakhsh et al. (2011), Liu (2008c), Redhe et al. (2002), Jansson et al. (2003), Kaya and Oeztuerk (2010), Liu (2008a), Hou et al. (2007), Liu (2010b)
	Conical/ Tapered	Liu (2010a), Chiandussi and Avalle (2002), Avalle and Chiandussi (2007), Hou et al. (2011), Avalle et al. (2002), Liu (2008b), Qi et al. (2012), Acar et al. (2011), Zhang et al. (2014b)
	Hat-sectional	Qi and Yang (2014), Najafi and Rais-Rohani (2012), Najafi et al. (2014), Xiang et al. (2006)
Multi-cell/honeycomb		Tang et al. (2013), Hou et al. (2014c), Zhang et al. (2008), Tran et al. (2014), Hou et al. (2008a), Sun et al. (2010b), Yin et al. (2011a), Esfahlani et al. (2013), Chen et al. (2014),Zarei and Kroger (2008a), Yin et al. (2011b)
Foam-filled		Yang and Qi (2013), Zarei and Kroger (2008b), Zarei and Kroger (2008c), Toksoy and Güden (2011), Hou et al. (2009), Zhang et al. (2012), Yin et al. (2014b), Bi et al. (2010), Hanssen et al. (2006), Kim (2001), Kim et al. (2002), Fang et al. (2014a), Zarei and Kroger (2008a), Sun et al. (2014a), Shariati et al. (2010), Hanssen et al. (2001), Yang and Qi (2013), Zarei and Kroger (2007), Zarei and Kroger (2008c), Zheng et al. (2014), Zarei and Kroger (2008b), Song et al. (2013), Zhang et al. (2014b)
TWB/ TRB		Qi and Yang (2014), Tang et al. (2013), Gedikli (2013), Zhu et al. (2008), Xu et al. (2013), Pan et al. (2010), Shi et al. (2007)
Composite		Chuang et al. (2008), Zarei et al. (2008), Belingardi et al. (2013), Duan et al. (2014), Paz et al. (2014),
FGT/FGF structure		Lanzi et al. (2004b), Fang et al. (2014b), Fang et al. (2016), Mohammadiha and Beheshti (2014), Yin et al. (2013), Yin et al. (2014a), Sun et al. (2010a), Sun et al. (2014b)

Table 7 Crashworthiness optimization for automotive structures

Loading scenario	Works
frontal impact	Zhang et al. (2014a), Wang et al. (2011b), Wang and Shi (2014), Kiani et al. (2014), Shi et al. (2013a), Shi et al. (2013b), Craig et al. (2005), Fang et al. (2005), Duddeck (2008), Liao et al. (2008a), Liao et al. (2008b), Acar and Solanki (2009b), , Acar and Solanki (2009a), Hou et al. (2012), Zhang et al. (2013b), Zhu et al. (2013), Abbasi et al. (2014), Hamza and Shalaby (2014), Yildiz and Solanki (2012), Kiani et al. (2013), Parrish et al. (2012), Gu et al. (2013), Redhe and Nilsson (2004), Zhu et al. (2012), Sun et al. (2011), Acar (2010a), Goel et al. (2010), Rais-Rohani et al. (2010), Wang et al. (2010), Acar (2010b), Redhe and Nilsson (2006), Pan and Zhu (2011b), Hamza and Saitou (2012), Zhu et al. (2009), Jansson et al. (2003), Zhu et al. (2011), Forsberg and Nilsson (2006), Sheldon et al. (2011), Stander (2012), Zhu et al. (2016)
Side impact	Yang et al. (2005), Bojanowski and Kulak (2011), Yildiz and Solanki (2012), Kiani et al. (2013), Parrish et al. (2012), Zhu et al. (2012), Rangavajhala and Mahadevan (2013), Zhang et al. (2013a), Baril et al. (2011), Yang et al. (2000), Fu and Sahin (2004), Koch et al. (2004), Youn et al. (2004), Sinha (2007), Sinha et al. (2007), Horstemeyer et al. (2009), Hou et al. (2014a), Wang et al. (2011a), Zhang et al. (2010), Xu et al. (2013) Zhu et al. (2008), Sheldon et al. (2011), Xu et al. (2015)
Roof strength and Rollover safety	Gu et al. (2001), Pan and Zhu (2011a), Zhang et al. (2013a), Zhu et al. (2012), Christensen et al. (2012), Christensen et al. (2013), Bojanowski and Kulak (2011), Su et al. (2011)

Table 8 Crashworthiness optimization for automotive components

Component	Works
Longitudinal rail	Liang and Le (2009), Yang et al. (1994), Soto (2004), Hanssen et al. (2006), Forsberg and Nilsson (2006), Redhe et al. (2002), Shi and Hagiwara (2000), Redhe et al. (2004), Cho et al. (2006), Zhang et al. (2007a), Zhang et al. (2007c), Cho et al. (2008), Aspenberg et al. (2012), Kim (2001), Wang et al. (2011b), Wang et al. (2011a), Redhe and Nilsson (2006), Mayer et al. (1996), Khakhali et al. (2010), Nguyen et al. (2014)
Pillars	Shi et al. (2007), Hanssen et al. (2006), Marklund and Nilsson (2001), Bae and Huh (2012)
Crash box	Pan et al. (2010), Lee et al. (2013)
Bumper	Costas et al. (2014), Farkas et al. (2012), Shin et al. (2008), Jeong et al. (2010), Gu et al. (2012)

References

Abbasi M, Ghafari-Nazari A, Reddy S, Fard M (2014) A new approach for optimizing automotive crashworthiness: concurrent usage of ANFIS and Taguchi method. Struct Multidisc Optim 49:485–499

Abdullah K, Wild P, Jeswiet J, Ghasempoor A (2001) Tensile testing for weld deformation properties in similar gage tailor welded blanks using the rule of mixtures. J Mater Process Technol 112:91–97

Abramowicz W, Jones N (1984) Dynamic axial crushing of square tubes. Int J Impact Eng 2:179–208

Abramowicz W, Jones N (1986) Dynamic progressive buckling of circular and square tubes. Int J Impact Eng 4:243–270

Abramowicz W, Wierzbicki T (1989) Axial crushing of multicorner sheet metal columns. J Appl Mech 56:113–120

Acar E (2010a) Optimizing the shape parameters of radial basis functions: an application to automobile crashworthiness. Proc Institution Mech Eng Part D-J Automobile Eng 224:1541–1553

Acar E (2010b) Various approaches for constructing an ensemble of metamodels using local measures. Struct Multidisc Optim 42:879–896

Acar E, Rais-Rohani M (2009) Ensemble of metamodels with optimized weight factors. Struct Multidisc Optim 37:279–294

Acar E, Solanki K (2009a) Improving the accuracy of vehicle crashworthiness response predictions using an ensemble of metamodels. Int J Crashworth 14:49–61

Acar E, Solanki K (2009b) System reliability based vehicle design for crashworthiness and effects of various uncertainty reduction measures. Struct Multidisc Optim 39:311–325

Acar E, Guler MA, Gerceker B, Cerit ME, Bayram B (2011) Multi-objective crashworthiness optimization of tapered thin-walled tubes with axisymmetric indentations. Thin-Walled Struct 49:94–105

Ahmetoglu MA, Brouwers D, Shulkin L, Taupin L, Kinzel GL, Altan T (1995) Deep drawing of round cups from tailor-welded blanks. J Mater Process Technol 53:684–694

Alexander J (1960) An approximate analysis of the collapse of thin cylindrical shells under axial loading. Quarter J Mech Appl Math 13:10–15

Allahbakhsh HR, Saetni J (2011) Design optimization of square and circular aluminium extrusion damage columns with crashworthiness criteria. Indian J Eng Mater Sci 18:341–350

Allahbakhsh HR, Saemi J, Hourali M (2011) Design optimization of square aluminium damage columns with crashworthiness criteria. Mechanika:187–192

Antony J (2002) Design for six sigma: a breakthrough business improvement strategy for achieving competitive advantage. Work Stud 51:6–8

Aspenberg D, Jergeus J, Nilsson L (2012) Robust optimization of front members in a full frontal car impact. Eng Optim 45:245–264

Astori P, Impari F (2013) Crash response optimisation of helicopter seat and subfloor. Int J Crashworth 18:570–578

Avalle M, Chiandussi G (2007) Optimisation of a vehicle energy absorbing steel component with experimental validation. Int J Impact Eng 34:843–858

Avalle M, Chiandussi G, Belingardi G (2002) Design optimization by response surface methodology: application to crashworthiness design of vehicle structures. Struct Multidisc Optim 24:325–332

Bae GH, Huh H (2012) Comparison of the optimum designs of center pillar assembly of an auto-body between conventional steel and ahss with a simplified side impact analysis. Int J Automot Technol 13:205–213

Bandi P, Schmiedeler JP, Tovar A (2013) Design of crashworthy structures with controlled energy absorption in the hybrid cellular automaton framework. J Mech Des 135:091002

Baril C, Yacout S, Clement B (2011) Design for six sigma through collaborative multiobjective optimization. Comput Ind Eng 60:43–55

Belingardi G, Beyene AT, Koricho EG (2013) Geometrical optimization of bumper beam profile made of pultruded composite by numerical simulation. Compos Struct 102:217–225

Beyer H-G, Sendhoff B (2007) Robust optimization–a comprehensive survey. Comput Methods Appl Mech Eng 196:3190–3218

Bi J, Fang H, Wang Q, Ren X (2010) Modeling and optimization of foam-filled thin-walled columns for crashworthiness designs. Finite Elem Anal Des 46:698–709

Bisagni C, Lanzi L, Ricci S (2002) Optimization of helicopter subfloor components under crashworthiness requirements using neural networks. J Aircr 39:296–304

Bishop CM (1995) Neural networks for pattern recognition. Oxford university press

Blumhardt R (2001) FEM - crash simulation and optimization. Int J Veh Des 26:331–347

Bojanowski C, Kulak RF (2011) Multi-objective optimisation and sensitivity analysis of a paratransit bus structure for rollover and side impact tests. Int J Crashworth 16:665–676

Borvik T, Hopperstad OS, Reyes A, Langseth M, Solomos G, Dyngeland T (2003) Empty and foam-filled circular aluminium tubes subjected to axial and oblique quasi-static loading. Int J Crashworth 8:481–494

Breyfogle III FW (2003) Implementing six sigma: smarter solutions using statistical methods. John Wiley & Sons

Chen WG (2001) Optimisation for minimum weight of foam-filled tubes under large twisting rotation. Int J Crashworth 6:223–241

Chen W, Allen JK, Tsui K-L, Mistree F (1996) A procedure for robust design: minimizing variations caused by noise factors and control factors. J Mech Des 118:478–485

Chen Z, Qiu H, Gao L, Li X, Li P (2014) A local adaptive sampling method for reliability-based design optimization using Kriging model. Struct Multidisc Optim 49:401–416

Chiandussi G, Avalle M (2002) Maximisation of the crushing performance of a tubular device by shape optimisation. Comput Struct 80:2425–2432

Cho Y-B, Bae C-H, Suh M-W, Sin H-C (2006) A vehicle front frame crash design optimization using hole-type and dent-type crush initiator. Thin-Walled Struct 44:415–428

Cho Y-B, Bae C-H, Suh M-W, Sin H-C (2008) Maximisation of crash energy absorption by crash trigger for vehicle front frame using the homogenisation method. Int J Veh Des 46:23–50

Choi W, Park G (1999) Transformation of dynamic loads into equivalent static loads based on modal analysis. Int J Numer Methods Eng 46: 29–43

Christensen J, Bastien C, Blundell MV (2012) Effects of roof crush loading scenario upon body in white using topology optimisation. Int J Crashworth 17:29–38

Christensen J, Bastien C, Blundell MV, Batt PA (2013) Buckling considerations and cross-sectional geometry development for topology optimised body in white. Int J Crashworth 18:319–330

Chuang C, Yang R (2012) Benchmark of topology optimization methods for crashworthiness design. In: 12th International LS-DYNA Users Conference, Dearborn, Michigan, USA

Chuang C, Yang R, Li G, Mallela K, Pothuraju P (2008) Multidisciplinary design optimization on vehicle tailor rolled blank design. Struct Multidisc Optim 35:551–560

Coello CAC, Pulido GT, Lechuga MS (2004) Handling multiple objectives with particle swarm optimization. Evol Comput, IEEE Trans 8: 256–279

Costas M, Díaz J, Romera L, Hernández S (2014) A multi-objective surrogate-based optimization of the crashworthiness of a hybrid impact absorber. Int J Mech Sci 88:46–54

Couckuyt I, Deschrijver D, Dhaene T (2014) Fast calculation of multiobjective probability of improvement and expected improvement criteria for Pareto optimization. J Glob Optim 60:575–594

Craig KJ, Stander N, Dooge DA, Varadappa S (2005) Automotive crashworthiness design using response surface-based variable screening and optimization. Eng Comput 22:38–61

Cristello N, Kim IY (2007) Multidisciplinary design optimization of a zero-emission vehicle chassis considering crashworthiness and hydroformability. Proc Instit Mech Eng Part D-J Automobile Eng 221:511–526

Das I, Dennis JE (1997) A closer look at drawbacks of minimizing weighted sums of objectives for Pareto set generation in multicriteria optimization problems. Struct Optimization 14:63–69

Davis SC, Diegel SW, Boundy RG (2013) Transportation energy data book. 32 edn., Oak Ridge National Lab

Deb K, Pratap A, Agarwal S, Meyarivan T (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. Evol Comput, IEEE Trans 6:182–197

Draper NR, Smith H (1981) Applied regression analysis 2nd ed

Du Bois P et al. (2004) Vehicle crashworthiness and occupant protection

Duan S, Tao Y, Han X, Yang X, Hou S, Hu Z (2014) Investigation on structure optimization of crashworthiness of fiber reinforced polymers materials. Composit Part B-Eng 60:471–478

Duddeck F (2008) Multidisciplinary optimization of car bodies. Struct Multidisc Optim 35:375–389

Esfahlani SS, Shirvani H, Shirvani A, Nwaubani S, Mebrahtu H, Chirwa C (2013) Hexagonal honeycomb cell optimisation by way of meta-model techniques. Int J Crashworth 18:264–275

Fang H, Rais-Rohani M, Liu Z, Horstemeyer MF (2005) A comparative study of metamodeling methods for multiobjective crashworthiness optimization. Comput Struct 83:2121–2136

Fang J, Gao Y, Sun G, Zhang Y, Li Q (2014a) Crashworthiness design of foam-filled bitubal Structures with uncertainty. Int J Non-Linear Mech

Fang J, Gao Y, Sun G, Zhang Y, Li Q (2014a) Parametric analysis and multiobjective optimization for functionally graded foam-filled thin-wall tube under lateral impact. Comput Mater Sci 90:265–275

Fang J, Gao Y, Sun G, Xu C, Li Q (2015a) Multiobjective robust design optimization of fatigue life for a truck cab. Reliab Eng Syst Saf 135: 1–8

Fang J, Gao Y, Sun G, Zheng G, Li Q (2015b) Dynamic crashing behavior of new extrudable multi-cell tubes with a functionally graded thickness. Int J Mech Sci 103:63–73

Fang J, Gao Y, An X, Sun G, Chen J, Li Q (2016) Design of transversely-graded foam and wall thickness structures for crashworthiness criteria. Compos Part B 92:338–349

Farkas L, Moens D, Donders S, Vandepitte D (2012) Optimisation study of a vehicle bumper subsystem with fuzzy parameters. Mech Syst Signal Process 32:59–68

Forrester A, Keane AJ (2009) Recent advances in surrogate-based optimization. Prog Aerosp Sci 45:50–79

Forrester A, Sobester A, Keane A (2008) Engineering design via surrogate modelling: a practical guide. John Wiley & Sons

Forsberg J, Nilsson L (2006) Evaluation of response surface methodologies used in crashworthiness optimization. Int J Impact Eng 32:759–777

Forsberg J, Nilsson L (2007) Topology optimization in crashworthiness design. Struct Multidisc Optim 33:1–12

Friedman JH (1991) Multivariate adaptive regression splines. Ann Stat: 1–67

Fu Y, Sahin KH (2004) Better Optimization of Nonlinear Uncertain Systems (BONUS) for vehicle structural design. Ann Oper Res 132:69–84

Gedikli H (2013) Crashworthiness optimization of foam-filled tailor-welded tube using coupled finite element and smooth particle hydrodynamics method. Thin-Walled Struct 67:34–48

Ghamarian A, Zarei H (2012) Crashworthiness investigation of conical and cylindrical end-capped tubes under quasi-static crash loading. Int J Crashworth 17:19–28

Goel T, Haftka R, Shyy W, Queipo NV (2007) Ensemble of surrogates. Struct Multidisc Optim 33:199–216

Goel T, Stander N, Lin Y-Y (2010) Efficient resource allocation for genetic algorithm based multi-objective optimization with 1,000 simulations. Struct Multidisc Optim 41:421–432

Gu L, Yang R, Tho CH, Makowski M, Faruque O, Li Y (2001) Optimization and robustness for crashworthiness of side impact. Int J Veh Des 26:348–360

Gu J, Li GY, Dong Z (2012) Hybrid and adaptive meta-model-based global optimization. Eng Optim 44:87–104

Gu X, Sun G, Li G, Mao L, Li Q (2013) A Comparative study on multiobjective reliable and robust optimization for crashworthiness design of vehicle structure. Struct Multidisc Optim 48:669–684

Guo L, Tovar A, Penninger CL, Renaud JE (2011) Strain-based topology optimisation for crashworthiness using hybrid cellular automata. Int J Crashworth 16:239–252

Gupta NK, Velmurugan R (1999) Axial compression of empty and foam filled composite conical shells. J Compos Mater 33:567–591

Haftka R, Villanueva D, Chaudhuri A (2016) Parallel surrogate-assisted global optimization with expensive functions–a survey. Struct Multidisc Optim 54:3–13

Hajela P, Lee E (1997) Topological optimization of rotorcraft subfloor structures for crashworthiness considerations. Comput Struct 64:65–76

Hamza K, Saitou K (2012) A co-evolutionary approach for design optimization via ensembles of surrogates with application to vehicle crashworthiness. J Mech Design 134

Hamza K, Shalaby M (2014) A framework for parallelized efficient global optimization with application to vehicle crashworthiness optimization. Eng Optim 46:1200–1221

Han J, Yamazaki K (2003) Crashworthiness optimization of S-shape square tubes. Int J Veh Des 31:72–85

Hanssen AG, Langseth M, Hopperstad OS (2000) Static and dynamic crushing of circular aluminium extrusions with aluminium foam filler. Int J Impact Eng 24:475–507

Hanssen AG, Langseth M, Hopperstad OS (2001) Optimum design for energy absorption of square aluminium columns with aluminium foam filler. Int J Mech Sci 43:153–176

Hanssen AG, Stobener K, Rausch G, Langseth M, Keller H (2006) Optimisation of energy absorption of an A-pillar by metal foam insert. Int J Crashworth 11:231–241

Hardy RL (1971) Multiquadric equations of topography and other irregular surfaces. J Geophys Res 76:1905–1915

Helton JC, Johnson JD, Sallaberry CJ, Storlie CB (2006) Survey of sampling-based methods for uncertainty and sensitivity analysis. Reliability Eng Syst Saf 91:1175–1209

Hohenbichler M, Rackwitz R (1982) First-order concepts in system reliability. Struct Saf 1:177–188

Hong UP, Park G (2003) Determination of the crash pulse and optimization of the crash components using the response surface approximate optimization. Proc Institution Mech Eng Part D-J Automobile Eng 217:203–213

Horstemeyer MF, Ren XC, Fang H, Acar E, Wang PT (2009) A comparative study of design optimisation methodologies for side-impact crashworthiness, using injury-based versus energy-based criterion. Int J Crashworth 14:125–138

Hou S, Li Q, Long S, Yang X, Li W (2007) Design optimization of regular hexagonal thin-walled columns with crashworthiness criteria. Finite Elem Anal Des 43:555–565

Hou S, Li Q, Long S, Yanga X, Li W (2008a) Multiobjective optimization of multi-cell sections for the crashworthiness design. Int J Impact Eng 35:1355–1367

Hou S, Li Q, Long SY, Yang XJ, Li W (2008b) Multiobjective optimization of multi-cell sections for the crashworthiness design. Int J Impact Eng 35:1355–1367

Hou S, Li Q, Long S, Yang X, Li W (2009) Crashworthiness design for foam filled thin-wall structures. Mater Des 30:2024–2032

Hou S, Han X, Sun G, Long S, Li W, Yang X, Li Q (2011) Multiobjective optimization for tapered circular tubes. Thin-Walled Struct 49:855–863

Hou S, Dong D, Ren L, Han X (2012) Multivariable crashworthiness optimization of vehicle body by unreplicated saturated factorial design. Struct Multidisc Optim 46:891–905

Hou S, Liu T, Dong D, Han X (2014a) Factor screening and multivariable crashworthiness optimization for vehicle side impact by factorial design. Struct Multidisc Optim 49:147–167

Hou S, Tan W, Zheng Y, Han X, Li Q (2014b) Optimization design of corrugated beam guardrail based on RBF-MQ surrogate model and collision safety consideration. Adv Eng Softw 78:28–40

Hou S, Zhang Z, Yang X, Yin H, Li Q (2014c) Crashworthiness optimization of new thin-walled cellular configurations. Eng Comput 31: 4–4

Huang X, Xie YM, Lu G (2007) Topology optimization of energy-absorbing structures. Int J Crashworth 12:663–675

Hunkeler S, Duddeck F, Rayamajhi M, Zimmer H (2013) Shape optimisation for crashworthiness followed by a robustness analysis with respect to shape variables. Struct Multidisc Optim 48:367–378

Ingrassia T, Nigrelli V, Buttitta R (2013) A comparison of simplex and simulated annealing for optimization of a new rear underrun protective device. Eng Comput 29:345–358

Jacob GC, Fellers JF, Simunovic S, Starbuck JM (2002) Energy absorption in polymer composites for automotive crashworthiness. J Compos Mater 36:813–850

Jansson T, Nilsson L, Redhe M (2003) Using surrogate models and response surfaces in structural optimization – with application to crashworthiness design and sheet metal forming. Struct Multidisc Optim 25:129–140

Jeong SB, Yi SI, Kan CD, Nagabhushana V, Park G (2008) Structural optimization of an automobile roof structure using equivalent static loads. Proc Institution Mech Eng Part D-J Automobile Eng 222: 1985–1995

Jeong SB, Yoon S, Xu S, Park G (2010) Non-linear dynamic response structural optimization of an automobile frontal structure using equivalent static loads. Proc Institution Mech Eng Part D-J Automobile Eng 224:489–501

Jiang Z, Gu M (2010) Optimization of a fender structure for the crashworthiness design. Mater Des 31:1085–1095

Jin R, Chen W, Simpson TW (2001) Comparative studies of metamodelling techniques under multiple modelling criteria. Struct Multidisc Optim 23:1–13

Jin R, Du X, Chen W (2003) The use of metamodeling techniques for optimization under uncertainty. Struct Multidisc Optim 25:99–116

Kang B, Choi W, Park G (2001) Structural optimization under equivalent static loads transformed from dynamic loads based on displacement. Comput Struct 79:145–154

Kaushik A, Ramani A (2014) Topology optimization for nonlinear dynamic problems: considerations for automotive crashworthiness. Eng Optim 46:487–502

Kaya N, Oeztuerk F (2010) Multi-objective crashworthiness design optimisation of thin-walled tubes. Int J Veh Des 52:54–63

Keane AJ (2006) Statistical improvement criteria for use in multiobjective design optimization. AIAA J 44:879–891

Khakhali A, Nariman-zadeh N, Darvizeh A, Masoumi A, Notghi B (2010) Reliability-based robust multi-objective crashworthiness optimisation of S-shaped box beams with parametric uncertainties. Int J Crashworth 15:443–456

Kiani M, Gandikota I, Parrish A, Motoyama K, Rais-Rohani M (2013) Surrogate-based optimisation of automotive structures under multiple crash and vibration design criteria. Int J Crashworth 18:473–482

Kiani M, Motoyama K, Rais-Rohani M, Shiozaki H (2014) Joint stiffness analysis and optimization as a mechanism for improving the structural design and performance of a vehicle. Proc Instit Mech Eng Part D-J Automobile Eng 228:689–700

Kim HS (2001) Analysis of crash response of aluminium foam-filled front side rail of a passenger car. Int J Crashworth 6:189–207

Kim HS (2002) New extruded multi-cell aluminum profile for maximum crash energy absorption and weight efficiency. Thin-Walled Struct 40:311–327

Kim HS, Chen W, Wierzbicki T (2002) Weight and crash optimization of foam-filled three-dimensional "S" frame. Comput Mech 28:417–424

Kim S-H, Kim H, Kim NJ (2015) Brittle intermetallic compound makes ultrastrong low-density steel with large ductility. Nature 518:77–79

Kinsey B, Liu Z, Cao J (2000) A novel forming technology for tailor-welded blanks. J Mater Process Technol 99:145–153

Koch PN, Simpson TW, Allen JK, Mistree F (1999) Statistical approximations for multidisciplinary design optimization: the problem of size. J Aircr 36:275–286

Koch PN, Yang R, Gu L (2004) Design for six sigma through robust optimization. Struct Multidisc Optim 26:235–248

Kurtaran H, Eskandarian A, Marzougui D, Bedewi NE (2002) Crashworthiness design optimization using successive response surface approximations. Comput Mech 29:409–421

Lanzi L, Bisagni C, Ricci S (2004a) Crashworthiness optimization of helicopter subfloor based on decomposition and global approximation. Struct Multidisc Optim 27:401–410

Lanzi L, Castelletti LML, Anghileri M (2004b) Multi-objective optimisation of composite absorber shape under crashworthiness requirements. Compos Struct 65:433–441

Lee Y, Ahn J-S, Park G Crash optimization of automobile frontal and side structures using equivalent static loads.In: 11th World Congress on Structural and Multidisciplinary Optimization, Sydney, NSW, Australia, 2015

Lee SH, Kim HY, Oh SI (2002) Cylindrical tube optimization using response surface method based on stochastic process. J Mater Process Technol 130:490–496

Lee I, Choi KK, Noh Y, Zhao L, Gorsich D (2011) Sampling-based stochastic sensitivity analysis using score functions for RBDO problems with correlated random variables. J Mech Des 133:021003

Lee S-J, Lee H-A, Yi S-I, Kim D-S, Yang HW, Park G (2013) Design flow for the crash box in a vehicle to maximize energy absorption. Proc Inst Mech Eng Part D-J Automobile Eng 227:179–200

Liang C-C, Le G-N (2009) Bus rollover crashworthiness under European standard: an optimal analysis of superstructure strength using successive response surface method. Int J Crashworth 14:623–639

Liao X, Li Q, Yang X, Li W, Zhang W (2008a) A two-stage multi-objective optimisation of vehicle crashworthiness under frontal impact. Int J Crashworth 13:279–288

Liao X, Li Q, Yang X, Zhang W, Li W (2008b) Multiobjective optimization for crash safety design of vehicles using stepwise regression model. Struct Multidisc Optim 35:561–569

Liu Y (2008a) Crashworthiness design of multi-corner thin-walled columns. Thin-Walled Struct 46:1329–1337

Liu Y (2008b) Design optimisation of tapered thin-walled square tubes. Int J Crashworth 13:543–550

Liu Y (2008c) Optimum design of straight thin-walled box section beams for crashworthiness analysis. Finite Elem Anal Des 44:139–147

Liu Y (2010a) Crashworthiness design of thin-walled curved beams with box and channel cross sections. Int J Crashworth 15:413–423

Liu Y (2010b) Thin-walled curved hexagonal beams in crashes - FEA and design. Int J Crashworth 15:151–159

Liu S-T, Tong Z-Q, Tang Z-L, Zhang Z-H (2014) Design optimization of the S-frame to improve crashworthiness. Acta Mech Sin 30:589–599

Lönn D, Öman M, Nilsson L, Simonsson K (2009) Finite element based robustness study of a truck cab subjected to impact loading. Int J Crashworth 14:111–124

Lönn D, Fyllingen Ø, Nilssona L (2010) An approach to robust optimization of impact problems using random samples and meta-modelling. Int J Impact Eng 37:723–734

Lönn D, Bergman G, Nilsson L, Simonsson K (2011) Experimental and finite element robustness studies of a bumper system subjected to an offset impact loading. Int J Crashworth 16:155–168

Marklund PO, Nilsson L (2001) Optimization of a car body component subjected to side impact. Struct Multidisc Optim 21:383–392

Marsolek J, Reimerdes HG (2004) Energy absorption of metallic cylindrical shells with induced non-axisymmetric folding patterns. Int J Impact Eng 30:1209–1223

Marzbanrad J, Ebrahimi MR (2011) Multi-objective optimization of aluminum hollow tubes for vehicle crash energy absorption using a genetic algorithm and neural networks. Thin-Walled Struct 49:1605–1615

Mayer RR, Kikuchi N, Scott RA (1996) Application of topological optimization techniques to structural crashworthiness. Int J Numer Methods Eng 39:1383–1403

McKittrick J et al (2010) Energy absorbent natural materials and bioinspired design strategies: A review. Mater Sci Eng C 30:331–342

Messac A, Puemi-Sukam C, Melachrinoudis E (2000a) Aggregate objective functions and Pareto frontiers: required relationships and practical implications. Optim Eng 1:171–188

Messac A, Sundararaj GJ, Tappeta RV, Renaud JE (2000b) Ability of objective functions to generate points on nonconvex Pareto frontiers. AIAA J 38:1084–1091

Milho JF, Ambrósio JAC, Pereira MFOS (2004) Design of train crash experimental tests by optimization procedures. Int J Crashworth 9:483–493

Mohammadiha O, Beheshti H (2014) Optimization of functionally graded foam-filled conical tubes under axial impact loading. J Mech Sci Technol 28:1741–1752

Montgomery DC (1996) Desing and analysis of experiments. Wiley, New York, EUA

Naceur H, Guo YQ, Ben-Elechi S (2006) Response surface methodology for design of sheet forming parameters to control springback effects. Comput Struct 84:1651–1663

Najafi A, Rais-Rohani M (2012) Sequential coupled process-performance simulation and multi-objective optimization of thin-walled tubes. Mater Des 41:89–98

Najafi A, Acar E, Rais-Rohani M (2014) Multi-objective robust design of energy-absorbing components using coupled process–performance simulations. Eng Optim 46:146–164

Nguyen VS, Wen G, Yin H, Van TP (2014) Optimisation design of reinforced S-shaped frame structure under axial dynamic loading. Int J Crashworth 19:385–393

Oman M, Nilsson L (2010) Structural optimization of product families subjected to multiple crash load cases. Struct Multidisc Optim 41:797–815

O'Neill B (2009) Preventing passenger vehicle occupant injuries by vehicle design—a historical perspective from IIHS. Traffic Injury Prevent 10:113–126

Ortmann C, Schumacher A (2013) Graph and heuristic based topology optimization of crash loaded structures. Struct Multidisc Optim 47:839–854

Pan F, Zhu P (2011a) Design optimisation of vehicle roof structures: benefits of using multiple surrogates. Int J Crashworth 16:85–95

Pan F, Zhu P (2011b) Lightweight design of vehicle front-end structure: contributions of multiple surrogates. Int J Veh Des 57:124–147

Pan F, Zhu P, Zhang Y (2010) Metamodel-based lightweight design of B-pillar with TWB structure via support vector regression. Comput Struct 88:36–44

Park G (2011) Technical overview of the equivalent static loads method for non-linear static response structural optimization. Struct Multidisc Optim 43:319–337

Park G, Lee T-H, Lee KH, Hwang K-H (2006) Robust design: an overview. AIAA J 44:181–191

Parkinson A, Sorensen C, Pourhassan N (1993) A general approach for robust optimal design. J Mech Des 115:74–80

Parrish A, Rais-Rohani M, Najafi A (2012) Crashworthiness optimisation of vehicle structures with magnesium alloy parts. Int J Crashworth 17:259–281

Patel NM, Kang B-S, Renaud JE, Tovar A (2009) Crashworthiness design using topology optimization. J Mech Design 131

Paz J, Díaz J, Romera L, Costas M (2014) Crushing analysis and multi-objective crashworthiness optimization of GFRP honeycomb-filled energy absorption devices. Finite Elem Anal Des 91:30–39

Pedersen CBW (2003a) Topology optimization design of crushed 2D-frames for desired energy absorption history. Struct Multidisc Optim 25:368–382

Pedersen CBW (2003b) Topology optimization for crashworthiness of frame structures. Int J Crashworth 8:29–39

Pedersen CBW (2004) Crashworthiness design of transient frame structures using topology optimization. Comput Methods Appl Mech Eng 193:653–678

Qi C, Yang S (2014) Crashworthiness and lightweight optimisation of thin-walled conical tubes subjected to an oblique impact. Int J Crashworth 19:334–351

Qi C, Yang S, Dong F (2012) Crushing analysis and multiobjective crashworthiness optimization of tapered square tubes under oblique impact loading. Thin-Walled Struct 59:103–119

Qu X, Haftka R (2004) Reliability-based design optimization using probabilistic sufficiency factor. Struct Multidisc Optim 27:314–325

Rais-Rohani M, Solanki KN, Acar E, Eamon CD (2010) Shape and sizing optimisation of automotive structures with deterministic and probabilistic design constraints. Int J Veh Des 54:309–338

Rakheja S, Balike M, Hoa SV (1999) Study of an energy dissipative under-ride guard for enhancement of crashworthiness in a car-truck collision. Int J Veh Des 22:29–53

Ramakrishna S (1997) Microstructural design of composite materials for crashworthy structural applications. Mater Des 18:167–173

Ramu P, Kim NH, Haftka R (2007) Error amplification in failure probability estimates of small errors in surrogates. In: SAE 2007 World Congress. Detroit, Michigan, USA

Rangavajhala S, Mahadevan S (2013) Design optimization for robustness in multiple performance functions. Struct Multidisc Optim 47:523–538

Raquel C, Naval P (2005) An effective use of crowding distance in multiobjective particle swarm optimization. In: Proceedings of the 2005 conference on Genetic and evolutionary computation, Washington DC, USA. New York, NY, USA, pp 257-264

Redhe M, Nilsson L (2004) Optimization of the new Saab 9-3 exposed to impact load using a space mapping technique. Struct Multidisc Optim 27:411–420

Redhe M, Nilsson L (2006) A multipoint version of space mapping optimization applied to vehicle crashworthiness design. Struct Multidisc Optim 31:134–146

Redhe M, Forsberg J, Jansson T, Marklund PO, Nilsson L (2002) Using the response surface methodology and the D-optimality criterion in crashworthiness related problems. Struct Multidisc Optim 24:185–194

Redhe M, Giger M, Nilsson L (2004) An investigation of structural optimization in crashworthiness design using a stochastic approach - a comparison of stochastic optimization and the response surface methodology. Struct Multidisc Optim 27:446–459

Reid SR, Reddy TY (1986) Static and dynamic crushing of tapered sheet metal tubes of rectangular cross-section. Int J Mech Sci 28:623–637

Reyes A, Hopperstad O, Hanssen A, Langseth M (2004) Modeling of material failure in foam-based components. Int J Impact Eng 30: 805–834

Roux W (2011) Topology Design using LS-TaSC™ Version 2 and LS-DYNA®

Rzesnitzek T, Müllerschön H, Günther FC, Wozniak M (2002) Two-Stage stochastic and deterministic optimization. In: 1st DYNAMORE Users' Forum, Bad Mergentheim, German

Sacks J, Welch WJ, Mitchell TJ, Wynn HP (1989) Design and analysis of computer experiments. Stat Sci 4:409–423

Santosa SP, Wierzbicki T, Hanssen AG, Langseth M (2000) Experimental and numerical studies of foam-filled sections. Int J Impact Eng 24: 509–534

Schonlau M (1998) Computer experiments and global optimization. University of Waterloo

Seitzberger M, Rammerstorfer FG, Degischer HP, Gradinger R (1997) Crushing of axially compressed steel tubes filled with aluminium foam. Acta Mech 125:93–105

Seitzberger M, Rammerstorfer FG, Gradinger R, Degischer HP, Blaimschein M, Walch C (2000) Experimental studies on the quasi-static axial crushing of steel columns filled with aluminium foam. Int J Solids Struct 37:4125–4147

Shakeri M, Mirzaeifar R, Salehghaffari S (2007) New insights into the collapsing of cylindrical thin-walled tubes under axial impact load. Proc Instit Mech Eng Part C-J Mech Eng Sci 221:869–885

Shariati M, Allahbakhsh HR, Saemi J, Sedighi M (2010) Optimization of foam filled spot-welded column for the crashworthiness design. Mechanika:10-16

Sheldon A, Helwig E, Cho Y-B (2011) Investigation and application of multi-disciplinary optimization for automotive body-in-white development. In: 8th European LS-DYNA Users conference, Strasbourg, 23–24

Shi Q, Hagiwara I (2000) Optimal design method to automobile problems using holographic neural network's approximation. Jpn J Ind Appl Math 17:321–339

Shi Y, Zhu P, Shen L, Lin Z (2007) Lightweight design of automotive front side rails with TWB concept. Thin-Walled Struct 45:8–14

Shi L, Yang R, Zhu P (2012) A method for selecting surrogate models in crashworthiness optimization. Struct Multidisc Optim 46:159–170

Shi L, Yang R, Zhu P (2013a) An adaptive response surface method for crashworthiness optimization. Eng Optim 45:1365–1377

Shi L, Zhu P, Yang R-J, Lin S-P (2013b) Adaptive sampling-based RBDO method for vehicle crashworthiness design using Bayesian metric and stochastic sensitivity analysis with independent random variables. Int J Crashworth 18:331–342

Shimoyama K, Shinkyu J, Obayashi S (2013) Kriging-surrogate-based optimization considering expected hypervolume improvement in non-constrained many-objective test problems. In: Evolutionary Computation (CEC), 2013 I.E. Congress on, 658–665

Shin MK, Park KJ, Park G (2007) Optimization of structures with non-linear behavior using equivalent loads. Comput Methods Appl Mech Eng 196:1154–1167

Shin MK, Yi SI, Kwon OT, Park G (2008) Structural optimization of the automobile frontal structure for pedestrian protection and the low-speed impact test. Proc Instit Mech Eng Part D-J Automobile Eng 222:2373–2387

Simpson TW, Poplinski J, Koch PN, Allen JK (2001) Metamodels for computer-based engineering design: survey and recommendations. Eng Comput 17:129–150

Simpson TW, Booker AJ, Ghosh D, Giunta AA, Koch PN, Yang R (2004) Approximation methods in multidisciplinary analysis and optimization: a panel discussion. Struct Multidisc Optim 27:302–313

Sinha K (2007) Reliability-based multiobjective optimization for automotive crashworthiness and occupant safety. Struct Multidisc Optim 33:255–268

Sinha K, Krishnan R, Raghavendra D (2007) Multi-objective robust optimisation for crashworthiness during side impact. Int J Veh Des 43: 116–135

Smola A, Schölkopf B (2004) A tutorial on support vector regression. Stat Comput 14:199–222

Sobester A, Leary SJ, Keane AJ (2004) A parallel updating scheme for approximating and optimizing high fidelity computer simulations. Struct Multidisc Optim 27:371–383

Song X, Sun G, Li G, Gao W, Li Q (2013) Crashworthiness optimization of foam-filled tapered thin-walled structure using multiple surrogate models. Struct Multidisc Optim 47:221–231

Soto CA (2004) Structural topology optimization for crashworthiness. Int J Crashworth 9:277–283

Stander N (2012) An efficient new sequential strategy for multi-objective optimization using LS-OPT®. Paper presented at the 12th International LS-DYNA Users Conference, Dearborn, Michigan, USA

Stander N An Adaptive Surrogate-Assisted Strategy for Multi-Objective Optimization. In: 10th World Congress on Structural and Multidisciplinary Optimization, Orlando, Florida, USA, 2013.

Su R, Gui L, Fan Z (2011) Multi-objective optimization for bus body with strength and rollover safety constraints based on surrogate models. Struct Multidisc Optim 44:431–441

Sun G, Li G, Hou S, Zhou S, Li W, Li Q (2010a) Crashworthiness design for functionally graded foam-filled thin-walled structures. Mater Sci Eng A-Struct Mater Properties Microstruct Process 527:1911–1919

Sun G, Li G, Stone M, Li Q (2010b) A two-stage multi-fidelity optimization procedure for honeycomb-type cellular materials. Comput Mater Sci 49:500–511

Sun G, Li G, Zhou S, Li H, Hou S, Li Q (2011) Crashworthiness design of vehicle by using multiobjective robust optimization. Struct Multidisc Optim 44:99–110

Sun G, Song X, Baek S, Li Q (2014a) Robust optimization of foam-filled thin-walled structure based on sequential Kriging metamodel. Struct Multidisc Optim 49:897–913

Sun G, Xu F, Li G, Li Q (2014b) Crashing analysis and multiobjective optimization for thin-walled structures with functionally graded thickness. Int J Impact Eng 64:62–74

Tang L, Wang H, Li G, Xu F (2013) Adaptive heuristic search algorithm for discrete variables based multi-objective optimization. Struct Multidisc Optim 48:821–836

Tarlochan F, Samer F, Hamouda AMS, Ramesh S, Khalid K (2013) Design of thin wall structures for energy absorption applications: Enhancement of crashworthiness due to axial and oblique impact forces. Thin-Walled Struct 71:7–17

Toksoy AK, Güden M (2011) The optimisation of the energy absorption of partially Al foam-filled commercial 1050H14 and 6061T4 Al crash boxes. Int J Crashworth 16:97–109

Tovar A, Patel NM, Kaushik AK, Renaud JE (2007) Optimality conditions of the hybrid cellular automata for structural optimization. AIAA J 45:673–683

Tran T, Hou S, Han X, Tan W, Nguyen N (2014) Theoretical prediction and crashworthiness optimization of multi-cell triangular tubes. Thin-Walled Struct 82:183–195

Tu J, Choi KK, Park YH (1999) A new study on reliability-based design optimization. J Mech Des 121:557–564

Viana FA, Haftka R, Steffen V Jr (2009a) Multiple surrogates: how cross-validation errors can help us to obtain the best predictor. Struct Multidisc Optim 39:439–457

Viana FAC, Haftka R, Steffen V (2009b) Multiple surrogates: how cross-validation errors can help us to obtain the best predictor. Struct Multidisc Optim 39:439–457

Viana FC, Haftka R, Watson L (2013) Efficient global optimization algorithm assisted by multiple surrogate techniques. J Glob Optim 56: 669–689

Viana FA, Simpson TW, Balabanov V, Toropov V (2014) Metamodeling in multidisciplinary design optimization: how far have we really come? AIAA J 52:670–690

Villa A, Strano M, Mussi V (2011) Optimization of Design and Manufacturing Process of Metal Foam Filled Anti-Intrusion Bars. 14th International Conference on Material Forming Esaform, 2011 Proceedings 1353:1656-1661

Vining GG, Myers RH (1990) Combining Taguchi and response surface philosophies: a dual response approach. J Qual Technol 22

Wang GG (2003) Adaptive response surface method using inherited latin hypercube design points. J Mech Des 125:210–220

Wang GG, Shan S (2007) Review of metamodeling techniques in support of engineering design optimization. J Mech Des 129:370–380

Wang X, Shi L (2014) A new metamodel method using Gaussian process based bias function for vehicle crashworthiness design. Int J Crashworth 19:311–321

Wang H, Li GY, Li E (2010) Time-based metamodeling technique for vehicle crashworthiness optimization. Comput Methods Appl Mech Eng 199:2497–2509

Wang H, Li E, Li GY (2011a) Probability-based least square support vector regression metamodeling technique for crashworthiness optimization problems. Comput Mech 47:251–263

Wang H, Shan S, Wang GG, Li G (2011b) Integrating least square support vector regression and mode pursuing sampling optimization for crashworthiness design. J Mech Des 133

Wierzbicki T, Abramowicz W (1983) On the crushing mechanics of thin-walled structures. J Appl Mech-T Asme 50:727–734

Witowski K, Erhart A, Schumacher P, Müllerschön H (2012) Topology optimization for crash

Wu JP, Nusholtz GS, Bilkhu S (2002) Optimization of vehicle crash pulses in relative displacement domain. Int J Crashworth 7:397–413

Xiang YJ, Wang Q, Fan ZJ, Fang HB (2006) Optimal crashworthiness design of a spot-welded thin-walled hat section. Finite Elem Anal Des 42:846–855

Xu F, Sun G, Li G, Li Q (2013) Crashworthiness design of multi-component tailor-welded blank (TWB) structures. Struct Multidisc Optim 48:653–667

Xu H, Majcher MT, Chuang C, Fu Y, Yang R (2014) Comparative benchmark studies of response surface model-based optimization and direct multidisciplinary design optimization. SAE Technical Paper

Xu H, Chuang C, Yang R (2015) A data mining-based strategy for direct multidisciplinary optimization. SAE Int J Mater Manufact 8:357–363

Yamazaki K, Han J (1998) Maximization of the crushing energy absorption of tubes. Struct Optim 16:37–46

Yang S, Qi C (2013) Multiobjective optimization for empty and foam-filled square columns under oblique impact loading. Int J Impact Eng 54:177–191

Yang R, Tseng L, Nagy L, Cheng J (1994) Feasibility study of crash optimization. ASME 2:549–556

Yang R, Akkerman A, Anderson DF, Faruque O, Gu L (2000) Robustness optimization for vehicular crash simulations. Comput Sci Eng 2:8–13

Yang R, Wang N, Tho C, Bobineau J, Wang B (2005) Metamodeling development for vehicle frontal impact simulation. J Mech Des 127:1014–1020

Yao W, Chen X, Luo W, van Tooren M, Guo J (2011) Review of uncertainty-based multidisciplinary design optimization methods for aerospace vehicles. Prog Aerosp Sci 47:450–479

Yi SI, Lee JY, Park G (2012) Crashworthiness design optimization using equivalent static loads. Proc Instit Mech Eng Part D-J Automobile Eng 226:23–38

Yildiz AR, Solanki KN (2012) Multi-objective optimization of vehicle crashworthiness using a new particle swarm based approach. Int J Adv Manuf Technol 59:367–376

Yin H, Wen G, Gan N (2011a) Crashworthiness design for honeycomb structures under axial dynamic loading. Int J Comput Methods 8: 863–877

Yin H, Wen G, Hou S, Chen K (2011b) Crushing analysis and multiobjective crashworthiness optimization of honeycomb-filled single and bitubular polygonal tubes. Mater Des 32:4449–4460

Yin H, Wen G, Hou S, Qing Q (2013) Multiobjective crashworthiness optimization of functionally lateral graded foam-filled tubes. Mater Des 44:414–428

Yin H, Wen G, Fang H, Qing Q, Kong X, Xiao J, Liu Z (2014a) Multiobjective crashworthiness optimization design of functionally graded foam-filled tapered tube based on dynamic ensemble metamodel. Mater Des 55:747–757

Yin H, Wen G, Liu Z, Qing Q (2014b) Crashworthiness optimization design for foam-filled multi-cell thin-walled structures. Thin-Walled Struct 75:8–17

Youn BD, Choi KK (2004a) A new response surface methodology for reliability-based design optimization. Comput Struct 82:241–256

Youn BD, Choi KK (2004b) Selecting probabilistic approaches for reliability-based design optimization. AIAA J 42:124–131

Youn BD, Choi KK, Yang R, Gu L (2004) Reliability-based design optimization for crashworthiness of vehicle side impact. Struct Multidisc Optim 26:272–283

Zabaras N, Ganapathysubramanian S, Li Q (2003) A continuum sensitivity method for the design of multi-stage metal forming processes. Int J Mech Sci 45:325–358

Zarei HR, Kroger M (2006) Multiobjective crashworthiness optimization of circular aluminum tubes. Thin-Walled Struct 44:301–308

Zarei HR, Kroger M (2007) Crashworthiness optimization of empty and filled aluminum crash boxes. Int J Crashworth 12:255–264

Zarei H, Kroger M (2008a) Optimum honeycomb filled crash absorber design. Mater Des 29:193–204

Zarei HR, Kroger M (2008b) Bending behavior of empty and foam-filled beams: structural optimization. Int J Impact Eng 35:521–529

Zarei HR, Kroger M (2008c) Optimization of the foam-filled aluminum tubes for crush box application. Thin-Walled Struct 46:214–221

Zarei H, Kröger M, Albertsen H (2008) An experimental and numerical crashworthiness investigation of thermoplastic composite crash boxes. Compos Struct 85:245–257

Zerpa LE, Queipo NV, Pintos S, Salager J-L (2005) An optimization methodology of alkaline–surfactant–polymer flooding processes using field scale numerical simulation and multiple surrogates. J Pet Sci Eng 47:197–208

Zhang Y, Zhu P, Chen G (2007a) Lightweight design of automotive front side rail based on robust optimisation. Thin-Walled Struct 45:670–676

Zhang Y, Zhu P, Chen G, Lin Z (2007b) Study on structural lightweight design of automotive front side rail based on response surface method. J Mech Des 129:553–557

Zhang Y, Zhu P, Chen G, Lin ZQ (2007c) Study on structural lightweight design of automotive front side rail based on response surface method. J Mech Des 129:553–557

Zhang X, Cheng G, Wang B, Zhang H (2008) Optimum design for energy absorption of bitubal hexagonal columns with honeycomb core. Int J Crashworth 13:99–107

Zhang Z, Liu S, Tang Z (2009) Design optimization of cross-sectional configuration of rib-reinforced thin-walled beam. Thin-Walled Struct 47:868–878

Zhang B, Yang J, Zhong Z (2010) Optimisation of vehicle side interior panels for occupant safety in side impact. Int J Crashworth 15:617–623

Zhang Y, Sun G, Li G, Luo Z, Li Q (2012) Optimization of foam-filled bitubal structures for crashworthiness criteria. Mater Des 38:99–109

Zhang S, Zhu P, Chen W (2013a) Crashworthiness-based lightweight design problem via new robust design method considering two sources of uncertainties. Proc Instit Mech Eng Part C-J Mech Eng Sci 227:1381–1391

Zhang S, Zhu P, Chen W, Arendt P (2013b) Concurrent treatment of parametric uncertainty and metamodeling uncertainty in robust design. Struct Multidisc Optim 47:63–76

Zhang X, Wen Z, Zhang H (2014a) Axial crushing and optimal design of square tubes with graded thickness. Thin-Walled Struct 84:263–274

Zhang Y, Sun G, Xu X, Li G, Li Q (2014b) Multiobjective crashworthiness optimization of hollow and conical tubes for multiple load cases. Thin-Walled Struct 82:331–342

Zheng G, Wu S, Sun G, Li G, Li Q (2014) Crushing analysis of foam-filled single and bitubal polygonal thin-walled tubes. Int J Mech Sci 87:226–240

Zhou XJ, Ma YZ, Li XF (2011) Ensemble of surrogates with recursive arithmetic average. Struct Multidisc Optim 44:651–671

Zhou Q, Wu X, Xia Y, Cai W (2014) Spot weld layout optimization of tube crash performance with manufacturing constraints. J Manufact Sci Eng-Trans Asme 136

Zhu P, Shi YL, Zhang KZ, Lin ZQ (2008) Optimum design of an automotive inner door panel with a tailor-welded blank structure. Proc Instit Mech Eng Part D-J Automobile Eng 222:1337–1348

Zhu P, Zhang Y, Chen G (2009) Metamodel-based lightweight design of an automotive front-body structure using robust optimization. Proc Instit Mech Eng Part D-J Automobile Eng 223:1133–1147

Zhu P, Zhang Y, Chen G (2011) Metamodeling development for reliability-based design optimization of automotive body structure. Comput Ind 62:729–741

Zhu P, Pan F, Chen W, Zhang S (2012) Use of support vector regression in structural optimization: application to vehicle crashworthiness design. Math Comput Simul 86:21–31

Zhu P, Pan F, Chen W, Viana FAC (2013) Lightweight design of vehicle parameters under crashworthiness using conservative surrogates. Comput Ind 64:280–289

Zhu F, Jiang B, Chou CC (2016) On the development of a new design methodology for vehicle crashworthiness based on data mining theory. SAE Technical Paper

Zurada JM (1992) Introduction to artificial neural systems vol 8. West publishing company, St. Paul