FEEDBACK CONTROL OF MEMS MICROMIRRORS SUBJECT TO PARAMETRIC UNCERTAINTY

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ABSTRACT

This paper presents research on the development of microelectromechanical systems (MEMS) micromirror arrays with precise and accurate positioning enabled by the use of closed-loop control techniques. The MEMS mirror arrays are one degree-of-freedom, electrostatically actuated and exhibit nonlinear actuation profiles that include pull-in and hysteresis. The device performance is subject to parametric uncertainties from the fabrication process. Preliminary proportional-integrator (PI) controllers are studied in simulation on the nonlinear system to explore issues in the control development. Two different model linearization methods are presented to examine the best way to approximate the nonlinear behaviors using linear models. The effects of parametric uncertainties on the open-loop plant response are considered. It is evident based on these studies that the open-loop response is very sensitive to model uncertainties, while using closed-loop control can achieve input-tracking despite uncertainties in the plant. The work-in-progress includes the development of an optical test bed for the experimental validation of the results presented in this paper.

1 INTRODUCTION

MEMS micromirror arrays are used in a variety of applications, including optical, scanning, diffraction gratings, and adaptive optics. It is desirable to ensure accurate positioning capabilities for each mirror in the array despite the presence of outside disturbances or variations from the fabrication process such as small deviations in dimensional or material properties across the array. Despite substantial advances in MEMS fabrication techniques, many micromachining processes offer dimensional tolerances that are considerably large by modern machining standards. This parametric uncertainty makes it difficult to develop truly reliable and high-precision MEMS devices. This is why many state-of-the-art micromirror arrays rely on discrete open-loop actuation that may limit the device to on/off binary operation [1, 2] or require extensive calibration to simulate analog performance. Many of today’s emerging technologies require true analog positioning capabilities. In order to guarantee precision and accuracy of the mirror position for analog operation, closed-loop feedback control techniques are required. These controllers must also be able to operate in the presence of parametric uncertainties.

This paper presents a study of feedback controller applied to a nonlinear system of MEMS electrostatic micromirror arrays that are subject to parametric variations from the fabrication process. Preliminary Proportional-Integral (PI) controllers are applied to investigate and characterize the critical issues of achieving successful feedback control. First, a literature review is presented discussing previous feedback control efforts applied to MEMS, followed by a description of the micromirror system that is the focus of this research. In order to use linear control design techniques to design a control system, the nonlinear plant model must be linearized. Here, two different linearization methods are discussed. The first uses a linear approximation in the modeled dynamics, and the second uses a Taylor Series Expansion (TSE) about an operating point. This paper presents work on a very simple control methodology to show the effects of parametric uncertainties on closed-loop performance.

2 LITERATURE REVIEW

Feedback control methods can be used to achieve traditional control objectives such as to improve transient response, reduce steady-state errors, reject disturbances and improve system bandwidth and stability. Within the limited amount of work that has been done to design and implement feedback control systems on MEMS devices, a wide array of techniques and methods have been employed, including linear-time-invariant (LTI) techniques such as PID, robust, adaptive, and nonlinear control design.
2.1 Classical Control Methods:

The use of classical, linear controller design such as PID, lead-lag, and state-variable is adequate for systems in which the nonlinearities are small enough to be neglected, or when the system is operated in a small range of motion avoiding nonlinear behavior [22-24]. Pannu et al. used simple PID control to improve the closed-loop response of a magnetically actuated micromirror by improving settling time, achieving steady-state tracking and increasing disturbance rejection [25]. Horsley et al. used linear PD and phase-lead closed-loop controllers for position control of an electrostatic lateral comb drive that was operated only within its stable range [24]. Messenger et al. performed PID and lead-control for position tracking on micromachined thermal actuators that exhibit approximately linear actuation [26, 27].

In the case of systems with large nonlinearities, such as those from electrostatics, it can be a challenge to apply linear control design and ensure that a controller designed for the linear system will be able to operate on the actual nonlinear plant. Despite the considerable nonlinearities associated with electrostatic actuation, linearization of the plant model is often done to allow for the use of linear-time-invariant (LTI) control methods. The nonlinear effects of electrostatic actuation are perhaps most evident for parallel-plate actuator systems. Lu and Fedder used a linearized plant model for a parallel-plate type actuator and designed a LTI controller for both extended range of travel and position control [20]. The LTI controller did not account for the higher order nonlinear effects of the actuator, initial conditions or external disturbances. By including disturbance inputs into the analysis, it was found that the LTI controller was unable to satisfy both the stability conditions and disturbance rejection for large deflections of the actuator, meaning that it could not attain the large deflections predicted for the given controller design [20]. This illustrates the importance of considering robust operation of the controller, especially when using a linearized plant model for a highly nonlinear system.

2.2 Robust and Adaptive Control:

In utilizing closed-loop feedback control techniques for MEMS devices, robustness becomes a common desired quality [7, 8, 19-21, 23, 28, 29]. Robustness is important in MEMS control systems as there can be many uncertainties introduced through variations in the device parameters, including geometry and material properties that arise from the fabrication process, as well as nonlinearities in the dynamics and disturbances from noise or other external influences. There are many ways to compensate for these uncertainties and develop robustly stable control methods.

Adaptive control techniques have been shown to be effective for disturbance rejection and improved performance that is robustly stable to plant uncertainties. When applying adaptive control it is very important to have an accurate system model. The actual system output is compared to the estimated output predicted by the model and this error is used to determine the controller gains during each step. If the predictive plant model does not reflect the actual system behavior well, then large errors can lead to poor performance and sometimes cause the system to go unstable [7, 8]. Calculating the controller gains at each step in real-time can be difficult to implement, requires computationally intensive algorithms and cannot be done compactly in an analog circuit.

Adaptive methods have been employed to account for parametric uncertainties within the plant that arise from variations from the fabrication process. For actuators with performance that is highly sensitive to fabrication variations, adaptive techniques may also be used for parameter estimation. In the case of [21], the actuator dynamics was sensitive to fabrication errors arising from the alignment tolerances of bulk-micromachining. Adaptive estimation of the uncertain dimensional parameter was used to adjust the controller gains allowing the controller to achieve very large, stable deflection past the pull-in point and good position tracking.

An advantage of adaptive control over other methods, like PID, is that the controller can compensate for uncertainties from fabrication, reject disturbance, and achieve desired tracking objectives by continuously updating the controller parameters according to the actual system performance. A direct comparison of PID and adaptive control for an electrostatic torsion micromirror was conducted by [23] and it was shown that closed-loop adaptive control achieved better transient response than a carefully tuned PID controller and was able to handle amplitude limited, time-varying disturbance signal.

References [7, 8] demonstrate the use of adaptive control techniques for rejecting disturbances that occur in adaptive optics applications when there is turbulence in the atmosphere that affects the optical wave front. Using an adaptive filter alone was effective at disturbance rejection of band limited noise less than 50 KHz [8]. Kim et al. showed that using a combination of LTI h-infinity control and adaptive control resulted in good disturbance rejection of 20 KHz bandwidth noise and the h-infinity controller improved performance by eliminating steady-state drift and reducing ‘jitter’ [7].

There are few examples of robust control design methods such as h-inf and mu-synthesis that have been applied to MEMS systems. In addition to the use of h-infinity control demonstrated by Kim, et al. [7], mu-synthesis controller design was applied to a dual-stage actuator system for tracking in a hard-disk drive [29]. In this case, the dual-stage system consisted of a piezoelectric actuator as its coarse positioner, and the fine position controlled by an electrostatic microactuator. The two actuators were treated as a single system for the controller designs to achieve desired performance and robustness for the combined system. Plant uncertainties were considered as 10% uncertainty in stiffness of the microactuator, and undisclosed uncertainties for the piezoelectric actuator. Disturbance signals to both actuators and sensor noise are also included in the model. The controller design was successful in simulations, but no experimental work has been done so far. The application of mu-synthesis to design robust controllers has not been specifically applied to a strictly MEMS device. Difficulties in implementing these types of controllers arise if the order of the controller is very high.

2.3 Nonlinear Control:

Nonlinear techniques that incorporate Lyapunov stability analysis have been employed to prove stability for control schemes applied to electrostatic MEMS actuators [10,18, 21,
An overview of Lyapunov stability analysis and how it applies to nonlinear controller design or MEMS is given by [30]. It is clear that this method is mathematically intensive and that proving global asymptotic stability of the Lyapunov function is not a trivial matter [11]. In addition, like many other advance control techniques discussed here, the resulting control laws may not be amenable to implementation in analog circuitry. There has been limited research in applying these nonlinear strategies to MEMS devices and little experimental application.

2.4 Other Methods:

Additional control techniques that have been used include sliding mode control (SMC), which can also be robust to plant variations, have good disturbance rejection and compact implementation schemes. SMC is a digital nonlinear control method generally good for systems with nonlinearities and parametric uncertainties and tends to produce low order controllers. Lee et al. used a discrete-time SMC for a dual-stage actuator for hard-disk drives [31]. For this application, sliding-mode control is used to track a desired trajectory so as to avoid unwanted excitation of any resonant modes. The controller performed well in simulations. SMC was also applied to the problem of electrostatic pull-in instability of two-axis torsion micromirrors [28]. SMC operates by lots of switching pulses that can result in chattering of the actuated device about the steady state value. Chiou et al. [32] examine the use of fuzzy control for a micromirror that is actuated using an array of electrodes that allow for a large number of positions using programmed digital operation. The fuzzy controller showed improvement in the transient response over the open-loop system in simulation, but issues concerning feedback signal and controller implementation are not addressed.

2.5 Remarks:

It is clear from examining these various control methods that as the techniques become more complex to account for robust performance and system nonlinearities, the implementation issues also become more complicated. While many of the papers in the literature discuss robustness of the control system, very few go into great depth of defining the system uncertainties and determining the acceptable margins for the uncertainty. Therefore it is not always clear if meaningful robustness is achieved for the system.

This paper presents work on a very simple control methodology to show the effects of parametric uncertainties on closed-loop performance.

3 SYSTEM MODEL

The device presented in this paper is a one-degree-of-freedom micro mirror actuated via a vertical comb drive and is shown schematically in Fig. 1. The mirrors are fabricated in polysilicon by surface micromachining and are actuated with electrostatic vertical comb drives located beneath the mirror surface. This allows for large arrays with high fill factors. Vertical comb drives are more easily controlled than parallel plate actuators, making them a good choice for analog scanning devices [4,5,8,13,14,16].

![Figure 1. Schematic of 1DOF micro mirror device with hidden vertical comb fingers (not to scale).](image)

The physical nature of electrostatic actuation introduces highly nonlinear behavior into the actuator function and results in a well-documented instability known as pull-in. Many researchers have sought ways to model and to alleviate this instability including the use of mechanical design alterations [6,15,16] and by using feedback control [17]. The implementation of control techniques can be used to control position of a single mirror. For mirrors in a large array, it can ensure uniform operation of mirrors throughout the array and alleviate the effects of uncertainties from fabrication errors in surface micromachining [3] and reject outside disturbances [7].

Like many electrostatically actuated devices, the micromirror presented here exhibits nonlinear behaviors such as pull-in instability. These nonlinearities are not considered here and will be addressed as future work. Only actuation within the stable range of motion is considered in this work.

For the electrostatic comb drive mirror presented here, there is a ground plane and a series of vertically offset comb fingers, all contained underneath a flat mirror surface. A voltage potential is applied across the fixed fingers and the moving fingers of the device creating an electrostatic force. This force causes the mirror to rotate about an axis supported by the hidden spring suspension, shown separately in Figure 1(a). The equation of motion is

\[ J \ddot{\theta} + b \dot{\theta} + k_m \theta = T_e(\theta, V) \]  

where \( J \) is the mass moment of inertia of the plate, \( b \) is the damping coefficient due to squeeze-film effect, \( k \) is the linear spring constant, and \( T_e(\theta, V) \) is the electrostatic torque that is a function of both position and voltage.

The determination of squeeze-film damping coefficient is dependent upon the geometry of the surfaces between which the fluid is trapped. Because of the vertical comb fingers under the surface of the mirror, determining this coefficient analytically is difficult. For the purpose of this discussion, consider the squeeze-film damping term for a torsional plate developed by Pan, et al [33].
\[ b = K_{\text{rot}} \frac{\mu L w^5}{g^3} \]  

where

\[ K_{\text{rot}} = \frac{48}{\pi^6 \left( \frac{L}{w} \right)^2 + 4} \]

L is the length of the plate, w the width, g is the gap between the plates, and \( \mu \) is the viscosity of the fluid. Theoretically, these parameters should be constants. However, the topology created by the comb drives creates varying gap distances that are also a function of the angle, \( \theta \). A more thorough treatment of damping characteristics is needed and will be considered in future work. Table 1 lists the values of parameters for this system.

### Table 1. List of parameters used for analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ), density of polysilicon</td>
<td>2331 kg/m³</td>
</tr>
<tr>
<td>( k_m ), mechanical spring constant</td>
<td>175.23 pN·µm/°rad</td>
</tr>
<tr>
<td>( \mu ), viscosity of air</td>
<td>1.73e-5 N·s/m²</td>
</tr>
<tr>
<td>L, length of mirror</td>
<td>10 µm</td>
</tr>
<tr>
<td>w, width of mirror</td>
<td>100 µm</td>
</tr>
<tr>
<td>g, gap between plates</td>
<td>2 µm</td>
</tr>
<tr>
<td>N, number of unit cells</td>
<td>27</td>
</tr>
</tbody>
</table>

The electrostatic torque \( T_e \) is given as

\[ T_e(\theta, V) = \frac{1}{2} \frac{\partial C}{\partial \theta} V^2 \]  

where \( V \) is the applied voltage, and \( C \) is the capacitance which can be determined from simulation to be a function of the rotation angle \([35,7]\). Because of the symmetry of the comb fingers, a unit cell is defined as shown in Figure 1. The results of the capacitance for one unit cell are plotted in Figure 2 along with a comparison of fourth, third, second and first order polynomial approximations of the data. The capacitance as a function of \( \theta \) approximated with an \( n \)th order polynomial curve fit is

\[ C(\theta) = N(P_1 \theta^n + P_2 \theta^{n-1} + \cdots + P_n \theta + P_{n+1}) \]  

where the coefficients of the polynomial are \( P_i \) \((i = 1, 2, \ldots, n, n+1)\), and \( N \) is the total number of unit cells. Table 2 compares the quality of the different order polynomial approximations compared to the FEA data points. One metric to evaluate the fit quality for a curve fit is the norm of the residuals, \( \text{normr} \). The smaller the value of \( \text{normr} \), the better the approximation. Another standard metric is the sum of the square of the residuals, \( r^2 \), which is calculated from \( \text{normr} \) by

\[ r^2 = 1 - \frac{\text{normr}^2}{(n-1)s^2} \]  

where \( n \) is the number of data points (FEA data), and \( s \) is the standard deviation of the curve fit approximation from the data. A value of \( r^2 \) equal to one indicates a perfect fit. It is clear that a higher order polynomial does a better job of capturing the nature of the capacitance data. However, the first order linear curve fit is still sufficient. The advantage of using the first order fit is that its derivative which is used in equation (4) is a constant, thus simplifying the plant model to a linear approximation.

The effects of using a higher order curve fit versus the first order are more apparent by looking at the static equilibrium relationship between the applied voltage, \( V \), and the rotation angle, \( \theta \). This is shown in Figure 3 for the fourth order fit, called the “nonlinear” model, and the first order fit, the “linear capacitance approximation” model. It is clear that by using the lower order model approximation there is a difference between the predicted static performances. The linear model is suitable to the design of the controller, but the resulting control must still be able to perform well on the nonlinear system.

For this particular system, it is clear that the linear capacitance approximation model presented above will be an adequate representation of the nonlinear micromirror system. This linearized model can be used in the control design stages and implemented on the nonlinear system as shown in Figs. 4 and 5.
For a system in which the capacitance cannot be adequately modeled as linear, such as the case of parallel plate electrostatics, a higher order approximation is required. In this case, it is possible to linearize the second order dynamic model in equation (1) about an operating point \((\theta_0, V_0)\) using the Taylor series expansion (TSE) [36]. This can be considered as the small signal model approximation about \(\delta\theta\) and \(\delta V\). Doing so yields the following linear system model,

\[
J \delta \ddot{\theta} + b \delta \dot{\theta} + k_m \delta \theta = k_e \delta \theta + C \delta V
\]  

The linearization in (7) includes a term that is dependent only on the rotation angle that can be considered the electrostatic spring force, \(k_e\) [20].

\[
k_e = \frac{1}{2} \left( \frac{d^2 C}{d \theta^2} \right) \theta_0^2
\]  

The nonlinear torque approximation is reduced to a constant.

\[
C_T = V_0 \frac{dC}{d\theta}\bigg|_{\theta_0}
\]  

When linearizing a function about an operating point, it is desirable that the linear model will provide an adequate estimate of the nonlinear function within a small range about that operating point. For systems that are operating over a large range or that have very nonlinear characteristics, this linearization may not provide a satisfactory estimate of the nonlinear function. To illustrate the effect of the small signal linearization, Fig. 6 shows the static equilibrium relationship between rotation angle and actuation voltage for the nonlinear system model and for the small signal model linearized about the operating point (7 degrees, 54 volts). The inset shows the small signal response for \(\delta\theta, \delta V\).

It is clear in Fig. 6 that this linear estimate of the nonlinear system does not capture all of the static performance characteristics over the entire range of operation, but is adequate enough for a portion of the range from 5 to 14 degrees. In order to cover the full range of actuation, a piecewise linearization can be done at different operating points. In this case, different controllers can be used for each operating point and implemented using gain scheduling [38].

The linearized models discussed above are important when considering control design techniques that require a linear transfer function or state-space model for the design process. While it is possible to choose PID control gains by trial and error, certain design methods, such as root locus, require a linear model. Other control methods, such as LQR or H-infinity, have design procedures that also rely on linear models to produce the controller.
4 CONTROLLER DESIGN
4.1 Open-loop System:

It is convenient to rewrite the model dynamics in equation (1) in terms of natural frequency, $\omega_n$, and the damping ratio, $\zeta$.

$$\omega_n = \sqrt{\frac{k_m}{J}}$$

(10)

$$\zeta = \frac{b}{2\sqrt{k_m J}}$$

(11)

Written in state-space from, the system is described as follows,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \frac{dc}{dt} \end{bmatrix}.$$  

(12)

For the dynamic model presented here, the natural frequency is approximately 188 kHz and the damping ratio is approximately 0.3. As stated previously in section 3, the squeeze-film damping coefficient is difficult to predict analytically for this model, and could vary widely from this estimated value. The damping ratio has a significant effect on the open-loop performance of the system, as seen in Fig. 7 for damping ratios ranging from 0.1 to 1.

![Figure 7. Open-loop plant response to a step input of 7 degrees for different damping ratios.](image)

To illustrate the effects of parametric uncertainty on the system response, the open-loop response of the plant model is considered using different values of stiffness, $k_m$. Figure 8 shows the response to a step input command of 7 degrees (0.12 radians) for the nominal stiffness value, and for variations of ±10%.

![Figure 8. Open-loop response to a step input of 7 degrees for the nonlinear plant dynamics and variations in spring stiffness, $k_m$.](image)

To further illustrate this concept, all of the parameters in the system described in equation (1) are subject to parametric variation, including the mass moment of inertia, $J$, the damping, $b$, the spring stiffness, $k_m$, and the electrostatic torque, $T_e$. If each of these parameters is allowed to vary by ±10% from the nominal value, there are a very large number of possible plants to consider. Figure 9 shows the open-loop plant responses to a step input of 7 degrees of the system model for 50 randomly generated sets of parameters $m$, $b$, $k_m$, and $T_e$ that are allowed to vary by ±10% of their nominal values.

![Figure 9. Open-loop plant response to a step input of 7 degrees for 50 random parameter variations.](image)

It is clear that with the presence of uncertainties, a step input to the open-loop plant will result in steady-state error in the response. The implementation of closed-loop feedback control is used to ensure performance even in the presence of such uncertainty.
4.2 PI Control:

A PI controller is implemented on the system in an effort to ensure zero steady-state error despite the presence of model uncertainty. Using only a simple proportional controller (P-controller) on the system is not sufficient to ensure zero steady-state error for different plant variations, therefore an integral term is included. The controller gains are chosen as the proportional gain, \( K_P = 100 \), and the integral gain, \( K_I = 300 \). Figure 10 shows step responses for different values ranging from 2 degrees to 16 degrees including plant variations of \( \pm 10\% \) variation in the spring stiffness, \( k_m \). The closed-loop system has no overshoot, which is important in electrostatic systems that experience pull-in. It is clear that the transient response of the system is affected by the magnitude of the step input. As the command input is changed, the settling time is affected. For a system application with strict transient performance requirements, this set of gains may not be sufficient at very low command angles.

Previously, the open-loop plant responses of the system for 50 randomly generated sets of parameters \( m, b, k_m, \) and \( T_e \) that are allowed to vary by \( \pm 10\% \) of their nominal values was presented in Fig. 9. The closed loop response of those same 50 plants is shown in Fig. 11. It is clear that the controller drives all of the plants to zero steady-state error, achieving the goal of static input tracking.

5 CONCLUSIONS AND FUTURE WORK

The work presented here is part of an ongoing research project to use closed-loop control techniques to ensure precise performance of MEMS micromirrors despite parametric uncertainty in the plant. This paper demonstrates the use of PI control and discusses the issues of using two different linearized models. The PI controller showed that it was able to correct for plant uncertainty and that the system response is sensitive to variations in input command. This paper does not address the ability of the PI control system to reject other noise and disturbances.

Future work will be the experimental implementation of these controllers on the actual micromirror devices using an optical bench testbed. Laser beam steering and an optical position sensor are used for position feedback.

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6 REFERENCES


